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Bin Packing and Related Problems: Pattern-Based Approaches



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Abstract

We present novel exact and approximate methods for solving bin packing and related problems, which show a large advantage in practice when compared with traditional methods. For cutting stock problems, we present pattern-based heuristics, which do not depend on the number of pieces; this allows the algorithms to find solutions for extremely large cutting stock instances, with billions of items, in a very short amount of time. For bin packing, cutting stock, cardinality constrained bin packing and two-constraint bin packing, we present a very simple and powerful method that allows solving exactly many thousands of benchmark instances in a few seconds, on average.

The conventional assignment-based first/best fit decreasing algorithms (FFD/BFD) are not polynomial in the cutting stock input size in its most common format. Therefore, even for small instances with large demands, it is difficult to compute FFD/BFD solutions. Our pattern-based methods overcome the main problems of conventional heuristics in cutting stock problems by representing the solution in a much more compact format. An off-line variant of the sum-of-squares heuristics is also studied and compared with the FFD/BFD heuristics.

The proposed exact method, based on an arc flow formulation with side constraints, solves bin packing problems — including multi-constraint variants — by simply representing all the patterns in a very compact graph. Conventional formulations for bin packing problems are usually highly symmetric and provide very weak lower bounds. The proposed arc flow formulation provides a very strong lower bound, and it is able to break symmetry completely. However, it appears that with this model symmetry is not the main problem. We present a graph compression algorithm that usually reduces the underlying graph substantially without weakening the bounds. We report computational results obtained with many benchmark test sets, all of them showing a large advantage of this formulation with respect to traditional ones.

Resumo

Apresentamos novos métodos exatos e aproximados para a resolução de problemas de *bin packing* e variantes, os quais mostram grande superioridade na prática quando comparados com métodos tradicionais. Para problemas de *cutting stock*, apresentamos heurísticas baseadas em padrões que não dependem do número de peças; permitindo assim aos algoritmos encontrar soluções para instâncias de *cutting stock* extremamente grandes, com bilhões de itens, num período muito curto de tempo. Para problemas de *bin packing*, *cutting stock*, *bin packing* com cardinalidade e *bin packing* com duas restrições, apresentamos um método muito simples e poderoso que permite resolver de forma exata milhares de instâncias de *benchmark* em poucos segundos, em média.

Os algoritmos *first/best fit decreasing* (FFD/BFD) convencionais baseados em atribuições não são polinomiais no tamanho do input de *cutting stock* no seu formato mais comum. Portanto, mesmo para instâncias pequenas com grandes procuras, é difícil calcular soluções FFD/BFD. Os nossos métodos baseados em padrões superam os maiores problemas das heurísticas convencionais em *cutting stock* através de uma representação mais compacta da solução. Uma variante *off-line* da heurística *sum-of-squares* é também estudada e comparada com as heurísticas FFD/BFD.

O método exato proposto, baseado numa formulação de fluxos em arcos com *side constraints*, resolve problemas *bin packing* — incluindo variantes de restrições múltiplas — simplesmente representando todos os padrões num grafo muito compacto. Formulações convencionais para *bin packing* são geralmente altamente simétricas e fornecem limites muito fracos. A formulação de fluxos em arcos proposta fornece *bounds* muito fortes e permite quebrar a simetria completamente. No entanto, neste modelo a simetria não parece ser o problema principal. Apresentamos um algoritmo de compressão de grafos que normalmente reduz o grafo subjacente substancialmente sem enfraquecer os *bounds*. Apresentamos resultados computacionais obtidos com muitos grupos de instâncias de *benchmark*, todos eles apresentando uma grande superioridade em relação a métodos tradicionais.

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Chapter 1

Introduction

There are problems whose solution can be computed in polynomial time. These problems are considered efficiently solvable, or tractable. On the other hand, there are provably intractable problems, e.g., undecidable or non-deterministic intractable (see, e.g., Garey et al. 1979). However, most of the apparently intractable problems encountered in practice are decidable and can be solved in polynomial time using a non-deterministic computer model that has the ability to do an unbounded number of independent computational sequences in parallel. There is neither a proof that verifies the apparent intractability of these problems nor an efficient algorithm to solve them. Problems that can be solved in polynomial time using a non-deterministic computer model are in the complexity class NP (non-deterministic polynomial time). The efficiently solvable problems belong to the class P (deterministic polynomial time), which is contained in NP. The NP class contains many important problems, the hardest of which are called NP-complete problems. A problem is NP-complete if it is in NP and any other NP problem can be reduced to it in polynomial time. It is not known whether every problem in NP can be quickly solved — this is called the $P = NP$ conjecture. However, if any single NP-complete problem can be solved in polynomial time, every problem in NP can also be solved in polynomial time. An optimization problem consists of finding the best solution from all the solutions satisfying all the problem's constraints, while the corresponding decision problem consists of finding a feasible solution better than a given value. When a decision version of a combinatorial optimization problem is NP-complete, the optimization version is NP-hard (non-deterministic polynomial-time hard). NP-hard problems are at least as hard as any problem in NP.

The bin packing problem (BPP) is a combinatorial NP-hard problem (see, e.g., Garey

et al. 1979) in which objects of different volumes must be packed into a finite number of bins, each with capacity W , in a way that minimizes the number of bins used. In fact, BPP is strongly NP-hard (see, e.g., Garey and Johnson 1978) since it remains so even when all of its numerical parameters are bounded by a polynomial in the length of the input. Therefore, BPP cannot even be solved in pseudo-polynomial time unless $P = NP$. Besides being strongly NP-hard, BPP is also hard to approximate within $3/2 - \varepsilon$. If such approximation exists, one could partition n non-negative numbers into two sets with the same sum in polynomial time. This problem — called the number partition problem — could be reduced to a bin packing problem with bins of capacity equal to half of the sum of all the numbers. Any approximation better than $3/2 - \varepsilon$ of the optimal value could be used to find a perfect partition, corresponding to a packing in $\lfloor 2(3/2 - \varepsilon) \rfloor = 2$ bins. However, the number partition problem is known to be NP-hard. Concerning the heuristic solution of BPP, Simchi-Levi (1994) showed that the first fit decreasing and the best fit decreasing heuristics have an absolute performance ratio of $3/2$, which is the best possible absolute performance ratio for the bin packing problem unless $P=NP$.

There are many variants of this problem and they have many applications, such as filling up containers, placing computer files with specified sizes into memory blocks of fixed size, loading trucks with volume or weight capacity limits, among others.

BPP can be seen as a special case of the cutting stock problem (CSP). In this problem we have a number of rolls of paper of fixed width waiting to be cut for satisfying demand of different customers, who want pieces of various widths. We have to cut the rolls in such a way that waste is minimized. Note that, in the paper industry, solving this problem to optimality can be economically significant; a small improvement in reducing waste can have a huge impact in yearly savings.

There are many similarities between BPP and CSP. However, in CSP, the items of equal size (which are usually ordered in large quantities) are grouped into orders with a required level of demand, while in BPP the demand for a given size is usually close to one. According to Wäscher et al. (2007), cutting stock problems are characterized by a weakly heterogeneous assortment of small items, in contrast with bin packing problems, which are characterized by a strongly heterogeneous assortment of small items.

One of the bin packing variants is the cardinality constrained BPP in which, in addition to the capacity constraint, the number of items per bin is also limited. This problem can be seen as a special case of two-constraint BPP (also called 2D-vector

bin packing by some authors) in which each item has a weight and a length. In the two-constraint BPP, there is a difficult problem in each dimension, whereas for the cardinality constrained BPP in one of the dimensions the problem is very easy: we just need to count the number of items.

Note that cutting stock and two-constraint bin packing are clearly strongly NP-hard by the hardness of the standard bin packing. Cardinality constrained bin packing is strongly NP-hard for any cardinality larger than 2 (see, e.g., Epstein and van Stee 2011); for cardinality 2, the cardinality constrained bin packing problem can be solved in polynomial time as a maximum non-bipartite matching problem in a graph where each item is represented by a node and every compatible pair of items is connect by an edge.

In the next chapters, we present new algorithms to compute heuristic solutions for large cutting stock problems and a powerful arc flow formulation that allowed us to solve every benchmark instance we found in the literature for BPP and CSP, with and without cardinality constraints, and many two-constraint bin packing open instances, in a few seconds, on average.

In the remainder of this chapter, we give account of previous approaches to bin packing problems with heuristic and exact methods. Chapter 2 presents the advances made in heuristic methods and Chapter 3 presents a new exact method. Finally, Chapter 4 presents the conclusions.

1.1 Previous work on heuristic methods

In the standard BPP, we assume that the entire list of items is known before the packing begin. However, there are some applications where this is not the case, e.g., when the items are physical objects and there are no intermediate space to store them. Algorithms that can construct packings under this regime are called on-line heuristics; in this context, methods for the previous case are called off-line.

Since BPP is hard to solve exactly, many different heuristic solution methods have been proposed. Coffman et al. (1997b) provide an excellent survey on on-line and off-line heuristics for BPP. In this section, we will focus on three heuristics: first fit decreasing (FFD), best fit decreasing (BFD) and sum-of-squares (SS). FFD and BFD are perhaps the most famous algorithms for off-line bin packing. SS is an on-line heuristics, proposed by Csirik et al. (1999), which we are going to test in an off-line

environment.

In the on-line heuristics category, some of the best known heuristics are: next, first, last, best, worst, and almost worst fit algorithms. These heuristics are well studied by Johnson (1973) and Johnson (1974). For clarity, let us assume that bins are positioned in a line. Next fit (NF) is probably the simplest on-line algorithm for bin packing. It can be implemented to run in linear time. This algorithm places every new object in the rightmost open bin, starting a new bin if necessary. Despite its simplicity, this heuristics will use at most twice as many bins as the optimal solution would require (see, e.g., Coffman et al. 1997b). The historically most important on-line heuristics is first fit (FF), where we place every object in the leftmost bin that still has room. This algorithm can be implemented to run in $\mathcal{O}(n \log n)$ using an appropriate data structure (see, e.g., Johnson 1973). The last fit (LF) algorithm is similar but it chooses the rightmost open bin that still has room instead of the leftmost. Best fit (BF), worst fit (WF), and almost worst fit (AWF) can also be implemented to run in $\mathcal{O}(n \log n)$ using self-balancing search trees (see Section 2.4.1). BF chooses the fullest bin that still has room, while WF chooses the emptiest existing bin. Finally, AWF algorithm places every object in the second-emptiest bin.

Johnson (1973) proves that for any implementation of the previously described on-line heuristics, there exists a list of length n that will take at least $\Omega(n \log n)$ time to compute a solution, except for the NF heuristics.

We are mainly interested in off-line heuristics where the sizes of the items to be packed are all known in advance. When the on-line restriction is removed, the previously described heuristics can also be used and dramatically better results are usually achieved by packing the largest objects first (see, e.g., Coffman et al. 1997b). In the off-line environment, the best known heuristics are first fit decreasing (FFD) and best fit decreasing (BFD). Coffman et al. (1997a) and Coffman et al. (1997b) analyze the average and worst case behavior of these two heuristics.

There are variants of many of these heuristics for the two-constraint BPP and cardinality constrained BPP (see, e.g., Caprara and Toth 2001). Garey et al. (1976) present an adaptation of the FFD heuristics for two-constraint BPP called 2FFD where items are considered in non-increasing order of $\max(w_i, v_i)$ where w_i and v_i are the normalized weights in each dimension.

1.1.1 First and Best Fit Decreasing heuristics

In the FFD algorithm, items are sorted in non-increasing order of weight and the FF heuristics is applied to assign each item to a bin. This can be implemented to run in $\mathcal{O}(n \log n)$ time using an appropriate data structure to identify in $\mathcal{O}(\log n)$ time the leftmost bin that still has room (see, e.g., Johnson 1973).

Algorithm 1 shows how a FFD solution can be computed. In this algorithm, B is the current number of bins, Sol is the list of assignments, where an assignment of an item of weight w_i to a bin k is represented as $w_i \rightarrow \text{bin } k$, and $Rem[j]$ is the current available space in the j -th bin. For each item, we look for the first bin with enough available space to which the item will be assigned. The complexity of this algorithm is limited by sorting the items and identifying, for every item, the leftmost bin with available space. It can be implemented to run in $\mathcal{O}(n \log n)$ time using appropriated algorithms and data structures (see Section 2.2.1).

Algorithm 1: First Fit Decreasing Algorithm

input : n - number of items; w - list of weights; W - bin capacity

output: B - number of bins needed; Sol - list of assignments

```

1 function FFD( $n, w, W$ ):
2    $w \leftarrow \text{reverse}(\text{sort}(w))$ ;                                // sort items in non-increasing order of weight
3    $B \leftarrow 0$ ;
4    $Sol \leftarrow []$ ;
5    $Rem \leftarrow []$ ;
6   for  $i \leftarrow 1$  to  $n$  do                                     // for each item
7      $S \leftarrow \{j \mid 1 \leq j \leq B, Rem[j] \geq w_i\}$ ;        // set of bins with enough available space
8     if  $S \neq \emptyset$  then
9        $j \leftarrow \min(S)$ ;                                       // first bin with enough available space
10       $Rem[j] \leftarrow Rem[j] - w_i$ ;
11       $Sol.append(w_i \rightarrow \text{bin } j)$ ;
12    else                                                         // there is no bin with enough available space
13       $B \leftarrow B + 1$ ;
14       $Rem.append(W - w_i)$ ;
15       $Sol.append(w_i \rightarrow \text{bin } B)$ ;
16 return ( $B, Sol$ );

```

In the BFD algorithm, items are sorted in non-increasing order of weight and the BF heuristics is applied to assign each item to a bin. This algorithm can be implemented to run in $\mathcal{O}(n \log(\min(n, W)))$ time using a self-balancing search tree to identify the fullest bin with enough available space (see Section 2.4.1).

1.1.2 Sum-of-Squares heuristics

The SS heuristics was proposed by Csirik et al. (1999). According to them, this heuristics performs very well on symmetric discrete distributions, i.e., distributions in which, for all sizes w , items of size w and $W - w$ occur with equal probability. They justify this with the fact that typically when the next item to be packed fits a partially filled bin, there already exists a bin with gap equal to the item size. Hence, most of the bins end up being packed perfectly and this leads to less total waste.

This algorithm is an on-line heuristics for BPP. In an on-line environment, we do not know in advance all the items to be packed. Therefore, on-line heuristics must assign each item at time to a bin without knowing anything about the subsequent items. The SS algorithm aims at minimizing the total waste by maximizing the number of perfectly packed bins. In order to accomplish this, it tries to maintain the number of bins for each possible gap as balanced as possible. Given a set of variables whose sum is fixed, the values of those variables is closer when the sum-of-squares is minimized. This is the idea behind the use of the sum-of-squares as a criterion for an on-line heuristics.

The SS heuristics works as follows. Let $N(g)$ be the current number of bins with gap g . Initially, $N(g) = 0$, for $1 \leq g < W$. To pack an item of size w_i , we choose a bin, with gap at least w_i , that yields the minimum updated value of $\sum_{1 \leq g < W} N(g)^2$. Completely filled bins are not considered in the sum-of-squares. If there is a tie, we choose the candidate with the smallest gap.

Despite its good performance, the SS algorithm does not always make “good” local decisions. Suppose that we are packing an item of size w_i and that there are two bins with gap w_i ($N(w_i) = 2$), three with gap $g' > 2w_i$ ($N(g') = 3$) and none with gap $g' - w_i$ ($N(g' - w_i) = 0$). In this case, it is possible to pack the item and perfectly fill a bin. If we choose to close a bin, the sum-of-squares will decrease 3 units, since $(N(w_i) - 1)^2 = N(w_i)^2 - 3$. However, the algorithm chooses to assign the item to a bin with gap g' , as the sum-of-squares will decrease 4 units; indeed, $(N(g') - 1)^2 = N(g')^2 - 5$ and $(N(g' - w_i) + 1)^2 = N(g' - w_i)^2 + 1$. This choice may allow a bin with gap $g'' = g' - w_i$ to be perfectly packed later and other items of size w_i may fill the two bins with gap w_i . This kind of decisions may lead to better or worse solutions depending on the subsequent items.

Algorithm 2 computes a SS solution in $\mathcal{O}(n \min(n, W))$ time. In this algorithm, \mathbf{B} is the current number of bins, \mathbf{Sol} is the list of assignments, $\mathbf{N}[g]$ is current the number

of bins with gap g and **Buckets** $[g]$ is the list of these bins. We assign each item of size w_i to a bin with the smallest gap that yields the minimum sum-of-squares.

Csirik et al. (1999) show that we do not really need to compute the sum-of-squares, though they do not present any algorithm faster than $\mathcal{O}(n \min(n, W))$. This is a drawback of this algorithm, since it is the slowest one when compared with FF and BF heuristics.

Algorithm 2: Sum-of-Squares Algorithm

input : n - number of items; w - list of weights; W - bin capacity

output: B - number of bins needed; **Sol** - list of assignments

```

1 function SS( $n, w, W$ ):
2    $B \leftarrow 0$ ;
3    $\text{Sol} \leftarrow []$ ;
4    $\text{Buckets}[g] \leftarrow []$ , for  $0 \leq g < W$ ;
5    $N[g] \leftarrow 0$ , for  $1 \leq g < W$ ;  $N[W] \leftarrow \infty$ ;
6   for  $i \leftarrow 1$  to  $n$  do                                     // for each item
7      $g \leftarrow \infty$ ;
8      $\text{min}_{ss} \leftarrow \infty$ ;
9     foreach  $g' \in \{g'' \mid w_i \leq g'' \leq W, N[g''] > 0\}$  do           // for each gap  $\geq w_i$ 
10       $N' \leftarrow N$ ;
11       $N'[g'] \leftarrow N'[g'] - 1$ ;
12       $N'[g' - w_i] \leftarrow N'[g' - w_i] + 1$ ;
13       $ss \leftarrow \sum_{1 \leq g'' < W} N'[g'']^2$ ;
14      if  $ss \leq \text{min}_{ss}$  and  $g' < g$  then           // choose the smallest gap that yields the minimum ss
15         $g \leftarrow g'$ ;
16         $\text{min}_{ss} \leftarrow ss$ ;
17     if  $g = W$  then
18        $B \leftarrow B + 1$ ;
19        $j \leftarrow B$ ;
20     else
21        $j \leftarrow \text{Buckets}[g].\text{pop}()$ ;
22        $N[g] \leftarrow N[g] - 1$ ;
23        $N[g - w_i] \leftarrow N[g - w_i] + 1$ ;
24        $\text{Buckets}[g - w_i].\text{append}(j)$ ;
25        $\text{Sol.append}(w_i \rightarrow \text{bin } j)$ ;
26 return ( $B, \text{Sol}$ );

```

1.2 Previous work on exact methods

In this section, we will give account of previous approaches with exact methods to bin packing and related problems. We will introduce Martello and Toth's formulation for BPP, Kantorovich's formulation for CSP, Caprara and Toth's formulation for two-constraint BPP and its adaptation to cardinality constrained BPP.

We will also describe the classical formulation of Gilmore and Gomory (1961) for CSP, which is equivalent to the model introduced by Valério de Carvalho (1999). Gilmore and Gomory's model provides a very strong linear relaxation, but it has a potentially exponential size; even though Valério de Carvalho's model is also potentially exponential, it is usually much smaller. While Gilmore and Gomory's model is exponential in the number of decision variables with respect with the input size, Valério de Carvalhos's model is pseudo-polynomial in terms of decision variables and constraints. In both models, we consider every valid packing pattern. However, in Valério de Carvalho's model, patterns are derived from paths in a graph, whereby the model is usually much smaller.

Valério de Carvalho (2002) provides an excellent survey on integer programming models for bin packing and cutting stock problems. Here we will just look at the most common and straightforward approaches.

1.2.1 Martello and Toth's formulation

Martello and Toth (1990) developed a branch-and-bound algorithm for BPP based on the following mathematical programming formulation:

$$\text{minimize} \quad \sum_{k=1}^K y_k \quad (1.1)$$

$$\text{subject to} \quad \sum_{k=1}^K x_{ik} = 1, \quad i = 1, \dots, n, \quad (1.2)$$

$$\sum_{i=1}^n w_i x_{ik} \leq W y_k, \quad k = 1, \dots, K, \quad (1.3)$$

$$y_k \in \{0, 1\}, \quad k = 1, \dots, K, \quad (1.4)$$

$$x_{ik} \in \{0, 1\}, \quad i = 1, \dots, n, \quad k = 1, \dots, K, \quad (1.5)$$

where K is a known upper bound to the number of bins needed (it may be obtained

using, for example, the FFD heuristics), n is the number of items, w_i is the weight of item i , W is the bin capacity, and the variables are:

$$y_k = \begin{cases} 1 & \text{if bin } k \text{ is used,} \\ 0 & \text{otherwise;} \end{cases}$$

$$x_{ik} = \begin{cases} 1 & \text{if item } i \text{ is assigned to bin } k, \\ 0 & \text{otherwise.} \end{cases}$$

Martello and Toth (1990) proved that the lower bound for the linear relaxation of this model, which is equal to the minimum amount of space that is necessary to accommodate all the items if they could be divided, can be very weak for instances with large waste.

Property 1 (linear relaxation) *The lower bound provided by the linear relaxation of the model (1.1)-(1.5) is equal to $\lceil \sum_{i=1}^n w_i / W \rceil$.*

Property 2 (worst case) *In the worst case, as W increases, when all the items have a size $w_i = \lfloor W/2 + 1 \rfloor$, the lower bound approaches $1/2$ of the optimal solution.*

Proof If $w_i = \lfloor W/2 + 1 \rfloor$ then $\sum_{i=1}^n w_i = n \lfloor W/2 + 1 \rfloor \leq nW/2 + n$. Therefore $\lceil \sum_{i=1}^n w_i / W \rceil \leq \lceil (nW/2 + n) / W \rceil = \lceil n/2 + n/W \rceil$. As W increases, this lower bound approaches $\lceil n/2 \rceil$ while the optimal solution is n . \square

This is a drawback of this model, as good quality lower bounds are vital in branch-and-bound procedures. Another drawback is due to the symmetry of the problem, which makes this model very inefficient in practice.

1.2.2 Kantorovich's formulation

BPP and CSP are equivalent, in the sense that from the solution of one we can derive the solution of the other; however, BPP takes a list of items as input, while CSP takes a list of different item sizes and the corresponding demands. The size of the input for BPP can be exponentially larger than the input for CSP. Therefore, a polynomial-size formulation for BPP is not necessarily polynomial-size for CSP.

Kantorovich (1960) introduced the following mathematical programming formulation for CSP, where the objective is to minimize the number of rolls used to cut all the items demanded:

$$\text{minimize} \quad \sum_{k=1}^K y_k \quad (1.6)$$

$$\text{subject to} \quad \sum_{k=1}^K x_{ik} \geq b_i, \quad i = 1, \dots, m, \quad (1.7)$$

$$\sum_{i=1}^m w_i x_{ik} \leq W y_k, \quad k = 1, \dots, K, \quad (1.8)$$

$$y_k \in \{0, 1\}, \quad k = 1, \dots, K, \quad (1.9)$$

$$x_{ik} \geq 0, \text{ integer}, \quad i = 1, \dots, m, \quad k = 1, \dots, K, \quad (1.10)$$

where K is a known upper bound to the number of rolls needed, m is the number of different item sizes, w_i and b_i are the weight and demand of item i , and W is the roll length. The variables are y_k , which is 1 if roll k is used and 0 otherwise, and x_{ik} , the number of times item i is cut in the roll k .

Property 3 (Kantorovich's formulation) *The lower bound provided by the linear relaxation of the model (1.6)-(1.10) is equal to $\lceil \sum_{i=1}^m w_i b_i / W \rceil$.*

Proof Defining $\sigma = \sum_{i=1}^m w_i b_i / W$, an optimal solution (x^*, y^*) of the linear relaxation of (1.6)-(1.10) can be computed in $\mathcal{O}(n)$ time as $y_i^* = \sigma / K$ for $i = 1, \dots, K$ and $x_{ik}^* = b_i / K$. Therefore, $\sum_{k=1}^K y_k^* = \sum_{k=1}^K \sigma / K = \sigma$. It is easy to check that this solution is feasible. By summing up inequalities (1.8) and considering $\sum_{k=1}^K x_{ik} = b_i$, we have $\sum_{k=1}^K \sum_{i=1}^m w_i x_{ik} \leq \sum_{k=1}^K W y_k \Leftrightarrow \sum_{i=1}^m \sum_{k=1}^K w_i x_{ik} \leq \sum_{k=1}^K W y_k \Leftrightarrow \sum_{i=1}^m w_i \sum_{k=1}^K x_{ik} \leq \sum_{k=1}^K W y_k \Leftrightarrow \sum_{i=1}^m w_i b_i \leq \sum_{k=1}^K W y_k \Leftrightarrow \sum_{i=1}^m w_i b_i / W \leq \sum_{k=1}^K y_k \Leftrightarrow \sigma \leq \sum_{k=1}^K y_k$. Therefore, the solution is optimal since $\sum_{k=1}^K y_k^* = \sigma$. By rounding up the lower bound obtained, we have $\lceil \sigma \rceil = \lceil \sum_{i=1}^m w_i b_i / W \rceil$. \square

In the worst case, the lower bound provided by this model approaches 1/2 of the optimal solution, since this lower bound is equivalent to the one provided by Martello and Toth's formulation.

Dantzig-Wolfe decomposition is an algorithm for solving linear programming problems with a special structure (see, e.g., Dantzig and Wolfe 1960). It is a powerful tool that can be used to obtain models for integer and combinatorial optimization problems

with stronger linear relaxations. Vance (1998) applied a Dantzig-Wolfe decomposition to model (1.6)-(1.10), keeping constraints (1.6), (1.7) in the master problem, and the subproblem being defined by the integer solutions to the knapsack constraints (1.8). Vance showed that when all the rolls have the same width, the reformulated model is equivalent to the classical Gilmore-Gomory model.

1.2.3 Gilmore-Gomory's formulation

Gilmore and Gomory (1961) proposed the following model for CSP. A combination of orders in the width of the roll is called a cutting pattern. Let column vectors $a^j = (a_1^j, \dots, a_m^j)^\top$ represent all possible cutting patterns j . The element a_d^j represents the number of rolls of width w_d obtained in cutting pattern j . Let x_j be a decision variable that designates the number of rolls to be cut according to cutting pattern j . CSP can be modeled in terms of these variables as follows:

$$\text{minimize} \quad \sum_{j \in J} x_j \quad (1.11)$$

$$\text{subject to} \quad \sum_{j \in J} a_i^j x_j \geq b_i, \quad i = 1, \dots, m, \quad (1.12)$$

$$x_j \geq 0, \text{ integer}, \quad \forall j \in J, \quad (1.13)$$

where J is the set of valid cutting patterns that satisfy:

$$\sum_{i=1}^m a_i^j w_i \leq W \text{ and } a_i^j \geq 0, \text{ integer}. \quad (1.14)$$

Since constraints (1.14) just accept integer linear combinations of items, the search space of the continuous relaxation is reduced and the lower bound provided is stronger when compared with Kantorovich's formulation.

1.2.3.1 Gilmore-Gomory's formulation with column generation

It may be impractical to enumerate all the columns in the previous formulation, as their number may be very large, even for moderately sized problems. To tackle this problem, Gilmore and Gomory (1963) proposed column generation.

Let Model (1.11)-(1.13) be the restricted master problem. At each iteration of the column generation process, a subproblem is solved and a column (pattern) is intro-

duced in the restricted master problem if its reduced cost is strictly less than zero. The subproblem, which is a knapsack problem, is the following:

$$\text{minimize} \quad 1 - \sum_{i=1}^m c_i a_i \quad (1.15)$$

$$\text{subject to} \quad \sum_{i=1}^m w_i a_i \leq W \quad (1.16)$$

$$a_i \geq 0, \text{ integer}, \quad i = 1, \dots, m, \quad (1.17)$$

where c_i is the shadow price of the demand constraint of item i obtained from the solution of the linear relaxation of the restricted master problem, and $a = (a_1, \dots, a_m)$ is a cutting pattern whose reduced cost is given by the objective function.

The column generation process for this method can be summarized as follows. We start with a small set of patterns (columns) obtained with, for example, a FFD heuristics. Then we solve the linear relaxation of the restricted master problem with the current set of columns. At each iteration, a subproblem is solved and the pattern a^* from the solution of the knapsack problem is introduced in the restricted master problem. Simplex iterations are then performed to update the solution of the master problem. This process is repeated until no pattern with negative reduced cost is found. At the end of this process, we have the optimal solution of the linear relaxation of the model (1.11)-(1.13).

1.2.4 Caprara and Toth's formulation

Two-constraint BPP can be formulated similarly to the standard version. Caprara and Toth (2001) introduced the following mathematical programming formulation for

this problem:

$$\text{minimize} \quad \sum_{k=1}^K y_k \quad (1.18)$$

$$\text{subject to} \quad \sum_{k=1}^K x_{ik} = 1, \quad i = 1, \dots, n, \quad (1.19)$$

$$\sum_{i=1}^n w_i x_{ik} \leq A y_k, \quad k = 1, \dots, K, \quad (1.20)$$

$$\sum_{i=1}^n v_i x_{ik} \leq B y_k, \quad k = 1, \dots, K, \quad (1.21)$$

$$y_k \in \{0, 1\}, \quad k = 1, \dots, K, \quad (1.22)$$

$$x_{ik} \in \{0, 1\}, \quad i = 1, \dots, n, \quad k = 1, \dots, K, \quad (1.23)$$

where K is a known upper bound to the number of bins needed, n is the number of items, w_i is the weight of item i in the first dimension, v_i is the weight of item i in the second dimension, and A and B are bin capacities in the first and second dimensions, respectively. Each variable y_k is 1 if bin k is used and 0 otherwise, and x_{ik} is 1 if item i is assigned to bin k , 0 otherwise.

Property 4 (Caprara and Toth's formulation) *The lower bound provided by the linear relaxation of the model (1.18)-(1.23) is equal to $\max(\lceil \sum_{i=1}^n w_i/A \rceil, \lceil \sum_{i=1}^n v_i/B \rceil)$.*

Proof Defining $\sigma = \max(\sum_{i=1}^n w_i/A, \sum_{i=1}^n v_i/B)$, Caprara (1998) proved that an optimal solution (x^*, y^*) of the linear relaxation of (1.18)-(1.23) can be computed in $\mathcal{O}(n)$ time as $y_i^* = \sigma/n$ for $i = 1, \dots, n$ and $x_{ik}^* = 1/n$. Therefore, $\sum_{k=1}^n y_k^* = \sum_{k=1}^n \sigma/n = \sigma$. It is easy to check that this result is also valid if we limit the number of bins by a known upper bound K . In this case, an optimal solution can be $y_i^* = \sigma/K$ for $i = 1, \dots, K$ and $x_{ik}^* = 1/K$. By rounding up the lower bound obtained, we have $\max(\lceil \sum_{i=1}^n w_i/A \rceil, \lceil \sum_{i=1}^n v_i/B \rceil)$. \square

Property 5 (Caprara and Toth's formulation) *The worst case performance ratio of the lower bound provided by Caprara and Toth's formulation is $1/3$.*

Proof Caprara (1998) proved, for the p -dimensional vector packing, that the worst case performance of the lower bound provided by the linear relaxation of the model:

$$\text{minimize} \quad \sum_{k=1}^n y_k \quad (1.24)$$

$$\text{subject to} \quad \sum_{k=1}^n x_{ik} = 1, \quad i = 1, \dots, n, \quad (1.25)$$

$$\sum_{i=1}^n a_{li} x_{ik} \leq y_k, \quad k = 1, \dots, n, \quad l = 1, \dots, p, \quad (1.26)$$

$$y_k \in \{0, 1\}, \quad k = 1, \dots, n, \quad (1.27)$$

$$x_{ik} \in \{0, 1\}, \quad i = 1, \dots, n, \quad k = 1, \dots, n \quad (1.28)$$

is equal to $1/(p+1)$. For $p = 2$, this model is equivalent to Caprara and Toth's formulation for two-constraint bin packing. Therefore, in this problem, the worst case performance ratio is equal to $1/3$. \square

In this model, the lower bound weakness is a huge drawback. The worst case performance may be asymptotically achieved with, for example, $A = B = W$, $n = 3W$, $2W$ items of size $(\lfloor W/2 + 1 \rfloor, \lfloor W/3 + 1 \rfloor)$ and W items of size $(0, \lfloor 2W/3 + 1 \rfloor)$. As W increases, the lower bound approaches $\lceil n/3 \rceil$, while the optimal solution is n .

Cardinality constrained BPP can be seen as a special case of two-constraint BPP; we can formulate this problem based on Caprara and Toth's formulation in the following way:

$$\text{minimize} \quad \sum_{k=1}^K y_k \quad (1.29)$$

$$\text{subject to} \quad \sum_{k=1}^K x_{ik} = 1, \quad i = 1, \dots, n, \quad (1.30)$$

$$\sum_{i=1}^n x_{ik} w_i \leq W y_k, \quad k = 1, \dots, K, \quad (1.31)$$

$$\sum_{i=1}^n x_{ik} \leq C y_k, \quad k = 1, \dots, K, \quad (1.32)$$

$$y_k \in \{0, 1\}, \quad k = 1, \dots, K, \quad (1.33)$$

$$x_{ik} \in \{0, 1\}, \quad i = 1, \dots, n, \quad k = 1, \dots, K, \quad (1.34)$$

where K is a known upper bound to the number of bins needed, n is the number of items, w_i is the weight of item i , W is the capacity and C is the cardinality limit.

Property 6 (Cardinality constrained version) *The lower bound provided by the linear relaxation of the model (1.29)-(1.34) is equal to $\max(\lceil \sum_{i=1}^n w_i/W \rceil, \lceil n/C \rceil)$.*

Proof Fixing $A = W$, $v_i = 1$, and $B = C$ in Caprara and Toth's formulation, we have the model for the cardinality constrained version. From Property 4 we know that the lower bound is equal to $\max(\lceil \sum_{i=1}^n w_i/A \rceil, \lceil (\sum_{i=1}^n v_i/B) \rceil)$. Since, in this case, $\sum_{i=1}^n v_i = n$, the lower bound is equal to $\max(\lceil \sum_{i=1}^n w_i/W \rceil, \lceil n/C \rceil)$. \square

In practice, these models, except Gilmore-Gomory's model, have at least the same limitations as Martello and Toth's formulation for standard BPP.

1.2.5 Other methods

Among other methods for solving BPP exactly, one of the most important ones is Valério de Carvalho (1999)'s arc flow formulation with side constraints. This model has a set of flow conservation constraints and a set of demand constraints to ensure that the demand of every item is satisfied. The corresponding path flow formulation is equivalent to the classical Gilmore-Gomory's formulation for CSP. In Chapter 3, we develop this idea and extend it to some variants of the bin packing problem; we also present a graph compression method that usually reduces dramatically the graph size, allowing the solution of even harder problems.

Chapter 2

Heuristic Methods

2.1 Introduction

The bin packing problem (BPP) can be seen as a special case of the cutting stock problem (CSP). In BPP, n objects of m different weights must be packed into a finite number of bins, each with capacity W , in a way that minimizes the number of bins used. However, in CSP, the items of equal size (which are usually ordered in large quantities) are grouped into orders with a required level of demand; in BPP, the demand for a given size is usually close to one.

The best known implementations of FFD and BFD heuristics run in $\mathcal{O}(n \log n)$ and $\mathcal{O}(n \log(\min(n, W)))$, respectively. Neither of these run times is polynomial in the CSP input size. Therefore, for large CSP instances, computing a FFD/BFD solution may take a very long time using any conventional algorithm. These algorithms are usually fast for BPP, but for large CSP instances better alternatives are necessary.

In this chapter, we present pattern-based algorithms for these two heuristics. Our best pattern-based FFD and BFD implementations run in $\mathcal{O}(\min(m^2, n) \log m)$ and $\mathcal{O}(\min(m^2, n) \log(\min(n, W)))$, respectively. Both run times are polynomial in the CSP input size. These algorithms allow us to find solutions for large CSP instances quickly; for example, solutions for instances with one thousand million pieces are found in much less than one second of CPU time on current computers. An off-line variant of the sum-of-squares heuristics is also studied and compared with the well-known FFD/BFD heuristics.

Pattern-based algorithms overcome the main problems of conventional approaches in

cutting stock problems by representing the solution in a much more compact format. The final solution is represented by a set of patterns with associated multiplicities that indicate the number of times the pattern is used. Moreover, each pattern is represented by the piece lengths it contains and the number of times each piece appears. This representation of the output allows us to avoid the $\Omega(n)$ limit introduced by the assignment piece-by-piece. Besides that, this output format is much more practical in some situations where we are just interested in knowing how to cut the rolls.

In the remainder of this chapter, we will use cutting stock notation since it is for this kind of problems that our pattern-based heuristics are intended to. Nevertheless, in BPP instances, the pattern-based algorithms are at least as fast as the conventional ones.

The input format of all the algorithms in this chapter is the following: m - the number of different lengths; l - the set of lengths; b - the demand for each length; and L - the roll length. The output format depends on the algorithm used. Nevertheless, all the output formats will provide enough information to obtain an assignment piece-by-piece in $\Theta(n)$.

The remainder of this chapter is organized as follows. Section 2.2 presents a straightforward assignment-based FFD algorithm. Section 2.3 presents the pattern-based FFD algorithm along with an illustrative example. A complexity analysis is presented in Section 2.3.5. Section 2.4 and Section 2.5 present assignment-based and pattern-based BFD algorithms, respectively. Section 2.6 presents the sum-of-squares decreasing algorithm. Some computational results are presented in Section 2.7. Finally, Section 2.8 presents the conclusions.

2.2 Conventional First Fit Decreasing

The straightforward Algorithm 3 starts by sorting the piece lengths in decreasing order, in line 2, and initializes the solution with a single empty roll, in lines 3-5. The current solution is represented by the number of rolls used (R), the list of assignments (Sol ; an assignment of a piece of length l_i to a roll k is represented as $l_i \rightarrow \text{roll } k$) and the list of available spaces (Rem). For each length l_i , the b_{l_i} pieces will be inserted one-by-one in the solution. For each piece, we seek for the first roll where the piece fits. If we find a roll where the piece fits (line 11), we add the piece to the roll and update its remaining space. If there is no roll with enough space (line 17), we create

a new one. The variable k' is used to improve the running time in CSP instances by starting to seek for the first roll where the last piece of the same length was inserted (since the length is the same, we know that none of the previous rolls had enough space). The running time of this algorithm is $\mathcal{O}(n + mR)$.

Algorithm 3: Straightforward First Fit Decreasing Algorithm

input : m - number of different lengths; l - set of lengths; b - demand for each length; L - roll length

output: R - number of rolls needed; Sol - list of assignments

```

1 function FFD( $m, l, b, L$ ):
2    $l \leftarrow \text{reverse}(\text{sort}(l))$ ;                                // sort lengths in decreasing order
3    $Sol \leftarrow []$ ;
4    $R \leftarrow 1$ ;
5    $Rem \leftarrow [L]$ ;
6   for  $i \leftarrow 1$  to  $m$  do                                     // for each length
7      $k' \leftarrow 1$ ;
8     for  $j \leftarrow 1$  to  $b_{l_i}$  do                               // for each piece of length  $l_i$ 
9        $assigned \leftarrow \text{False}$ ;
10      for  $k \leftarrow k'$  to  $R$  do                               // try each roll
11        if  $Rem[k] > l_i$  then                                     // if there is enough space
12           $Rem[k] \leftarrow Rem[k] - l_i$ ;
13           $Sol.append(l_i \rightarrow \text{roll } k)$ ;
14           $assigned \leftarrow \text{True}$ ;
15           $k' \leftarrow k$ ;
16          break;
17      if not  $assigned$  then                                       // if the piece was not assigned to any roll
18         $R \leftarrow R + 1$ ;
19         $k' \leftarrow R$ ;
20         $Rem.append(L - l_i)$ ;
21         $Sol.append(l_i \rightarrow \text{roll } R)$ ;
22 return ( $R, Sol$ );

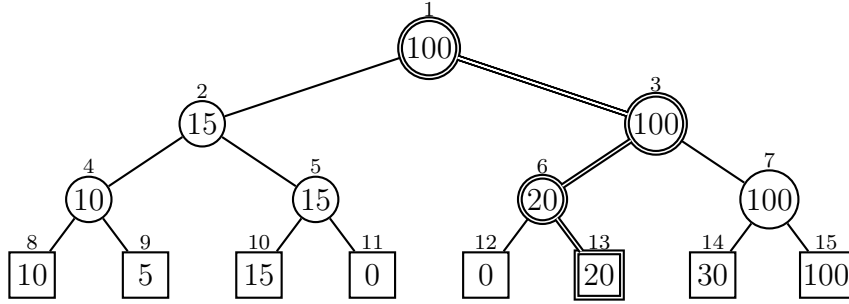
```

2.2.1 Efficient implementation

Johnson (1973) shows how to implement the FFD algorithm to run in $\mathcal{O}(n \log n)$ time using a tree. His tree was generalized for other applications and nowadays it is known as tournament tree/winner tree. A winner tree is a complete binary tree in which each node represents the best of its two children. For BPP, we need a tree with at least n leaves corresponding to bins. The value of each leaf is the remaining available space.

Figure 2.1 shows a winner tree applied to a small BPP instance. To find the leftmost bin with at least a certain gap g , we merely start at the root node and traverse to a leaf by choosing always the leftmost child that leads to a node with gap $\geq g$. After the insertion of the item, we update the label of the leaf node and then we proceed back up the path to the root relabeling each node if necessary. Since the tree has height at most $\lceil \log_2 n \rceil$, it takes $\mathcal{O}(\log n)$ time to identify the bin and $\mathcal{O}(\log n)$ time to update the tree. Therefore, this operation of insertion of an item takes $\mathcal{O}(\log n)$ time. Since we are dealing with a complete binary tree, we can represent this data structure efficiently using a simple array.

Figure 2.1: Winner tree.



Winner tree before an assignment of a piece of length 18 when the first 7 rolls, of capacity 100, are filled with 90, 95, 85, 100, 100, 80 and 70, respectively. The highlighted path indicates the search path along the tree that finds the leftmost bin with gap at least 18. The number inside each node is the key value and the number above is the index in the array representation.

Algorithm 4 shows a possible implementation of a winner tree for the FFD heuristics. For the sake of simplicity, we round the number of leaves to the smallest power of two greater than or equal to n . The procedure `initWTree` initializes the winner tree; `winner(g)` finds the leftmost roll with gap at least g ; and `update(j, g)` updates the available space of the roll j with the value g .

Algorithm 4: Winner tree implementation for the First Fit Decreasing heuristics

```

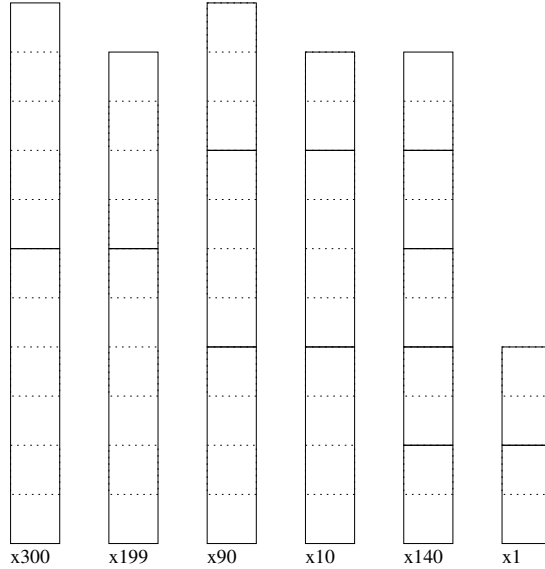
1 function initWTree( $n, W$ ): // initializes the tree
2    $n' \leftarrow 2^{\lceil \log_2 n \rceil};$  //  $n'$  is the smallest power of 2 greater than or equal to  $n$ 
3    $\text{size} \leftarrow 2n' - 1;$  // number of nodes in the tree
4    $\text{key}[i] \leftarrow W$ , for  $1 \leq i \leq \text{size}$ ; // initialize all the nodes with  $W$ 
5 function winner( $g$ ): // finds the leftmost leaf with value at least  $g$ 
6    $i \leftarrow 1;$ 
7   while  $2i \leq \text{size}$  do
8     if  $\text{key}[2i] \geq g$  then  $i \leftarrow 2i;$  // try the left child first
9     else  $i \leftarrow 2i + 1;$ 
10  return  $i - n' + 1;$  // return the index of the leaf
11 function update( $j, g$ ): // updates the  $j$ -th leaf with value  $g$ 
12   $i \leftarrow \text{size} - n' + j;$  // compute the array-index of the  $j$ -th leaf
13   $\text{key}[i] \leftarrow g;$  // update the leaf
14  while  $i > 1$  do // propagate the update
15     $p \leftarrow i/2;$ 
16     $t \leftarrow \max(\text{key}[2p], \text{key}[2p + 1]);$ 
17    if  $\text{key}[p] = t$  then break;
18     $\text{key}[p] \leftarrow t;$ 
19     $i \leftarrow p;$ 

```

2.3 Pattern-Based First Fit Decreasing

To represent pattern-based solutions, we will use the following notation: a set of pairs (m_i, p_i) where the multiplicity m_i is the number of times the pattern p_i appears in the solution, and each pattern p_i is a list $[a_1 \times (l = l_1), a_2 \times (l = l_2), \dots]$ where each $a_j \times (l = l_j)$ represents an item of length l_j repeated a_j times.

Figure 2.2 shows a FFD solution for an instance with $L = 11$, and pieces of lengths 6, 5, 4, 3, 2 with demands 499, 300, 399, 90, 712, respectively; hence, $n = 2000$ and $m = 5$. Despite having 2000 items, the solution can be represented in the pattern format as follows: $\{(300, [1 \times (l = 6), 1 \times (l = 5)]), (199, [1 \times (l = 6), 1 \times (l = 4)]), (90, [2 \times (l = 4), 1 \times (l = 3)]), (10, [2 \times (l = 4), 1 \times (l = 2)]), (140, [5 \times (l = 2)]), (1, [2 \times (l = 2)])\}$.

Figure 2.2: An example of a first fit decreasing solution.

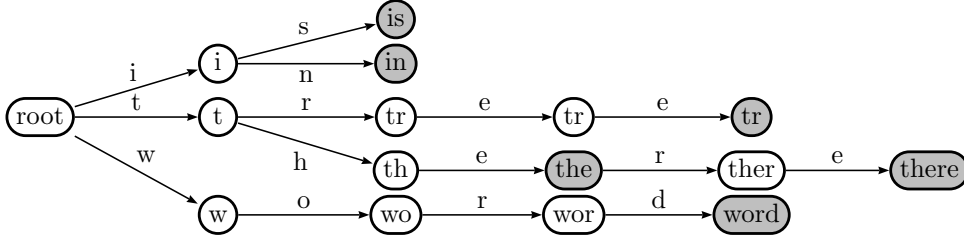
First fit decreasing solution for an instance with $L = 11$ and pieces of lengths 6, 5, 4, 3, 2 with demands 499, 300, 399, 90, 712, respectively; Despite having 2000 items, there are only 6 different patterns in the solution.

2.3.1 Prefix tree

A trie, also called a prefix tree, is a tree structure that is usually used to store words. Our algorithm uses a variant of this data structure to represent patterns and their multiplicities. In this section, we will explain how to interpret this kind of trees.

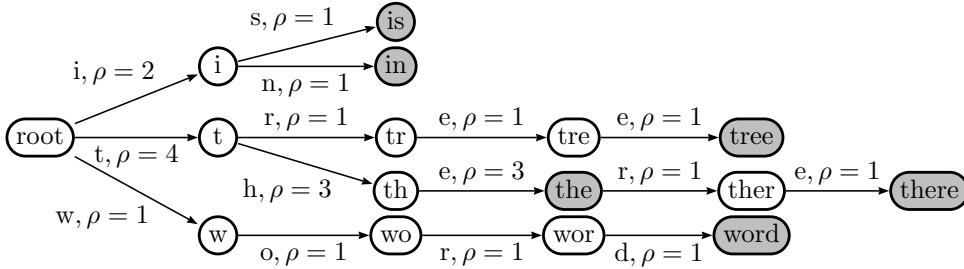
Figure 2.3 shows a small prefix tree containing the words {"there", "the", "word", "is", "in", "tree"}. Every path from the root to a node represents a prefix of words stored in the tree. The gray nodes are called terminal nodes. Paths from the root to terminal nodes correspond to words stored in the tree. The list of words can be retrieved by storing every path that ends in a terminal node while traversing the tree.

In the same way we store words in a trie, we can store patterns of the form $[a_1 \times (l = l_1), a_2 \times (l = l_2), \dots]$ considering each $a_i \times (l = l_i)$ as a symbol. However, we also need to know their multiplicities; but it is easy to modify the trie in order to count the number of occurrences of each prefix. Figure 2.4 shows a prefix tree with prefix count. The words from the list ["there", "the", "the", "word", "is", "in", "tree"] are stored in this tree. Every word appears once except the word "the" that appears twice. The ρ value of an arc can be seen as the number of words that use the arc. The number of times a prefix occurs is given by the ρ value of its last arc. The number of times a

Figure 2.3: A simple prefix tree.

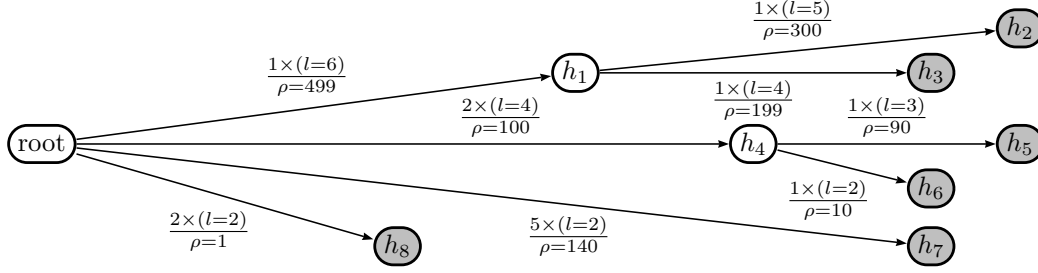
A simple prefix tree containing the words: {"there", "the", "word", "is", "in", "tree"}. Every path from the root to a node represents a prefix of the words stored in the tree. The gray nodes are called terminal nodes and contain the stored words.

prefix appears in the original list is given by the difference between the ρ value of the arc from its parent and the sum of the ρ values of arcs to its children. If this difference is greater than zero, the word exists by itself in the original list; otherwise, it is just a prefix of other words.

Figure 2.4: A prefix tree with prefix count.

A prefix tree containing the prefixes and prefix occurrence counts (ρ) of the words in the list ["there", "the", "the", "word", "is", "in", "tree"]. The ρ value of an arc can be seen as the number of words that use the arc.

With the introduction of the prefix count, we can now store solutions, in the previously described format, in a trie. Figure 2.5 shows the solution from Figure 2.2 stored in a trie. Every path from the root to a node represents a sub-pattern and the ρ of the last arc in the path indicates its multiplicity. The gray nodes are terminal nodes corresponding to patterns in the solution. The list of patterns and their multiplicities can be retrieved by storing every path that ends in a terminal node, along with the ρ of its last arc, while traversing the tree. We do not store patterns in the nodes, since we can always obtain them from the paths.

Figure 2.5: Tree representation of the first fit decreasing solution from Figure 2.2.

Every path from the root to a node represents a sub-pattern and the ρ value of the last arc in the path indicates its multiplicity. Sub-patterns ending in terminal nodes are patterns that appear by itself in the solution. Paths that end in non-terminal nodes are just sub-patterns of patterns in the solution. The root node represents the empty pattern.

2.3.2 Algorithm

In the pattern-based Algorithm 5, the solution is represented by a tree as described in the previous section. The tree is represented by the following data structures: $\text{Label}[u, v]$ indicates which piece appears between nodes u and v and the number of times it appears; $\text{Count}[u, v]$, which corresponds to the ρ value of the arc, indicates how many times the arc is used. Each node is identified by two values: h and s . The value s indicates the amount of roll used by the corresponding sub-pattern and h is just an identifier to distinguish nodes with the same value s . The solution is initialized with an empty tree, in lines 2-3. The piece lengths are sorted in decreasing order, in line 4. The variable h' is just the value h of the last node created. In this algorithm we only modify patterns that end in terminal nodes since non-terminal nodes are just a sub-patterns of other patterns. Q^* is a list of terminal nodes and their multiplicities, initialized with the root node ($h = 0, s = 0$) with multiplicity ∞ ; this means that the empty pattern (root) is considered to appear an infinity number of times in the solution and hence it is always possible to start new patterns from the empty pattern. Recall that the multiplicity of every node except the root will be equal to the difference between the ρ of the arc from its parent and the sum of the ρ values of arcs to its children, as described in the previous section.

In the main loop of line 8, for each length l_i we insert the b_{l_i} pieces in the solution by creating new patterns starting from the current terminal nodes. Q is the list of terminal nodes in the following iteration, and it is initialized empty, in line 9. In the loop 11, we try each terminal node in the order they appear in Q^* , checking if there is available space to introduce some pieces with the current length l_i . If there is available

Algorithm 5: Pattern-Based First Fit Decreasing Algorithm

input : m - number of different lengths; l - set of lengths; b - demand for each length; L - roll length

output: R - number of rolls needed; **Count** - ρ value of each arc; **Label** - length of the piece associated with each arc and the number of times it is repeated

```

1 function FFDTree( $m, l, b, L$ ):
2   Count[ $u, v$ ]  $\leftarrow 0$ , for all  $u, v$  ;                                // initialize auxiliary data structures
3   Label[ $u, v$ ]  $\leftarrow \text{NIL}$ , for all  $u, v$ ;
4    $l \leftarrow \text{reverse}(\text{sort}(l))$ ;                                       // sort lengths in decreasing order
5    $h' \leftarrow 0$ ;                                                         // index of the last node
6   root  $\leftarrow ((h, 0), \infty)$ ;                                         // empty pattern
7    $Q^* \leftarrow [\text{root}]$ ;                                               // list of terminal nodes
8   for  $i \leftarrow 1$  to  $m$  do                                             // for each piece length
9      $Q \leftarrow []$ ;
10     $\beta \leftarrow b_{l_i}$ ;
11    foreach  $((h_j, s_j), r_j) \in Q^*$  do                                // for each terminal node in the order they appear in  $Q^*$ 
12      if  $\beta > 0$  then
13         $\gamma \leftarrow (L - s_j) \text{ div } l_i$ ;
14        if  $\gamma > 0$  then
15           $u \leftarrow (h_j, s_j)$ ;
16           $\rho' = \min(r_j, \beta \text{ div } \gamma)$ ;
17          if  $\rho' > 0$  then                                              // if there is remaining demand to use the piece
18             $\beta \leftarrow \beta - \gamma \rho'$ ;  $r_j \leftarrow r_j - \rho'$ ;      the maximum number of times it fits
19             $h' \leftarrow h' + 1$ ;  $v \leftarrow (h', s_j + \gamma l_i)$ ;
20            Count[ $u, v$ ]  $\leftarrow \rho'$ ; Label[ $u, v$ ]  $\leftarrow (\gamma \times l_i)$ ;
21             $Q.\text{append}((v, \rho'))$ ;
22           $\gamma' \leftarrow \beta \bmod \gamma$ ;
23          if  $r_j > 0$  and  $\gamma' > 0$  then                                // if there remains demand and multiplicity to
24             $\beta \leftarrow \beta - \gamma'$ ;  $r_j \leftarrow r_j - 1$ ;          insert the remaining pieces
25             $h' \leftarrow h' + 1$ ;  $v \leftarrow (h', s_j + \gamma' l_i)$ ;
26            Count[ $u, v$ ]  $\leftarrow 1$ ; Label[ $u, v$ ]  $\leftarrow (\gamma' \times l_i)$ ;
27             $Q.\text{append}((v, 1))$ ;
28      if  $r_j > 0$  then                                                  // check if the current node remains as a terminal node
29         $Q.\text{append}(((h_j, s_j), r_j))$ ;
30     $Q^* \leftarrow Q$ 
31   $R \leftarrow \sum_{((h_j, s_j), r_j) \in Q^* \setminus \{\text{root}\}} r_j$ ;
32  return ( $R$ , Count, Label);

```

space, the value γ (line 13) will be larger than 0 and will indicate the maximum number of times the current piece fits (**div** denotes integer division and **mod** is its remainder). If γ is larger than 0 (line 14), the value ρ' (line 16) will indicate the number of rolls that can use the current piece the maximum number of times it fits. If ρ' is zero, no new pattern can be added; otherwise (line 17), the multiplicity of the current terminal node is decreased and a new pattern is created by adding a new terminal node. Then, in line 23, we check if there remains multiplicity and demand to add the remaining pieces of the current length. If this is the case, another pattern is formed by adding a new terminal node, and the multiplicity of the current node is decreased. In line 28, we check if current node remains as a terminal node. This process is repeated for every length and finally, in line 31, we compute the number of rolls needed by summing the multiplicities of the terminal nodes except the root. This algorithm runs in $\mathcal{O}(m^2)$ time, see Section 2.3.5 for more details.

Algorithm 6 receives the output of Algorithm 5 and produces an output in the format described in the previous section. This algorithm uses a simple recursive function that performs a depth-first traversal of the tree and whenever it finds a terminal node it adds the corresponding pattern to the solution (lines 3-9). In line 2, an adjacency list for the tree is obtained. The recursive function is called, in line 10, starting at the node $(h = 0, s = 0)$ with an empty pattern with multiplicity R . This algorithm is linear in the length of the output, which is limited by $\mathcal{O}(\min(m^2, n))$, see Section 2.3.5 for more details.

Algorithm 6: Solution Extraction Algorithm

input : R - number of rolls needed; **Count** - ρ value of each arc; **Label** - length of the piece associated with each arc and the number of times it is repeated

output: Set of patterns and multiplicities

```

1 function patternExtraction( $R$ , Count, Label):
2    $Adj \leftarrow \text{getAdjList}(\text{Label});$                                 // obtain an adjacency list of the tree
3   function getPatterns( $u, \rho', \text{path}$ ):                               // recursive function to extract patterns
4      $\text{patterns} \leftarrow \{ \};$ 
5     foreach  $v \in Adj[u]$  do
6        $\text{patterns} \leftarrow \text{patterns} \cup \text{getPatterns}(v, \text{Count}[u, v], \text{path} + \text{Label}[u, v])$ 
7        $\rho' \leftarrow \rho' - \text{Count}[u, v];$ 
8       if  $\rho' > 0$  then                                              // if it is a terminal node, add the pattern to the solution
9          $\text{patterns} \leftarrow \text{patterns} \cup \{(\rho', \text{path})\};$ 
9       return  $\text{patterns};$ 
10  return  $\text{getPatterns}((0, 0), R, [ ]);$ 

```

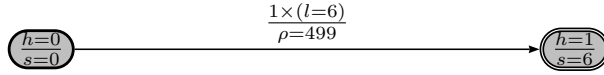
2.3.3 Example

Consider an instance with $L = 11$ and pieces of lengths 6, 5, 4, 3, 2 with demands 499, 300, 399, 90, 712, respectively; hence, $n = 2000$ and $m = 5$.

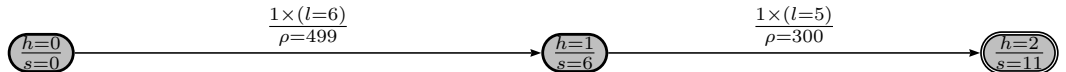
- We start with a tree with only one node corresponding to the empty pattern. This pattern has infinity multiplicity, which means that there is no limit on the number of patterns that can be created starting from it.



- After the insertion of the 499 pieces of length 6, we have a tree with two nodes; one corresponding to the empty pattern $[]$ and another corresponding to the pattern $[1 \times (l = 6)]$. Both nodes are terminal nodes since their multiplicity is greater than zero. Recall that the multiplicity of every node except the root is equal to the difference between the ρ of the arc from its parent and the sum of the ρ values of arcs to its children. The node $(h = 0, s = 0)$ remains with infinity multiplicity, and the node $(h = 1, s = 6)$ has multiplicity 499, meaning that it is possible to create more patterns starting from it by adding more pieces.

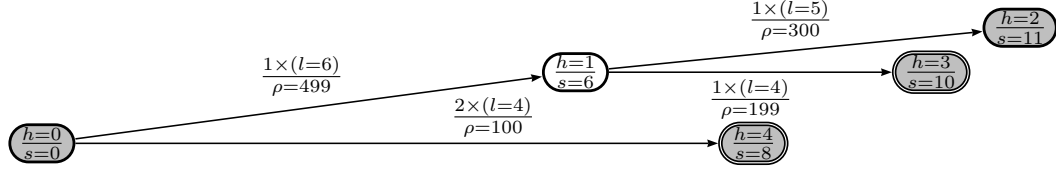


- At the third iteration, we insert the pieces of length 5. The first terminal node in the list Q^* with available space is the node $(h = 1, s = 6)$. This node has multiplicity 499, allowing it to hold all the pieces with $l = 5$, so we just add a child node. The tree is now composed by three terminal nodes. The solution represented by this tree is $\{(300, [1 \times (l = 6), 1 \times (l = 5)]), (199, [1 \times (l = 6)])\}$, i.e., there are 300 rolls with two pieces, one piece of length 6 and another of length 5, and 199 rolls with one piece of length 6. Note that the node $(h = 1, s = 6)$ remains as a terminal node but its multiplicity decreased to 199, since a pattern that uses 300 units of its multiplicity has been created starting from it.

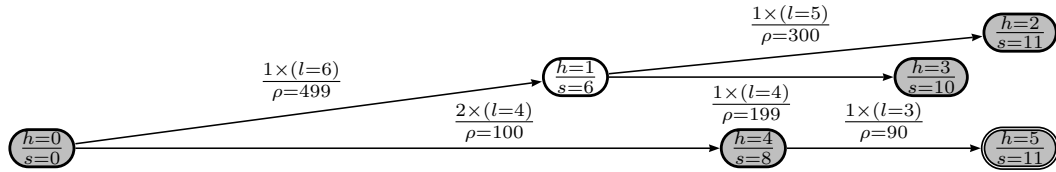


- After the insertion of the 399 pieces of length 4, the node $(h = 1, s = 6)$ is no longer a terminal node since the pattern $(199, [1 \times (l = 6), 1 \times (l = 4)])$

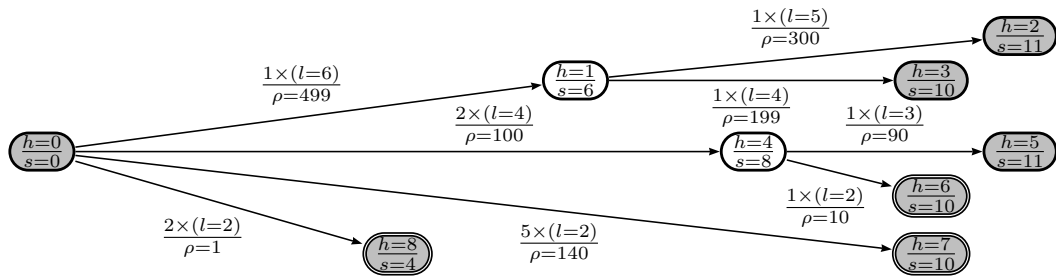
used the remaining multiplicity. The first terminal node with available space was $(h = 1, s = 6)$ and it was used to hold 199 pieces. For the remaining 100 pieces of length $l = 4$, a new pattern starting with two pieces of length 4 has been formed, since the next terminal node with available space was $(h = 0, s = 0)$ and $11 \text{ div } 4 = 2$. The current solution represented by this tree is $\{(300, [1 \times (l = 6), 1 \times (l = 5)]), (199, [1 \times (l = 6), 1 \times (l = 4)]), (100, [2 \times (l = 4)])\}$.



- At the fifth iteration, we add the pieces of length 3. The first terminal node in Q^* with available space is the node $(h = 4, s = 8)$. Since the multiplicity of this node is 100 and we have 90 pieces of length 3, we just add a new pattern $[2 \times (l = 4), 1 \times (l = 3)]$ by adding a child node to $(h = 4, s = 8)$.



- Now, to add the 712 pieces of length 2, things are a little more complicated. The first terminal node with available space is the node $(h = 4, s = 8)$. The multiplicity of this node is 10, so we replace the pattern $[2 \times (l = 4)]$ by $[2 \times (l = 4), 1 \times (l = 2)]$ adding a child node that uses the remaining multiplicity. Now, there remain 702 pieces to add. Since there are more pieces to add, we proceed to the next terminal node, which is $(h = 0, s = 0)$. Starting from this node, it is possible to use 140 times the pattern $[5 \times (l = 2)]$ since $11 \text{ div } 2 = 5$ and $702 \text{ div } 5 = 140$. Since, we still have two items of length 2 to add and the multiplicity of the node $(h = 0, s = 0)$ is infinity, we just add the pattern $[2 \times (l = 2)]$.



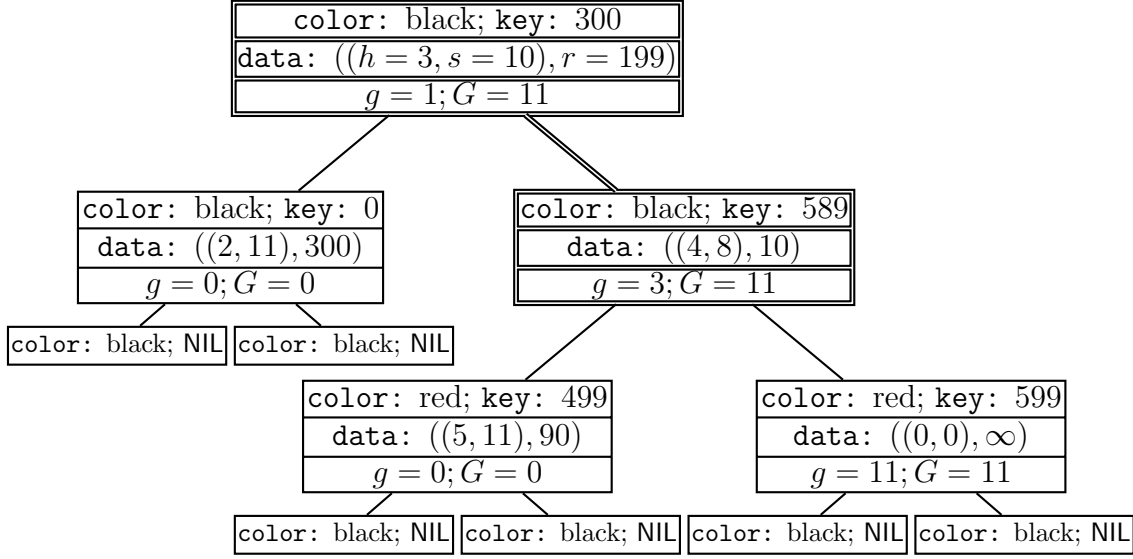
- The final solution represented by this tree is $\{(300, [1 \times (l = 6), 1 \times (l = 5)]), (199, [1 \times (l = 6), 1 \times (l = 4)]), (90, [2 \times (l = 4), 1 \times (l = 3)]), (10, [2 \times (l = 4), 1 \times (l = 2)]), (140, [5 \times (l = 2)]), (1, [2 \times (l = 2)])\}$ and it uses 6 different patterns and 740 rolls.

2.3.4 Efficient implementation

The pattern-based algorithm presented in the previous section runs in $\mathcal{O}(m^2)$ time and it is much faster than any conventional algorithm in CSP instances. However, in BPP instances, when $m = n$, it is quadratic. To overcome this problem, we introduce a new data structure that we call red-black winner tree, since it results from the combination of these two data structures. This data structure will allow us to compute any FFD solution in $\mathcal{O}(\min(m^2, n) \log m)$ time, which will always be at least as fast as the most efficient conventional FFD implementation in BPP instances and extremely fast in CSP instances.

A red-black tree is a self-balancing binary search tree that is typically used to implement associative arrays. Since it is a balanced tree, it guarantees insertion, search and delete to be logarithmic in the number of elements in the tree. This type of tree was initially proposed by Bayer (1972) with the name symmetric binary B-tree. In this data structure, each node has a color attribute, which is either red or black, and there is a set of rules that need to be always satisfied: every node is either red or black; the root is black; the leaves are sentinel nodes with no data, and their color is black; every node has two children; both children of every red node are black; every simple path from a given node to any of its descendant leaves contains the same number of black nodes. It can be proved that these constraints guarantee that the height of the tree is no longer than $2 \log(n + 1)$, where n is the total number of elements stored in the tree (see, e.g., Cormen et al. 2001).

The red-black winner tree is a self-balancing search tree in which each node contains a key, a data field, a value (g), and G , the maximum value in the subtree below the node including the node value. The key is used to keep the patterns sorted by position (i.e., by the index of the first roll that uses the pattern), the data field keeps the node for the pattern-based FFD algorithm and the value is simply the gap of the pattern represented by the node. Figure 2.6 shows a small red-black winner tree applied to the example for the pattern-based FFD algorithm before the assignment of the pieces of length 2.

Figure 2.6: Red-black winner tree.

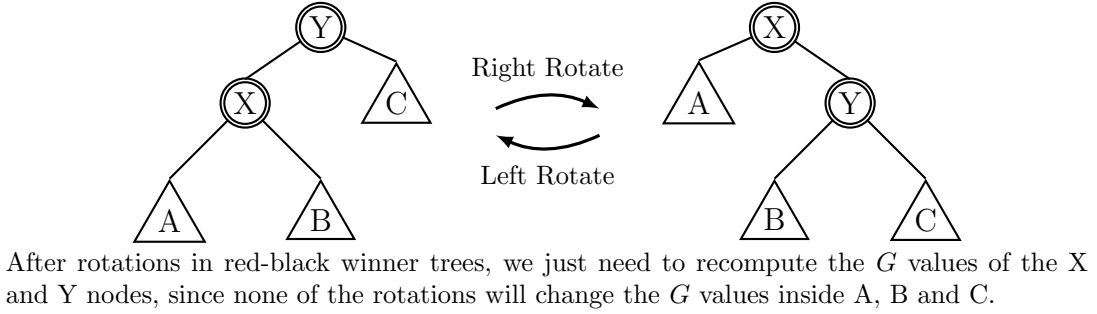
Red-black winner tree corresponding to the example for the pattern-based FFD algorithm before the assignment of the pieces of length 2. The highlighted path indicates the search path along the tree that finds the leftmost pattern with gap at least 2. Note that each pattern is represented by its corresponding terminal node in the pattern prefix tree. The key is the index of the first roll that uses the pattern stored in the node. The winner is the node with $\text{key} = 589$, and the corresponding node in the pattern tree is $(h = 4, s = 8)$, which has multiplicity 10. The insertion of the pieces of length 2 in this pattern modifies its data field and its g value. In this particular situation, the G value remains unchanged and we do not need to proceed back up the tree updating the G values; these updates stop at the first node whose G value remains unchanged since it is the G value that is propagated. The leaves are the sentinel nodes.

A normal red-black tree will only have in each node a key and a data field. We need to modify slightly some of the operations because of the G field. More specifically, we need to modify the update, insertion and the rotations. Moreover, we need to add a winner operation. Note that if we also wanted to delete keys we would also need to modify the delete operation. However, our algorithm only requires insertions and hence we will not specify the details about the delete operation.

When inserting a new node in a red-black tree, we start at the root and follow one of the child nodes depending on the key value. We stop when we find a leaf that will be replaced by the new node. When inserting a node with gap g' in a red-black winner tree, on the way down, we need to update $G \leftarrow \max(G, g')$ of every node along the path to the leaf where the node will initially be inserted. After the insertion of the node, it is colored red and two black leaves are added. Depending on the color of other nearby nodes it may be necessary to recolor nodes and/or apply rotations along the path up to the root in order to maintain the invariants, see, e.g., Cormen et al.

(2001) for more details. Rotations interfere with the G field since it is the maximum g value in the subtree below a node. Therefore, we need to modify this operation too. Figure 2.7 shows what happens during each type of rotation. None of the rotations will change the G values inside A, B and C. Therefore, we just need to recompute the G values of the X and Y nodes.

Figure 2.7: Red-black tree rotations.



Algorithm 7 shows how to implement the update and the winner operations. The operation `winner(g)` finds the leftmost pattern with gap at least g ; and `update(key, data, g)` replaces the pattern at position `key` by a new pattern `data` with gap g . Recall that each pattern is represented by its corresponding terminal node in the pattern prefix tree. When we update the data field of a node, we do not need to do anything. However, when we update the g value, we need to propagate this update along the path to the root. The `winner` operation is somewhat similar to the `winner` operation of a normal winner tree, we just look for the node with the smallest key and enough available space, since the key is the index of the first roll that uses the pattern stored in the node.

Algorithm 8 shows how to use the red-black winner tree in conjunction with Algorithm 5. This algorithm is very similar to the Algorithm 5, the main difference is the fact that we store the patterns in a red-black winner tree instead of using lists. For each length, while there remains demand, we use the winner operation to find the leftmost pattern with enough available space. The winner pattern will be replaced by a new pattern, another new pattern may appear, and if there remains multiplicity in the original pattern, it will be inserted again in the tree with the remaining multiplicity. We store in a list the patterns that appear; we update the winner with the first pattern; and if there are other patterns in the list, we insert them in the tree.

Algorithm 7: Red-black winner tree update and winner operations

```

1 function winner( $g$ ):                                     // finds the node with the smallest key and value at least  $g$ 
2    $x \leftarrow \text{root}$ ;
3   while True do
4     if  $x.\text{left} \neq \text{NIL}$  and  $x.\text{left}.G \geq g$  then  $x \leftarrow x.\text{left}$ ;           // try the left subtree
5     else if  $x.g \geq g$  then break;                                           // try the current node
6     else  $x \leftarrow x.\text{right}$ ;                                           // try the right subtree
7   return  $x$ ;

8 function update( $\text{key}, \text{data}, g$ ):                             // updates the value and the data field of a node
9    $x \leftarrow \text{search}(\text{key})$ ;
10   $x.g \leftarrow g$ ;
11   $x.\text{data} \leftarrow \text{data}$ ;
12  while True do
13     $t \leftarrow \max(x.\text{left}.G, x.\text{right}.G, x.g)$ ;
14    if  $x.G = t$  or  $x = \text{root}$  then break; // stop at the root or when the  $G$  value remains unchanged
15     $x.G \leftarrow t$ ;
16     $x \leftarrow x.\text{parent}$ ;

```

Algorithm 8: Efficient Pattern-Based First Fit Decreasing Algorithm

input : m - number of different lengths; l - set of lengths; b - demand for each length; L - roll length

output: R - number of rolls needed; **Count** - ρ value of each arc; **Label** - length of the piece associated with each arc and the number of times it is repeated

```

1 function FFDTree( $m, l, b, L$ ):
2    $\text{Count}[u, v] \leftarrow 0$ , for all  $u, v$ ; // initialize auxiliary data structures
3    $\text{Label}[u, v] \leftarrow \text{NIL}$ , for all  $u, v$ ;
4    $l \leftarrow \text{reverse}(\text{sort}(l))$ ; // sort lengths in decreasing order
5    $h' \leftarrow 0$ ;  $\text{root} \leftarrow ((h', 0), \infty)$ ;
6    $T \leftarrow \text{RBWT}([(key = 0, data = \text{root}, g = L)])$ ; // initialize the red-black winner tree with the root
7   for  $i \leftarrow 1$  to  $m$  do // for each piece length
8      $\beta \leftarrow b_{l_i}$ ;
9     while  $\beta > 0$  do // while there remains demand
10       $L \leftarrow []$ ;
11       $\text{key} \leftarrow T.\text{winner}(l_i)$ ;
12       $((h, s), r) \leftarrow T[\text{key}]$ ;
13       $\gamma \leftarrow (L - s) \text{ div } l_i$ ;
14      if  $\gamma > 0$  then
15         $u \leftarrow (h, s)$ ;
16         $\rho' = \min(r, \beta \text{ div } \gamma)$ ;
17        if  $\rho' > 0$  then // if there is remaining demand to use the piece
18          // the maximum number of times it fits
19           $\beta \leftarrow \beta - \gamma\rho'$ ;  $r \leftarrow r - \rho'$ ;
20           $h' \leftarrow h' + 1$ ;  $v \leftarrow (h', s + \gamma l_i)$ ;
21           $\text{Count}[u, v] \leftarrow \rho'$ ;  $\text{Label}[u, v] \leftarrow (\gamma \times l_i)$ ;
22           $L.\text{append}(((v, \rho'), L - v_2))$ ;
23           $\gamma' \leftarrow \beta \bmod \gamma$ ;
24          if  $r > 0$  and  $\gamma' > 0$  then // if there remains demand and multiplicity to
25            // insert the remaining pieces
26             $\beta \leftarrow \beta - \gamma'$ ;  $r \leftarrow r - 1$ ;
27             $h' \leftarrow h' + 1$ ;  $v \leftarrow (h', s + \gamma' l_i)$ ;
28             $\text{Count}[u, v] \leftarrow 1$ ;  $\text{Label}[u, v] \leftarrow (\gamma' \times l_i)$ ;
29             $L.\text{append}(((v, 1), L - v_2))$ ;
30          if  $r > 0$  then // if there remains multiplicity for the original pattern
31             $L.\text{append}(((h, s), r), L - s)$ ;
32             $(v, f), g \leftarrow L.\text{pop}(0)$ ;
33             $T.\text{update}(\text{key}, (v, r), g)$ ;
34            foreach  $((v, r), g) \in L$  do
35               $T.\text{insert}(\text{key}, (v, r), g)$ ;
36               $\text{key} \leftarrow \text{key} + r$ ;
37    $R \leftarrow T.\text{max}()$ ; // the maximum key value corresponds to the position of the empty pattern
38   return ( $R$ , Count, Label);

```

2.3.5 Complexity analysis

Theorem 1 *The number of different patterns p_m in a FFD/BFD solution is at most $2m$, where m is the number of different piece lengths.*

Proof The proof is made by induction on m .

- The case $m = 1$ is verified as follows: using a single length we have at most two patterns, one composed by the maximum number of times that the pieces fit the roll and another composed by the remaining pieces.
- Assume $p_m \leq 2m$ holds. There are three (non-exclusive) situations that can happen when introducing a new piece length:
 1. replace entirely some patterns by new patterns that include the current length (the total number of patterns remains the same);
 2. modify some pattern by reducing its multiplicity and adding:
 - one pattern where the piece is used the maximum number of times it fits the roll;
 - and/or, one pattern with multiplicity one with the remaining pieces.

By the induction hypothesis $p_m \leq 2m$. Since we add at most 2 patterns by introducing a new piece length, with $m + 1$ piece lengths we will have $p_{m+1} \leq 2m + 2 = 2(m + 1)$ patterns. This result is also valid for BFD solutions.

□

The pattern-based FFD Algorithm 5 has complexity $\mathcal{O}(mp)$ where m is the number of different piece lengths and p is the number of different patterns in the final solution. Theorem 1 ensures that the number of patterns will be at most $2m$. The complexity of the Algorithm 5 is therefore $\mathcal{O}(m^2)$. For BPP instances we still have $\mathcal{O}(n^2)$ if all the pieces have different lengths, i.e., $m = n$, but this algorithm is intended to CSP instances where the number of different lengths is usually much smaller than the total number of pieces. Also, this algorithm is polynomial in the CSP input size while a conventional $\mathcal{O}(n \log n)$ FFD algorithm is pseudo-polynomial.

Let us consider each $a_i \times (l = l_i)$ (i.e., piece of length l_i repeated a_i times) as a symbol. Let χ be the total number of symbols in the output and k the average number of

symbols per pattern (i.e., $k = \chi/p$). The number of patterns p is limited by $2m$, and k is usually very small in FFD/BFD solutions when the demands are high, since each length is used as many times as possible leaving little space for other pieces. The pattern extraction algorithm, Algorithm 6, runs in $\mathcal{O}(\min(m^2, n))$ time since it is linear in the length of the output (i.e., in the number of symbols). More precisely, it runs in $\mathcal{O}(kp)$. An assignment piece-by-piece can be obtained from the pattern representation in $\Theta(n)$.

Algorithm 8 runs in $\mathcal{O}(\min(m^2, n) \log m)$ time since it performs $\mathcal{O}(\log m)$ operations per arc. More precisely, since the number of arcs is no more than the number of symbols in the output, this algorithm runs in $\mathcal{O}(kp \log m)$ where p is limited by $2m$ and k is usually very small in cutting stock instances. This algorithm is as fast as the most efficient known implementation of the FFD heuristics in BPP instances, and it is extremely fast in cutting stock instances.

2.4 Conventional Best Fit Decreasing

The BFD Algorithm 9 starts by sorting the piece lengths in decreasing order (line 2) and it initializes the solution with a single empty roll (lines 3-5). The current solution is represented by the number of rolls used (**R**), the list of assignments (**Sol**; an assignment of a piece of length l_i to a roll k is represented as $l_i \rightarrow \text{roll } k$) and the list of the available spaces (**Rem**). For each length l_i , the b_{l_i} pieces will be inserted one-by-one in the solution. For each piece, we seek for the roll with smallest gap greater or equal to the piece length. If we find rolls where the piece fits (line 9), we add the piece to one with the smallest gap and we update its remaining space. If there is no roll with enough space (line 13), we create a new one. The naive implementation of this algorithm runs in $\mathcal{O}(n \min(n, L))$ where n is the number of pieces and L the length of the rolls, since it takes $\mathcal{O}(\min(n, L))$ time to identify the “best” roll. However, it can be implemented to run in $\mathcal{O}(n \log(\min(n, L)))$ time using a self-balancing search tree such as a red-black tree.

2.4.1 Efficient implementation

Algorithm 10 shows how to implement the upper bound operation that finds the smallest key greater than or equal to a certain value in a binary search tree. This operation allows us to find the smallest gap sufficient for the item size such that at least one

Algorithm 9: Straightforward Best Fit Decreasing Algorithm

input : m - number of different lengths; l - set of lengths; b - demand for each length; L - roll length

output: R - number of rolls needed; Sol - list of assignments

```

1 function BFD( $m, l, b, L$ ):
2    $l \leftarrow \text{reverse}(\text{sort}(l));$                                 // sort lengths in decreasing order
3    $Sol \leftarrow [];$ 
4    $R \leftarrow 1;$ 
5    $Rem \leftarrow [L];$ 
6   for  $i \leftarrow 1$  to  $m$  do                                     // for each length
7     for  $j \leftarrow 1$  to  $b_{l_i}$  do                               // for each piece of length  $l_i$ 
8        $S \leftarrow \{k \mid 1 \leq k \leq R, \text{Rem}[k] \geq l_i\};$ 
9       if  $S \neq \emptyset$  then
10         $bst \leftarrow \text{argmin}_{k \in S} \text{Rem}[k];$ 
11         $\text{Rem}[bst] \leftarrow \text{Rem}[bst] - l_i;$ 
12         $Sol.append(l_i \rightarrow \text{roll } bst);$ 
13      else                                                       // if there is no roll with enough available space
14         $R \leftarrow R + 1;$ 
15         $\text{Rem}.append(L - l_i);$ 
16         $Sol.append(l_i \rightarrow \text{roll } R);$ 
17 return ( $R, Sol$ );

```

roll has such gap, or an indication that no such roll exists, in $\mathcal{O}(\log(\min(n, L)))$ time when using a self-balancing binary search tree. By keeping the rolls grouped by gap in a tree, such as a red-black tree, the n items will be handled in $\mathcal{O}(n \log(\min(n, L)))$.

Algorithm 10: Upper Bound Operation

```

1 function upperBound(value):
2    $x \leftarrow \text{root};$ 
3    $\text{last} \leftarrow \text{NIL};$ 
4   while  $x \neq \text{NIL}$  do
5     if  $x.\text{key} = \text{value}$  then return  $x.\text{key}$  ;
6     else if  $x.\text{key} > \text{value}$  then
7        $\text{last} \leftarrow x.\text{key};$ 
8        $x \leftarrow x.\text{left};$ 
9     else
10       $x \leftarrow x.\text{right};$ 
11 return  $\text{last};$ 

```

2.5 Pattern-Based Best Fit Decreasing

The pattern-based BFD can be implemented in the same way as the pattern-based FFD Algorithm 5. We just need to sort, in non-increasing order of gap, the list of terminal nodes before iterating through them in line 11. Theorem 1 ensures that the number of patterns in a BFD solution is limited by $2m$. The complexity of the previous algorithm is therefore $\mathcal{O}(m^2 \log m)$. For CSP instances, this is probably enough. However, in BPP instances, this algorithm runs in $\mathcal{O}(n^2 \log n)$. By storing the list of terminal nodes in a self-balanced binary search tree and using the upper bound operation, this algorithm can be implemented to run in $\mathcal{O}(\min(m^2, n) \log(\min(m, L)))$. Moreover, it is usually much faster than $\mathcal{O}(\min(m^2, n) \log(\min(m, L)))$ in cutting stock instances since the number of arcs is usually much smaller than $\mathcal{O}(\min(m^2, n))$ (see Section 2.3.5).

2.6 Sum-of-Squares Decreasing

The sum-of-squares (SS) heuristics is an on-line heuristics for BPP. Since we are interested in off-line heuristics for large cutting-stock problems, we are going to modify this heuristics to an off-line environment and show how to implement a reasonably fast algorithm. Csirik et al. (1999) present the on-line version of this heuristics and show that we do not really need to compute the sum-of-squares. However, they do not present any algorithm faster than $\mathcal{O}(n \min(n, L))$. For CSP instances, it is really hard to compute solutions using this algorithm. Nevertheless, in an off-line environment, it is possible to implement a reasonably fast algorithm. Unfortunately, for this heuristics we did not find any pattern-based algorithm since the decisions for consecutive items usually vary a lot because of the selection criterion that is very sensible to the number of rolls with each gap. Nevertheless, we created an algorithm that can compute a sum-of-squares decreasing (SSD) solution in $\mathcal{O}(\max(mL, n \log L))$. In CSP instances, where m is much smaller than n , this complexity is dominated by $\mathcal{O}(n \log L)$, which is reasonably fast for n up to a few millions of items. This is much slower than the pattern-based algorithms that we presented for FFD and BFD, but in practice it makes possible to compute solutions for instances with one thousand million pieces in approximately one hour; using, for example, a $\mathcal{O}(n \min(n, L))$ algorithm, it would take much longer.

The SS heuristics works as follows. Let $N(g)$ be the current number of rolls with gap

g . Initially, $N(g) = 0$, for $1 \leq g < L$. To pack a piece of length l_i , we choose a roll with gap at least l_i that yields the minimum updated value of $\sum_{1 \leq g < L} N(g)^2$. If there is a tie, we choose the candidate with the smallest gap. In SSD, we just start by sorting the items and then we apply the SS heuristics as described.

2.6.1 Reasonably efficient implementation

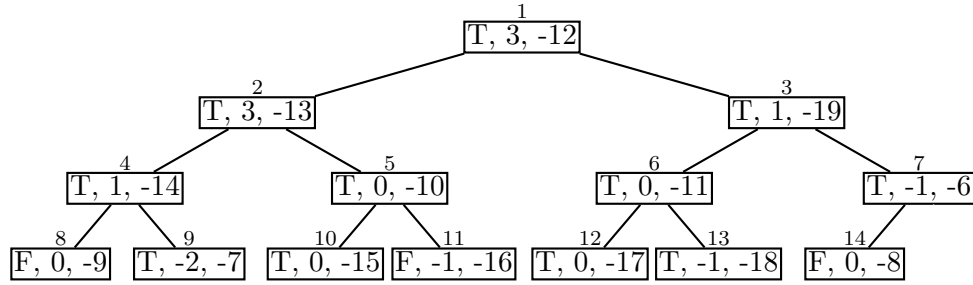
It is not really needed to compute the sum-of-squares for each possible choice and select the minimum; there are three decision cases, as follows. Let l_i be the length of the piece we want to cut. Case 1: if $N(l_i) > 0$, we can close a roll, decreasing the sum-of-squares by $2N(l_i) - 1$, since $N(l_i)$ decreases by one unit. Case 2: we can open a new roll, $N(L - l_i)$ increases by 1 and the sum-of-squares increases by $2N(L - l_i) + 1$. Or, case 3: we add the piece to a roll with gap g , $l_i < g < L$, in which case the best choice is g , such that $N(g) > 0$, that maximizes $N(g) - N(g - l_i)$. In the third case, the sum-of-squares decreases by $2(N(g) - N(g - l_i)) - 2$. The first two cases are easy; the third case needs something better than exhaustive search to achieve a reasonable algorithm for cutting stock problems.

A binary max-heap is a tree data structure that guarantees insertion, deletion and finding the maximum value in logarithmic time. This data structure can be seen as binary tree with two additional constraints: the tree is a complete binary tree (i.e., all levels of the tree, except possibly the last one, are fully filled); if B is a child node of A, then the key of A is greater than or equal to the key of B. The second property implies that the element with the greatest key is always in the root node. This data structure is usually implemented using an array where the left and right children of the node at position i are at positions $2i$ and $2i + 1$, respectively. There are two basic operations in this data structure: up-heap and down-heap. The up-heap/down-heap operations push a node up/down the tree until the heap-property is satisfied. The up-heap operation works as follows: compare the node with its parent; if they do not violate the heap property, stop; If not, swap the node with its parent and repeat the previous step. The down-heap operation is similar: if the node and its two direct children are not violating the heap property, stop; If not, swap the node with its greater child and repeat the previous step. For more details about this data structure see, e.g., Cormen et al. (2001).

The main idea to improve the run time of this algorithm in cutting stock problems is to keep the gaps, such that $N(g) > 0$, sorted in decreasing lexicographical order

by the $(N(g) - N(g - l_i), -g)$ value, since in case of a tie we want the smallest gap. Defining triplets $(N(g) > 0, N(g) - N(g - l_i), -g)$ for each gap, $\operatorname{argmax}_{l_i < g < L} \{(N(g) > 0, N(g) - N(g - l_i), -g)\}$ solves the third case. By storing these triplets in a max-heap, the third case can be solved in constant time. However, we need to add an auxiliary data structure and an operation to locate nodes in constant time and to update their value, which may increase or decrease. Figure 2.8 shows a small max-heap applied to the sum-of-squares decreasing algorithm in a situation where we have rolls with length $L = 20$, we are inserting pieces of length $l_i = 5$, and currently $N = \{1: 2, 2: 3, 3: 0, 4: 0, 5: 3, 6: 1, 7: 1, 8: 0, 9: 0, 10: 3, 11: 1, 12: 4, 13: 3, 14: 1, 15: 3, 16: 0, 17: 4, 18: 2, 19: 2\}$.

Figure 2.8: Max heap example.



This tree is a max heap applied to the sum-of-squares decreasing algorithm in a situation where we have rolls with length $L = 20$, we are inserting pieces of length $l_i = 5$, and currently $N = \{1: 2, 2: 3, 3: 0, 4: 0, 5: 3, 6: 1, 7: 1, 8: 0, 9: 0, 10: 3, 11: 1, 12: 4, 13: 3, 14: 1, 15: 3, 16: 0, 17: 4, 18: 2, 19: 2\}$. The number inside each node is the key value and the number above is the index in the in array representation. We have one key for every gap between $l_i + 1$ and $L - 1$. For each gap g the key is $(N(g) > 0, N(g) - N(g - l_i), -g)$. Since we want to find a roll with gap g that maximizes $N(g) - N(g - l_i)$ and that, in case of a tie, has smallest gap, $\operatorname{argmax}_{l_i < g < L} \{(N(g) > 0, N(g) - N(g - l_i), -g)\}$ gives exactly what we are looking for.

Algorithm 11 shows the main modifications we made in a standard binary max-heap implementation in order to implement the operations we needed for Algorithm 12. Our keys will be the triplets $(N(g) > 0, N(g) - N(g - l_i), -g)$, for $l_i < g < L$. Since the third element of the key is $-g$, it is used to identify to which gap the node is associated with. The third element of a triplet is assessed using the function `third`. The `up-heap` and `down-heap` operations must use the `swap` operation to swap the nodes in order to keep track of the position of the node associated with each gap. The `buildSSHeap` procedure initializes the heap data structure in linear time (see, e.g., Cormen et al. 2001). The `top` operation returns the root of the heap or the triplet `(False, NIL, NIL)` as an indication that the heap is empty. Finally, the `update` operation updates the node associated with a given gap g and restores the heap property using the `up-heap/down-heap` operations.

Algorithm 11: Main modifications in the max-heap data structure

```

1 function buildSSHeap(lst):                                     // initialize the heap with a list of triplets
2   size  $\leftarrow$  len(lst);
3   for  $i \leftarrow 1$  to size do                                // insert the triplets in the array
4     key[i]  $\leftarrow$  lst[i];
5     pos[-third(key[i])]  $\leftarrow i$ ;                          // record where is the triplet associated with gap  $g$  is stored
6   for  $i \leftarrow$  size/2 to 1 do down-heap( $i$ ) ;              // build the heap in linear time
7 function swap( $i, j$ ):                                          // used by up-heap and down-heap to swap nodes
8    $g_i \leftarrow$  -third(key[i]);  $g_j \leftarrow$  -third(key[j]); // the third element of the key is  $-g$ 
9   pos[ $g_i$ ]  $\leftarrow j$ ; pos[ $g_j$ ]  $\leftarrow i$ ;
10  key[i], key[j] = key[j], key[i];
11 function update( $g, value$ ):                                   // updates the value of the node associated with gap  $g$ 
12    $p \leftarrow$  pos[ $g$ ];                                       // locates the key associated with gap  $g$ 
13   cur  $\leftarrow$  key[ $p$ ];
14   key[ $p$ ]  $\leftarrow$  value;
15   if value > cur then up-heap( $p$ );                            // moves the node up/down
16   else down-heap( $p$ );
17 top top():                                                  // returns the top of the heap
18   if size  $\geq 1$  then return key[1];
19   else return (False, NIL, NIL);

```

Algorithm 12 shows how to compute SSD solutions in $\mathcal{O}(\max(mL, n \log L))$ time. In this algorithm, for each item of length l_i , we test which case leads to a smaller updated sum-of-squares. In lines 10 and 12, we test the first two cases. In line 15, we test the third case by assessing the top of the heap. In line 17, we check if the best option according the sum-of-squares criterion is to open a new roll. In this case, we need to update $N(L - l_i)$, which will increase and hence the corresponding node in the heap needs to be updated. Note that the only node that is affected is the one associated with gap $L - l_i$, no other node depends on this value. If the best option is to insert the item in an existing roll (line 24) with gap g , we need to update $N(g)$, which will decrease, and $N(g - l_i)$, which will increase. Since the node associated with gap $g + l_i$ also depends on the $N(g)$ value, we may also need to update the node associated with this gap too. This leads to at most three key updates in the heap.

Algorithm 12 computes SSD solutions in $\mathcal{O}(\max(mL, n \log L))$ time. For each different length, we need to build a heap, which takes $\mathcal{O}(L)$ time. For each item, we need to update the heap at most 3 times, and this can be performed in $\mathcal{O}(\log L)$ time. The same idea can also be used in the on-line version of this problem; however, we may need to keep m heaps updated, which leads to the complexity $\mathcal{O}(\max(mL, nm \log L))$.

Algorithm 12: Sum-of-Squares Decreasing Algorithm

input : m - number of different lengths; l - set of lengths; b - demand for each length; L - roll length

output: R - number of rolls needed; Sol - list of assignments

```

1 function SSD( $m, l, b, L$ ):
2    $l \leftarrow \text{reverse}(\text{sort}(l));$                                      // sort lengths in decreasing order
3    $R \leftarrow 0;$ 
4    $Sol \leftarrow [ ];$ 
5    $N[g] \leftarrow 0, \text{for } 1 \leq g < L;$ 
6    $\text{Buckets}[g] \leftarrow [ ], \text{for } 0 \leq g < L;$ 
7   for  $i \leftarrow 1$  to  $m$  do                                         // for each length
8      $\text{buildSSHeap}([(N[g] > 0, N[g] - N[g - l_i], -g) \mid l_i < g < L]);$ 
9     for  $j \leftarrow 1$  to  $b_{l_i}$  do                                     // for each piece of length  $l_i$ 
10      if  $N[l_i] > 0$  then                                           // Case 1: close
11         $\text{inc} \leftarrow -2N[l_i] + 1; \text{best} \leftarrow l_i;$ 
12      else                                                         // Case 2: open
13         $\text{inc} \leftarrow 2N[L - l_i] + 1; \text{best} \leftarrow L;$ 
14         $(\text{nonzero}, d, -g) \leftarrow \text{top}();$ 
15        if  $\text{nonzero}$  and  $-2d + 2 < \text{inc}$  then                       // Case 3: add
16           $\text{inc} \leftarrow -2d + 2; \text{best} \leftarrow g;$ 
17        if  $\text{best} = L$  then                                           // open a new roll
18           $g \leftarrow L - l_i;$ 
19          if  $g > l_i$  then
20             $\text{update}(g, (1, N[g] + 1 - N[g - l_i], -g));$ 
21             $N[g] \leftarrow N[g] + 1; R \leftarrow R + 1;$ 
22             $\text{Buckets}[g].\text{append}(R);$ 
23             $Sol.\text{append}(l_i \rightarrow \text{roll } R);$ 
24          else                                                         // insert in an existing roll
25             $g \leftarrow \text{best};$ 
26             $j \leftarrow \text{Buckets}[g].\text{pop}();$ 
27             $\text{Buckets}[g - l_i].\text{append}(j);$ 
28            if  $g - l_i > l_i$  then
29               $\text{update}(g - l_i, (1, N[g - l_i] + 1 - N[g - 2l_i], -(g - l_i)));$ 
30            if  $g > l_i$  then
31               $\text{update}(g, (N[g] - 1 > 0, N[g] - 2 - N[g - l_i], -g));$ 
32            if  $l_i < g + l_i < L$  then
33               $\text{update}(g + l_i, (N[g + l_i] > 0, N[g + l_i] + 1 - N[g], -(g + l_i)));$ 
34             $N[g] \leftarrow N[g] - 1;$ 
35             $N[g - l_i] \leftarrow N[g - l_i] + 1;$ 
36             $Sol.\text{append}(l_i \rightarrow \text{roll } j);$ 
37   return ( $R, Sol$ );

```

2.7 Computational results

All the algorithms were implemented in `Python` 2.6.1. In the following tables, we present average results over 20 instances for each class u120, u250, ..., t501 of OR-LIBRARY (2012)'s data set. These instances were introduced by Falkenauer (1996) for BPP and are divided in two classes. Uniform classes are composed by randomly generated instances and triplets classes are harder problems in which the optimal solution is composed of rolls completely filled with exactly three pieces. Table 2.1 presents the meaning of each column in subsequent tables. In the Table 2.2, we present the results using these instances with the original demand values. For these instances, the FFD solution can be obtained quickly using both assignment-based and pattern-based algorithms. In the Table 2.3, we present the results using the same piece lengths, but the demand for each length was multiplied by one million. In this case, the FFD/BFD solutions could only be obtained quickly (much less than one second in a standard desktop machine) using pattern-based algorithms, since the number of pieces is too large to use any conventional approach. Even a $\mathcal{O}(n \log n)$ algorithm is too slow since there are instances with one thousand million pieces. Finally, in Table 2.4 we present the results obtained using the SSD algorithm. To compute the SSD solutions it took many hours, since it is not a pattern based algorithm; it would take approximately the same time to compute FFD/BFD solutions using conventional algorithms. The exact solutions were obtained using the method described in the next chapter.

In our experiments, FFD and BFD usually lead to solutions with approximately the same number of rolls. When this does not happen, the BFD is usually better. The SSD heuristics presents good results, but, in our experiments, they are not as good as the ones obtained using the well known FFD/BFD heuristics.

Table 2.1: Meaning of the data displayed in subsequent tables.

Label	Description
L	roll length
n	number of pieces
m	number of different piece lengths
R^*	optimum number of rolls
$R^{\text{FFD}}, R^{\text{BFD}}, R^{\text{SSD}}$	number of rolls in the FFD/BFD/SSD solution
$\%g^{\text{FFD}}, \%g^{\text{BFD}}, \%g^{\text{SSD}}$	relative gap $(R - R^*)/R^*$
$\#p^{\text{FFD}}, \#p^{\text{BFD}}$	number of different patterns in the FFD/BFD solution

Table 2.2: Bin packing results.

class	L	n	m	R^*	R^{FFD}	$\%g^{\text{FFD}}$	$\#p^{\text{FFD}}$	R^{BFD}	$\%g^{\text{BFD}}$	$\#p^{\text{BFD}}$
u120	150	120	63.20	49.05	49.75	1.4	40.95	49.75	1.4	40.90
u250	150	250	77.20	101.60	103.10	1.5	62.20	103.10	1.5	62.20
u500	150	500	80.80	201.20	203.90	1.3	80.60	203.90	1.3	80.70
u1000	150	1,000	81.00	400.55	405.40	1.2	95.25	405.40	1.2	95.25
t60	1,000	60	50.00	20.00	23.20	16.0	23.20	23.20	16.0	23.20
t120	1,000	120	86.20	40.00	45.80	14.5	45.80	45.80	14.5	45.80
t249	1,000	249	140.10	83.00	95.00	14.5	93.30	95.00	14.5	93.30
t501	1,000	501	194.20	167.00	190.05	13.8	171.10	190.05	13.8	171.10

Table 2.3: Cutting stock results.

class	n	R^*	R^{FFD}	$\%g^{\text{FFD}}$	$\#p^{\text{FFD}}$	R^{BFD}	$\%g^{\text{BFD}}$	$\#p^{\text{BFD}}$
u120	120,000,000	48,496,486.10	49,369,514.25	1.8	66.10	49,369,514.25	1.8	66.75
u250	250,000,000	101,089,303.40	102,636,835.85	1.5	88.55	102,636,835.85	1.5	88.85
u500	500,000,000	200,636,290.20	203,496,389.45	1.4	94.90	203,496,389.45	1.4	94.90
u1000	1,000,000,000	400,006,333.65	404,912,242.50	1.2	96.30	404,912,242.50	1.2	96.30
t60	60,000,000	20,000,000.00	22,783,333.75	13.9	65.95	22,783,333.75	13.9	65.95
t120	120,000,000	40,000,000.00	45,410,417.10	13.5	111.85	45,410,417.10	13.5	111.85
t249	249,000,000	83,000,000.00	94,579,166.95	14.0	176.35	94,579,166.95	14.0	176.35
t501	501,000,000	167,000,000.00	189,714,583.65	13.6	239.40	189,714,583.65	13.6	239.40

Table 2.4: Sum-of-squares decreasing results.

class	n	R^{SSD}	$\%g^{\text{SSD}}$	class	n	R^{SSD}	$\%g^{\text{SSD}}$
u120	120	50.75	3.5	u120	120,000,000	50,856,623.90	4.9
u250	250	104.70	3.0	u250	250,000,000	104,980,602.95	3.9
u500	500	206.35	2.6	u500	500,000,000	207,139,880.30	3.2
u1000	1,000	410.80	2.6	u1000	1,000,000,000	411,773,546.25	2.9
t60	60	23.20	16.0	t60	60,000,000	22,972,797.10	14.9
t120	120	45.85	14.6	t120	120,000,000	45,758,198.85	14.4
t249	249	95.10	14.6	t249	249,000,000	95,337,207.75	14.9
t501	501	190.65	14.2	t501	501,000,000	191,389,790.95	14.6

2.8 Conclusions

We presented pattern-based first and best fit decreasing (FFD/BFD) algorithms for cutting stock problems (CSP) that overcome the limitations of assignment-based algorithms. Conventional assignment-based algorithms are not polynomial in the CSP input size in its most common format (i.e., items of same size grouped into orders with a required level of demand). Therefore, even for small CSP instances with large demands it is difficult to compute FFD/BFD solutions. In CSP, n pieces of m different lengths must be cut from rolls with length L . The pattern-based algorithms presented do not make assignments piece-by-piece to rolls and, therefore, they can be faster than linear in the number of items. The efficient pattern-based FFD/BFD implementations run in polynomial time in the cutting stock input size. These algorithms are as fast as the best known FFD/BFD implementations in BPP instances and they are much

faster than any conventional assignment-based algorithm in CSP instances.

We also presented a reasonably efficient algorithm for the sum-of-squares decreasing (SSD) algorithm, an off-line variant of the on-line sum-of-squares heuristics. The proposed algorithm is not polynomial in the cutting stock input size, since it is not a pattern-based algorithm. Nevertheless, it is enough to solve CSP instances up to a few millions of items in a reasonable amount of time. Our algorithm can be adapted to the on-line sum-of-squares heuristics and it can be faster than the original algorithm when L is very large and m is reasonably small. In our experiments, the SSD heuristics presents good results, but not as good as the results obtained using the well known FFD/BFD heuristics.

Chapter 3

Exact Methods

Valério de Carvalho (1999) proposed an arc flow formulation with side constraints for the bin packing problem (BPP). In this chapter we develop his idea and extend it to some variants of the bin packing problem. In Section 3.1, we describe Valério de Carvalho's method. In Section 3.2, we show how we improved and extended his formulation to some variants of the bin packing problem. Graph construction and solution extraction algorithms are presented in Sections 3.3 and 3.4, respectively. In Section 3.5, we show how to break symmetry completely using the new model. A graph compression method that usually reduces dramatically the graph size is presented in Section 3.6. Section 3.7 shows some results for the integrality gap of Gilmore and Gomory's model that are valid for the arc flow formulations that we present in this chapter, and some rounding properties that usually guarantee very good heuristic solutions. The results obtained with many benchmark data sets are summarized in Section 3.8. In Section 3.9, we present a run time analysis. Finally, Section 3.10 presents the conclusions.

3.1 Valério de Carvalho's arc flow formulation

Consider a bin packing instance with bins of capacity W and items of sizes w_1, w_2, \dots, w_m with demands b_1, b_2, \dots, b_m , respectively. The problem of determining a valid solution to a single bin can be modeled as the problem of finding a path in an acyclic directed graph $G = (V, A)$ with $V = \{0, 1, 2, \dots, W\}$ and $A = \{(i, j) \mid j - i = w_d, \text{ for } 1 \leq d \leq m \text{ and } 0 \leq i < j \leq W\}$, meaning that there exists an arc between two vertices i and $j > i$ if there is an item of size $w_d = j - i$. The number of vertices and arcs are bounded

by $\mathcal{O}(W)$ and $\mathcal{O}(mW)$, respectively. Additional arcs $(k, k+1)$, for $k = 0, \dots, W-1$, are included for representing unoccupied portions of the bin.

Property 7 (Flow formulation for a packing) *There is a packing of the n' items in a single bin if and only if there is a path between vertices 0 and W corresponding to the packing pattern. The length of arcs that constitute the path (excepting loss) define the item sizes to be packed.*

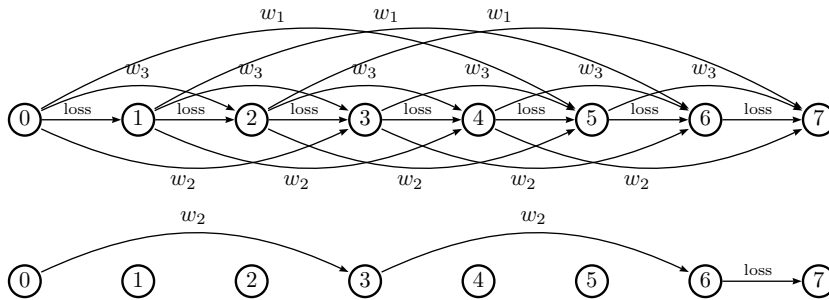
Proof

(\Rightarrow) Let $(a_1, a_2)(a_2, a_3), \dots, (a_{n'-1}, a_{n'})$ be a path where each a_i is a vertex. If there is a packing of the n' items in a single bin then, by construction, the graph will contain a path from 0 to W starting with $(0, a_1)(a_1, a_2) \dots (a_{n'-1}, a_{n'})$ and corresponding to the packing $(a_1, a_2 - a_1, \dots, a_{n'} - a_{n'-1})$.

(\Leftarrow) If there is a path $(0, a_1)(a_1, a_2) \dots (a_{n-1}, W)$ between vertices 0 and W with n arcs (with $n' \leq n$ arcs corresponding to items and $n - n'$ loss arcs), it is always possible to obtain the corresponding packing pattern from the differences $j - i$ from each arc (i, j) in the path (excluding loss). \square

Example 1 *Figure 3.1 shows the graph associated with an instance with bins of capacity $W = 7$ and items of sizes 5, 3, 2 with demands 3, 1, 2, respectively; the path shown below corresponds to a bin with two items of size $w_2 = 3$ and one unit of loss.*

Figure 3.1: Arc flow graph associated with Example 1.



Graph associated with an instance with bins of capacity $W = 7$ and items of sizes 5, 3 and 2 (top) and a path between vertices 0 and W that corresponds to a possible packing (bottom).

This kind of formulation has been used initially by Shapiro (1968) to model the knapsack problem as that of determining the longest path in a directed graph. The

same idea can be used to model bin packing problems. A solution to a single bin corresponds to a flow of one unit between vertices 0 and W . A path carrying a larger flow will correspond to using the same packing solution in multiple bins. Different paths correspond to different packing patterns.

The bin packing problem is thus equivalently formulated as that of determining the minimum flow between vertex 0 and vertex W , with additional constraints enforcing the sum of the flows in the arcs of each order to be greater than or equal to the corresponding demand. Consider decision variables x_{ij} (associated with the arcs defined above) corresponding to the number of items of size $j - i$ placed in any bin at a distance of i units from the beginning of the bin. A variable z , representing the number of bins required, aggregates the flow in the graph, and it can be seen as a feedback arc from vertex W to vertex 0. The model is as follows:

$$\text{minimize} \quad z \tag{3.1}$$

$$\text{subject to} \quad \sum_{(i,j) \in A} x_{ij} - \sum_{(j,k) \in A} x_{jk} = \begin{cases} -z & \text{if } j = 0, \\ z & \text{if } j = W, \\ 0 & \text{for } j = 1, \dots, W-1, \end{cases} \tag{3.2}$$

$$\sum_{(k,k+w_i) \in A} x_{k,k+w_i} \geq b_i, \quad i = 1, \dots, m, \tag{3.3}$$

$$x_{ij} \geq 0, \text{ integer}, \quad \forall (i, j) \in A. \tag{3.4}$$

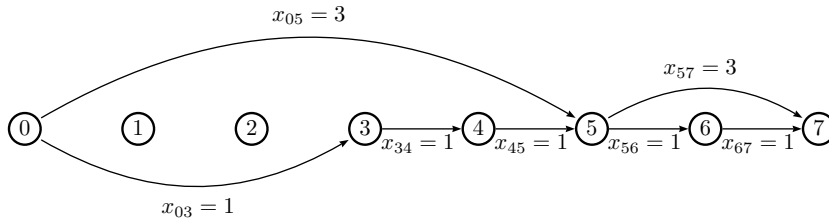
Property 8 (equivalence to the classical Gilmore-Gomory) *This model is equivalent to the classical Gilmore-Gomory model (1.11)-(1.14) for the cutting stock problem and hence the linear relaxation bounds are identical.*

Proof Valério de Carvalho (1999) proved that this formulation is equivalent to the classical Gilmore-Gomory model (1.11)-(1.14) by applying Dantzig-Wolfe decomposition to model (3.1)-(3.4), keeping (3.1) and (3.3) in the master problem, and (3.2) and (3.4) in the subproblem. As the subproblem is a flow problem that will only generate valid patterns, we can substitute (3.2) and (3.4) by patterns and obtain the classical model. From this equivalence follows that lower bounds provided by both models are the same. \square

Any feasible solution to model (3.1)-(3.4) can be transformed into a feasible solution to BPP using z bins. By the flow decomposition properties (see, e.g., Ahuja et al. 1993), non-negative flows can be represented by paths and cycles. Since the graph

G is acyclic, any flow can be decomposed into directed paths connecting the only excess node (node 0) to the only deficit node (node W); see Section 3.2 for more details. The graph in Figure 3.2 shows an optimal solution for Example 1. Let $(a_1, a_2)(a_2, a_3), \dots, (a_n, a_{n+1})$ be a path of length n where each a_i is a vertex. The optimal solution for BPP can be obtained by decomposing the flow into four paths: three paths $(0, 5)(5, 7)$ corresponding to three bins with one item of size 5 and one of size 2; and one path $(0, 3)(3, 4)(4, 5)(5, 6)(6, 7)$ corresponding to one bin with one item of size 3.

Figure 3.2: Optimal solution for Example 1.



The optimal BPP solution for Example 1 can be obtained by decomposing the flow into four paths: three paths $(0, 5)(5, 7)$ corresponding to three bins with one item of size 5 and one of size 2; and one path $(0, 3)(3, 4)(4, 5)(5, 6)(6, 7)$ corresponding to one bin with one item of size 3.

3.1.1 Symmetry reduction

In order to reduce the symmetry of the solution space and the size of the model, Valério de Carvalho introduced some rules. The idea is to consider only a subset of arcs from A . If we search for a solution in which the items are ordered by decreasing values of width, the following criteria may be used to reduce the number of arcs that are taken into account.

Criterion 1 *An arc $(k, k + w_e)$ of size w_e can only leave a node $k > 0$ if there is another arc $(k - w_d, k)$ of size $w_d \geq w_e$ entering k ; any arc can leave node $k = 0$.*

Criterion 2 *All the loss arcs $(k, k + 1)$ can be removed for $k < w_m$ (recall that w_m is the smallest item).*

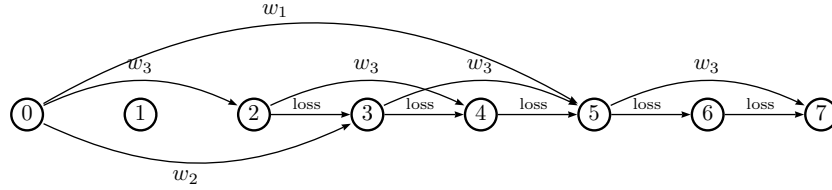
Criterion 3 *Given any node k that is the head of an arc of size w_d or $k = 0$, the only valid arcs for size w_e ($w_e < w_d$) are those that start at nodes $k + sw_e$, for $s = 0, 1, 2, \dots, b_e - 1$, with $k + (s + 1)w_e \leq W$, where b_e is the demand of items of size w_e .*

Property 9 *Valério de Carvalho's symmetry reduction rules do not exclude any valid packing pattern.*

Proof Criterion 1 just tries to impose an order on the placement of a bin's items according to their size; Criterion 2 only imposes that a bin never starts with loss; and Criterion 3 just tries to limit the number of consecutive arcs of a certain size by the demand. \square

The graph in Figure 3.3 (with 11 arcs) results from applying these criteria to the graph in Figure 3.1 (with 21 arcs). Note that this set of rules may not break completely the symmetry of the solution space. For instance, in the graph of Figure 3.3 the paths $(0, 3)(3, 5)(5, 6)(6, 7)$ and $(0, 3)(3, 4)(4, 5)(5, 7)$ correspond to the same pattern (one item of size w_2 and one of size w_3).

Figure 3.3: Graph corresponding to Example 1 after applying symmetry reduction.



Valério de Carvalho (1999) developed a branch-and-price procedure that combines column generation and branch-and-bound. At each iteration, the subproblem generates a set of variables, which altogether correspond to an attractive valid packing for a single bin.

As we will see in Section 3.8, thanks to symmetry reduction the memory required to hold the entire graph, even for reasonably large instances, is less than a few gigabytes. This eliminates the requirement of using column generation as has been done in the original paper and, more importantly, opens the possibility of using general-purpose mixed-integer programming solvers to tackle this problem directly. We used **Gurobi** (Gu et al. (2011)) to solve every instance of Falkenauer (1996) using this model. The average run time was less than 2 seconds. Using the same solver and Martello and Toth's model, we were able to solve only 7 out of the 160 instances within a 10 minute time limit (even allowing **Gurobi** to use its powerful heuristics).

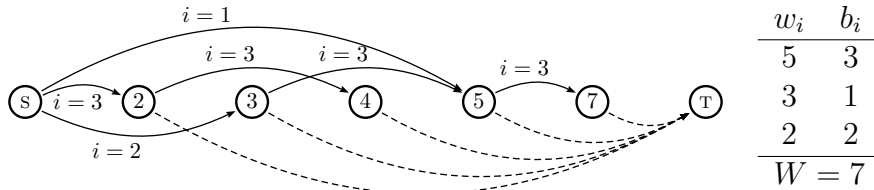
Valério de Carvalho's model proved to be very efficient for bin packing and cutting stock problems. The main ideas behind his formulation described in this section are the starting point for our methods.

3.2 A new arc flow formulation

It would be interesting to use Valério de Carvalho's model for cardinality constrained bin packing, or, more generally, for two-constraint bin packing. However, his model does not allow us to control the number of items we assign to each bin; in this section, we propose a generalization overcoming this issue.

Figure 3.4 shows an alternative graph to model the bin packing instance from Example 1 (Section 3.1). Item weights are sorted in decreasing order. In this graph, an arc (u, v, i) corresponds to an arc between nodes u and v associated with the i -th weight. Note that, for each pair of nodes (u, v) , multiple arcs associated with different items are allowed. The dashed arcs are the loss arcs that connect every node (except the source (s)) to the target (t). Since the loss arcs connect the nodes directly to the target (instead of connecting consecutive nodes) we do not always need to have a node for every integer value less than or equal to the capacity. Note that each path between s and t in this graph corresponds to a valid packing pattern for the Example 1 and all the patterns are represented in this graph.

Figure 3.4: Another possible graph for a bin packing instance.



In this graph, $V = \{s, 2, 3, 4, 5, 7, t\}$ is the set of vertices, $A = \{(s, 5, 1), (s, 3, 2), (s, 2, 3), (2, 4, 3), (3, 5, 3), (5, 7, 3), (2, t, 0), (3, t, 0), (4, t, 0), (5, t, 0), (7, t, 0)\}$ is the set of arcs. Item weights are sorted in decreasing order. An arc (u, v, i) corresponds to an arc between nodes u and v associated with the i -th weight. The dashed arcs are loss arcs that connect every node (except the source (s)) to the target (t). Note that each path between s and t in this graph corresponds to a valid packing pattern for the Example 1 and all the valid patterns are represented in this graph. In this graph, a node label w' means that every sub-pattern from the source to the node has at most weight w' .

We propose a generalization of Valério de Carvalho's model, still based on representing the packing patterns by means of flow in a graph, though we assign a more general

meaning to vertices and arcs. The formulation is the following:

$$\text{minimize} \quad z \quad (3.5)$$

$$\text{subject to} \quad \sum_{(u,v,i) \in A} f_{uvi} - \sum_{(v,u',i) \in A} f_{vu'i} = \begin{cases} -z & \text{if } v = \text{S}, \\ z & \text{if } v = \text{T}, \\ 0 & \text{for } v \in V \setminus \{\text{S}, \text{T}\}, \end{cases} \quad (3.6)$$

$$\sum_{(u,v,i) \in A} f_{uvi} \geq b_i, \quad i \in \{1, \dots, m\} \setminus J, \quad (3.7)$$

$$\sum_{(u,v,i) \in A} f_{uvi} = b_i, \quad i \in J, \quad (3.8)$$

$$f_{uvi} \leq b_i, \quad \forall (u, v, i) \in A, \text{ if } i \neq 0, \quad (3.9)$$

$$f_{uvi} \geq 0, \text{ integer}, \quad \forall (u, v, i) \in A, \quad (3.10)$$

where m is the number of different items, b_i is the demand of the i -th item, V is the set of vertices, S is the source vertex and T is the target; A is the set of arcs, where each arc has three components (u, v, i) corresponding to an arc between nodes u and v that contributes to the demand of the i -th item; arcs $(u, v, i = 0)$ are the loss arcs; f_{uvi} is the amount of flow along the arc (u, v, i) ; and $J \subseteq \{1, \dots, m\}$ is a subset of items whose demands are required to be satisfied exactly. In our experiments, $J = \{i = 1, \dots, m \mid b_i = 1\}$.

In Valério de Carvalho's model, a variable x_{ij} contributes to an item with weight $j - i$. In our model, a variable f_{uvi} contributes to an item with the i -th weight; the label of u and v may have no direct relation to the item weight. Moreover, it is possible to have more than one arc (associated with different items) between two vertices. This new model is more general; Valério de Carvalho's model is a sub-case, where an arc between nodes u and v can only contribute to the demand of an item of weight $v - u$. As in Valério de Carvalho's model, each arc can only contribute to an item, but the new model has several differences with respect to the original formulation:

- nodes are more general (e.g., they can encompass two-dimensions);
- there may be more than one arc between two vertices (multigraph);
- demands in general may be satisfied with excess but for some items they are required to be satisfied exactly (this allows us, for example, to take advantage of multiple-choice constraints when requiring the demands of items with demand one to be satisfied exactly);

- arcs have upper bounds equal to the total demand of the associated item (which allows excluding many feasible solutions that would exceed the demand);
- arc lengths may be unrelated to the corresponding item weight (i.e., $(u, v, i) \in A$ even if $v - u \neq w_i$).

More details on algorithms for graph construction are given in Section 3.3. Using this model it is possible to use more general graphs, but we always need to ensure that it is a directed acyclic graph whose paths between S and T correspond to every valid packing pattern of the original problem. Since **Gurobi** works very well with Valério de Carvalho's model, it is expected that it will also work very well with this model and graphs similar to the one in Figure 3.4. Note that by requiring the demands of some items to be satisfied exactly and by introducing upper bounds on variable values, the problem may sometimes become harder to solve. For instance, if the demand of a very small item is required to be satisfied exactly, many optimal solutions will probably be excluded unless the optimal solution has waste smaller than the item size. In some instances, results can be obtained more quickly by choosing carefully the variable upper bounds and the set of items whose demand must be satisfied exactly. Nevertheless, our choices regarding these two aspects overall proved to work very well, as we show in Section 3.8.

Property 10 (equivalence to the classical Gilmore-Gomory) *For a graph with all valid packing patterns represented as paths from S to T , model (3.5)-(3.10) is equivalent to the classical Gilmore-Gomory model (1.11)-(1.14) with the same patterns as the ones obtained from paths between S and T in the graph.*

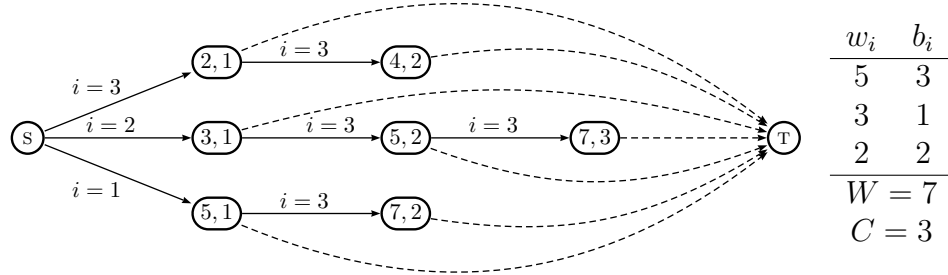
Proof Extending Valério de Carvalho's proof, we apply Dantzig-Wolfe decomposition to model (3.5)-(3.10) keeping (3.5), (3.7) and (3.8) in the master problem and (3.6), (3.9) and (3.10) in the subproblem. As the subproblem is a flow model that will only generate patterns resulting from paths in graph, we can substitute (3.6), (3.9) and (3.10) by the patterns and obtain the classical model. From this equivalence follows that lower bounds provided by both models are the same when the same set of patterns is considered. The equality constraints (3.8) and the upper bound on variable values (3.9) have no effect on the lower bounds since for every optimal solution with some excess there is a solution with the same objective value that satisfies the demands exactly; this solution can be obtained by replacing the use of some patterns by other patterns that do not include the items whose demands are being satisfied with excess (recall that every valid packing pattern is represented in the graph). \square

The new type of graph allows us to have less nodes in instances with very large bin capacities. We used **Gurobi** to solve every instance provided in Falkenauer (1996) using this type of graph, and the average run time was less than 2 seconds. This new graph also has the advantage of reducing symmetry in some situations.

3.2.1 Cardinality constrained bin packing

If there is a constraint limiting the number of items that can be placed in a bin, not every path in the graphs from the previous sections are feasible. For example, if we have a 2-item cardinality limit, we need to exclude paths containing three or more items. We could add constraints for excluding paths that violate cardinality, but this would make the problem much harder to solve. An alternative idea is to modify the graph in order to include cardinality information in the nodes, as in Figure 3.5; here, if we want to limit the cardinality to 2, we just need to remove the node $(7, 3)$ and the incident arcs.

Figure 3.5: Graph associated with Example 1, but now with cardinality limit 3.



In this graph, a node label (w', c') means that any path from the source will reach the node with at most c' items whose sum of weights is at most w' . Let w be the list of different item weights sorted in decreasing order, and w_i be the i -th element of w . We connect a node (w', c') to a node $(w' + w_i, c' + 1)$ using an arc $((w', c'), (w' + w_i, c' + 1), i)$.

Property 11 (Cardinality constrained bin packing bounds) *The lower bound provided by the linear relaxation of the arc flow formulation in the cardinality constrained BPP is at least $\max(z_{bp}^*, n/C)$ where C is the cardinality limit and z_{bp}^* is the lower bound obtained from the linear relaxation of the arc flow formulation for the standard BPP without cardinality constraint.*

Proof Let z^* be the lower bound of the cardinality constrained BPP. Suppose that $z^* < \max(z_{bp}^*, n/C)$. We have to consider two cases. If $z_{bp}^* \geq n/C$ then we have

a contradiction, since we can use the same solution for standard BPP (i.e., with no cardinality constraint), and by assumption $z^* < \max(z_{bp}^*, n/C) = z_{bp}^*$; but z_{bp}^* is optimal. If $n/C > z_{bp}^*$ we also have a contradiction, since by assumption $z^* < \max(z_{bp}^*, n/C) = n/C$; but we cannot have more than C items in each bin. \square

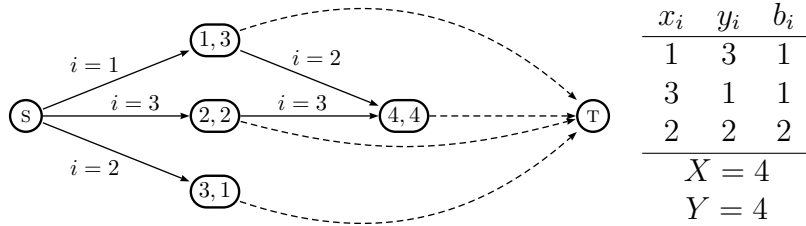
For testing this model, we have done the following experiment. For each instance i of Falkenauer (1996), we determined the maximum number of items C_i that fit into a single bin. In order to study the effect of cardinality even when it does not exclude any packing pattern, we then created instances with cardinality limits $C = C_i, C_i - 1, C_i - 2, \dots, 2$. As we show in Section 3.8, our model works very well in cardinality constrained BPP, for every cardinality.

3.2.2 Two-constraint bin packing

We can extend the idea we used for cardinality constrained BPP for the more general case of two-constraint BPP. For a given item i , there will be arcs connecting nodes (a', b') to nodes $(a' + x_i, b' + y_i)$, where x_i and y_i are the weight of the item in each dimension.

Consider an instance with bins of capacity $(X, Y) = (4, 4)$ and items of sizes $(x_1, y_1) = (1, 3)$, $(x_2, y_2) = (3, 1)$, $(x_3, y_3) = (2, 2)$ with demands 1, 1, 2, respectively. Figure 3.6 shows the corresponding graph.

Figure 3.6: Two-constraint bin packing example.



In this graph, a node label (a', b') means that every sub-pattern from the source to that node uses a' space in the first dimension and b' in the second. Item weights are sorted in decreasing order by the sum of the normalized weights (using decreasing lexicographical order in case of a tie). Let x_i and y_i be the weight of i -th item in the first and second dimensions, respectively. We connect a node (a', b') to a node $(a' + x_i, b' + y_i)$ using an arc $((a', b'), (a' + x_i, b' + y_i), i)$.

Property 12 (Two-constraint BPP bounds) *The lower bound provided by the linear relaxation of the arc flow formulation for two-constraint BPP is at least $\max(z_1^*, z_2^*)$*

where z_1^* and z_2^* are the lower bounds obtained from the linear relaxation using the arc flow formulation for standard BPP restricted only with one constraint concerning the the first and second dimensions, respectively.

Proof Let z^* be the lower bound for the two-constraint BPP. Every valid packing pattern for the two-constraint bin packing problem is also valid for the standard bin packing problems of each dimension. Therefore, when considering just one of the dimensions, the solution with objective value z^* is always feasible, and thus $z_1^* \leq z^*$ and $z_2^* \leq z^*$. \square

Note that the lower bound z^* in this problem can be much better than $\max(z_1^*, z_2^*)$. For example, consider an instance with bins of capacity $(X, Y) = (3, 3)$ and items of sizes $(1, 3)$, $(3, 1)$, $(2, 2)$ with demands 1, 1, 2, respectively. The lower bounds provided by z_1^* and z_2^* are equal to 3, while z^* is equal to 4.

In the cardinality constrained BPP, the number of nodes in the arc flow formulation is bounded by $\mathcal{O}(WC)$, where W is the bin capacity and C is the cardinality limit, and as C is usually very small the bound is fair. However, in the two-constraint bin packing problem, the number of nodes is bounded by $\mathcal{O}(XY)$ and both X, Y may be very large, and thus the graph may become too large.

Note that this idea can be generalized for the p -dimensional vector packing problem. In this problem there are p dimensions with capacity constraints. The generalization is simple, but depending on the number of dimensions and capacities, the size of the graph may be a problem.

3.3 Graph construction algorithms

In this section, we describe an algorithm to construct a graph respecting symmetry reduction criteria, which is applicable to both standard bin packing and the two-constraint variants.

For the sake of simplicity, we will consider that items will always have weights in two dimensions. We consider the weight in the second dimension as zero in standard bin packing problems and one in cardinality constrained bin packing problems. Item weights (with two dimensions) are sorted in decreasing order by the sum of normalized weights ($\alpha_i = (x_i/X + y_i/Y)$), using decreasing lexicographical order in case of a tie. We define x_i, y_i as the weight of the i -th item in each dimension and b_i as its demand.

All nodes are labeled with pairs (a', b') except the target (T). The source node (S) is labeled $(0, 0)$. We connect a node (a', b') to a node $(a' + x_i, b' + y_i)$ using an arc $((a', b'), (a' + x_i, b' + y_i), i)$ associated with the i -th item. In bin packing/cutting stock problems, there are arcs between nodes $(a', 0)$ and $(a' + w', 0)$ where w' is the weight of an item. In cardinality constrained BPP, there are arcs between nodes (a', b') and $(a' + w', b' + 1)$ since the weight in the second dimension is always one. Finally, in two-constraint bin packing, there are arcs between nodes (a', b') and $(a' + x', b' + y')$ where x' and y' are weights of some item in each dimension.

One of the symmetry reduction rules states that we can have an arc with tail in a node only if it is either the source node or the head of an arc associated with a larger item (according to the value of α_i). Our algorithm to construct the graph relies on this criterion. Initially, there is only the source node. For each item, we insert in the graph arcs associated with the item starting from previously existing nodes that include nodes created by being heads of arcs associated with previous items in the order. After processing an item, we add to the graph the set of nodes that appeared as heads of new arcs. Figure 3.7 shows a small example of the construction of a graph respecting symmetry reduction criteria using this method.

The graph construction process is summarized in Algorithm 13. This algorithm receives as input: m , the number of different item sizes; x_i and y_i , the weights of the i -th item in each dimension as described before; b_i , the demand of the i -th item; and, X and Y , the capacity limits of each dimension. In this algorithm, V and A are the current sets of vertices and arcs in the graph. This algorithm produces a graph that contains all the valid packing patterns represented by paths starting from S. We build the graph size by size (line 4) since for each size we can only have an arc starting from a node u if there is an arc with a previous size entering u . Initially, we just have the source node. After processing each size we add the new nodes to the set of vertices (in line 14). Using line 11 we avoid processing the same arc twice, since if its tail is already on the list of vertices, we have to process that node, with the current size, sooner or later. Therefore, we process each node at most m times and each arc once.

Using Algorithm 13, the graph can be constructed in pseudo-polynomial time $\mathcal{O}(|V|m)$; note that $|V|$ is limited by $\mathcal{O}(XY + X + Y)$, but usually is much smaller than its maximum possible value. For standard BPP and cardinality constrained BPP, it is easy to improve this algorithm to run in $\mathcal{O}(|V| + |A|)$ by indexing the nodes by the first item that fits. Nevertheless, Algorithm 13 is simple and $\mathcal{O}(|V|m)$ is almost as good as $\mathcal{O}(|V| + |A|)$ since in this type of graph $|A| \simeq |V|m$.

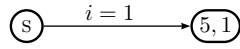
Figure 3.7: Graph construction example.

Consider a cardinality constrained bin packing instance with bins of capacity 7, cardinality limit 3, and items of sizes 5, 3, 2 with demands 3, 1, 2, respectively. This instance corresponds to adding cardinality limit 3 to Example 1.

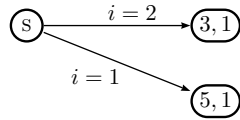
a) We start with a graph with only the source node:



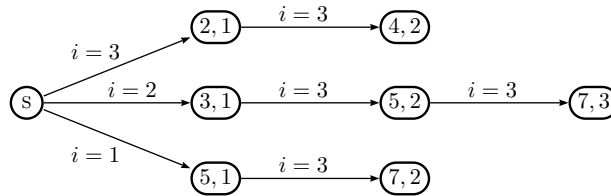
b) At the first iteration, we add arcs associated with the first item, which has weight 5. There is space only for one item of this size.



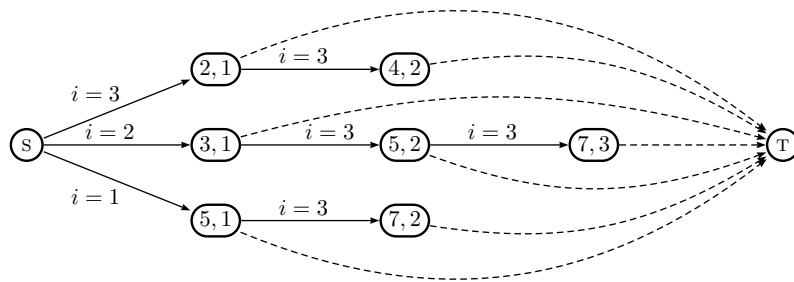
c) At the second iteration, we add arcs for the second item, which has weight 3. The demand does not allow us to form paths with more than 1 consecutive arc of this size.



d) At the third iteration, we add the arcs associated with the third item, which has weight 2. Since the demand of this item is 2, we can add paths (starting from previously existing nodes) with at most 2 items of size 2.



e) Finally, we add the loss arcs connecting each node, except the source, to the target. The resulting graph corresponds to the model to be solved by a general-purpose mixed-integer optimization solver.



Algorithm 13: Graph Construction Algorithm

input : m - number of different item sizes; x, y - vectors of weights in the first and second dimension, respectively; b - demand vector; X, Y - capacity limits of each dimension

output: V - set of vertices; A - set of arcs

```

1 function buildGraph( $m, x, y, b, X, Y$ ):
2    $V \leftarrow \{(0, 0)\};$                                      // the label of the source node (s) is (0, 0)
3    $A \leftarrow \{ \};$ 
4   for  $i = 1$  to  $m$  do                                     // for each size
5      $T \leftarrow \{ \};$ 
6     foreach  $(a', b') \in V$  do                               // for each node already in V
7       for  $k \leftarrow 1$  to  $b_i$  do
8          $(a'', b'') \leftarrow (a' + x_i, b' + y_i);$ 
9         if  $a'' > X$  or  $b'' > Y$  then break;                // check if it is valid node
10         $A \leftarrow A \cup \{(a', b'), (a'', b''), i\};$ 
11        if  $(a'', b'') \in V$  then break;                    // to avoid repeating work
12         $T \leftarrow T \cup \{(a'', b'')\};$ 
13         $(a', b') \leftarrow (a'', b'');$ 
14    $V \leftarrow V \cup T;$ 
15 return  $G = (V, A);$ 

```

Algorithm 14 receives the graph produced by Algorithm 13 and adds the loss arcs connecting each node, except the source, to the target. The resulting graph is the graph of the flow problem to be solved by a general-purpose mixed-integer optimization solver.

Algorithm 14: Add the final loss arcs

input : V - set of vertices; A - set of arcs

output: V - set of vertices including T ; A - set of arcs including the final loss arcs

```

1 function addLossArcs( $G = (V, A)$ ):
2   foreach  $v \in V$  do
3     if  $v \neq s$  then
4        $A \leftarrow A \cup \{(v, T, 0)\};$ 
5    $V \leftarrow V \cup \{T\};$ 
6 return  $G = (V, A);$ 

```

3.4 Solution extraction algorithms

After having the solution of the arc flow integer optimization model, we use a flow decomposition algorithm to obtain the bin packing solution.

Property 13 *Any integer solution to the arc flow model can be transformed into an integer solution to BPP.*

Proof Valério de Carvalho (1999) shows that any integer solution to his arc flow model can be transformed into an integer solution to BPP. His proof is also valid for our model. By the flow decomposition properties (see, e.g., Ahuja et al. 1993), non-negative flows can be represented by paths and cycles. Since we require an acyclic graph, any flow can be decomposed into directed paths connecting the only excess node (node S) to the only deficit node (node T). \square

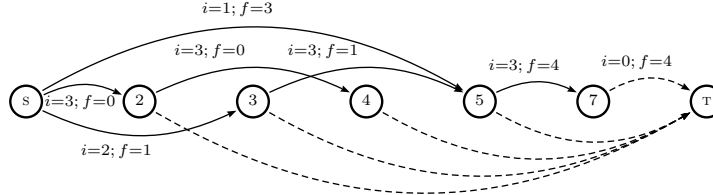
In Example 1, we have items of sizes 5, 3, 2 with demands 3, 1, 2, respectively. Figure 3.8 shows an arc flow solution for this example and how to obtain a bin packing solution from it. The bin packing solution uses 4 bins and it is optimal. The same method as the one used in the solution extraction example is used by Algorithm 15 to extract bin packing solutions from arc flow solutions.

Algorithm 15 extracts a bin packing solution from an arc flow solution. We use dynamic programming to find the path with largest flow; nodes are sorted in decreasing lexicographical order since it corresponds to a topological order of graphs created using Algorithm 13; dynamic programming allow us to compute for every node the largest flow (**dp**) to T and the next arc (**best**) in a path with such flow; this information allow us to rebuilt the path; then, we extract the pattern and reduce the flow along the arcs in the path. This process is repeated until the flow is zero. The output of this algorithm is formed by a list of pairs (m, p) where p is a pattern and m is its multiplicity.

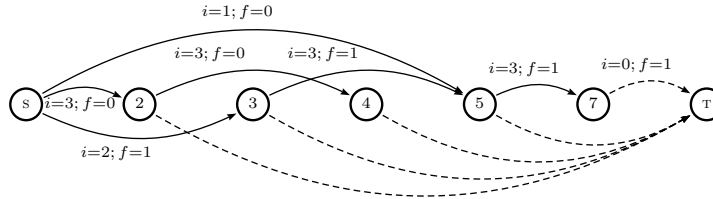
The bin packing solutions obtained from arc flow solutions may satisfy with excess the demand of some items since the model does not require the demand of every item to be satisfied exactly. In Figure 3.8, the bin packing solution is $[(3, [1, 3]), (1, [2, 3, 3])]$ and contains 3 items of the first size, 1 item of the second size and 5 items of the third size. This solution satisfies the demand of items of the third size with excess since the demand of items of this size is 2. However, it is always possible to obtain from a solution with some excess a solution that satisfies the demands exactly; the exact

Figure 3.8: Solution extraction example.

Consider the instance from Example 1. In this instance, we have bins of capacity 7 and items of sizes 5, 3, 2 with demands 3, 1, 2, respectively. The following graph shows an optimal solution for this instance:



a) The path with largest flow in the graph above is $(s, 5, 1)(5, 7, 3)(7, t, 0)$ with 3 units of flow. This path corresponds to the pattern $[1, 3]$ repeated 3 times. The pattern $[1, 3]$ corresponds to an item of the first size and another of the third size. By decreasing the flow along this path we obtain the following graph:



b) The path with largest flow in the graph above is $(s, 3, 2)(3, 5, 3)(5, 7, 3)(7, t, 0)$ with 1 unit of flow. This path corresponds to the pattern $[2, 3, 3]$. By decreasing the flow along this path we obtain a graph with zero units of flow between s and t . Therefore, there are no more patterns in the solution. The solution is $[(3, [1, 3]), (1, [2, 3, 3])]$ and corresponds to 3 bins with one item of size 5 and one of size 2, and 1 bin with one item of size 3 and two of size 2.

Note that the second node dimension was omitted in this example since it is always zero.

solution can be obtained by replacing some patterns by other patterns that do not include items whose demands are being satisfied with excess.

Algorithm 16 processes solutions from Algorithm 15 and creates a solution that satisfies the demand of every item exactly. In this algorithm, **div** denotes integer division, and $i \times c_i$ denotes the value i repeated c_i times on a list. For each pattern (line 3), we count the maximum number of times it can be used including all the items in it (considering its multiplicity and the demands already satisfied by previous patterns); we process the pattern, decrease the demands, and remove from the pattern the items that are not necessary anymore; we repeat this process while there remains multiplicity in the pattern. In Example 1, we have items of sizes 5, 3, 2 with demands 3, 1, 2, respectively. Consider the solution from Figure 3.8, which is $[(3, [1, 3]), (1, [2, 3, 3])]$. We start by processing the pattern $[1, 3]$, which has multiplicity 3. Since the demand of

Algorithm 15: Solution Extraction Algorithm**input** : V - set of vertices; A - set of arcs; **flow** - amount of flow in each arc**output**: L - list of patterns and multiplicities

```

1 function getPatterns( $G = (V, A)$ , flow):
2    $\text{adj}_u \leftarrow \{(v, i) \mid (u, v, i) \in A, \text{flow}[u, v, i] > 0\}$ , for all  $u \in V$ ; // adjacency list of the graph
3    $\text{lst} \leftarrow \text{reversed}(\text{sort}(V \setminus \{s, t\}))$ ; // lst is the list of vertices sorted in decreasing lexicographical order
4    $L \leftarrow []$ ; // L is the list of patterns
5   while True do
6      $\text{dp}[t] \leftarrow \infty$ ;  $\text{best}[t] \leftarrow (\text{NIL}, \text{NIL})$ ;
7     foreach  $u \in \text{lst}$  do // find the path with largest flow using dynamic programming
8        $\text{dp}[u] \leftarrow 0$ ;  $\text{best}[u] \leftarrow (\text{NIL}, \text{NIL})$ ;
9       foreach  $(v, i) \in \text{adj}_u$  do
10         $m \leftarrow \min(\text{dp}[v], \text{flow}[u, v, i])$ ;
11        if  $m > \text{dp}[u]$  then
12           $\text{dp}[u] \leftarrow m$ ;  $\text{best}[u] \leftarrow (v, i)$ ;
13    $f \leftarrow \text{dp}[s]$ ;
14   if  $f = 0$  then break; // stop when no more paths can be found
15    $u \leftarrow s$ ;
16    $\text{pattern} \leftarrow []$ ;
17   while  $u \neq \text{NIL}$  do // process the path along the graph starting from s
18      $(v, i) \leftarrow \text{best}[u]$ ;
19      $\text{flow}[u, v, i] \leftarrow \text{flow}[u, v, i] - f$ ; // decrease the flow along path
20     if  $i \neq 0$  then  $\text{pattern.append}(i)$ ; // add to the pattern if it is not a loss arc
21      $u \leftarrow v$ ;
22    $L.append((f, \text{pattern}))$  // add the pattern to the list of patterns
23 return  $L$ ;

```

items of the third size is only 2, for satisfying it exactly this pattern is only used 2 times. The pair $(2, [1, 3])$ enters to the solution; and the pattern with one item of the first size and multiplicity 1 remains, since items of the third size are not necessary anymore. The remaining demand of items of the first size is 1 and hence this pattern can be used once; the pair $(1, [1])$ enters to the solution. At this point, the demands of items of the first and third sizes are satisfied. We proceed to process the pattern $[2, 3, 3]$, which has multiplicity 1. Since items of the third size are not necessary anymore, we remove them from the pattern obtaining the pattern $[2]$. The remaining demand of items of the second size is 1 and hence the pair $(1, [2])$ enters to the solution. The final solution satisfying the demands exactly is $[(2, [1, 3]), (1, [1]), (1, [2])]$. This solution contains exactly 3 items of the first size, 1 item of the second size and 2 items of the third size.

Algorithm 16: Remove Excess**input** : L - list of patterns and multiplicities; b - demand vector**output**: L' - list of patterns and multiplicities satisfying the demand exactly

```

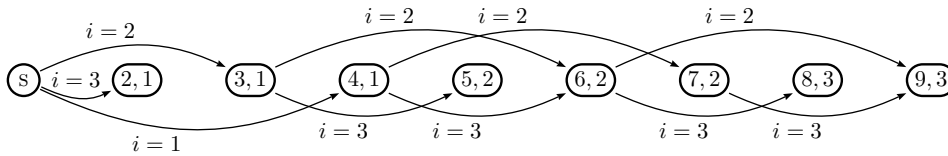
1 function removeExcess( $L, b$ ):
2    $L' \leftarrow []$ ;
3   foreach ( $f, \text{pattern}$ )  $\in L$  do
4      $c_i \leftarrow 0$ , for all  $i$ ;
5     foreach  $i \in \text{pattern}$  do                                // count the number of occurrences of each item in the pattern
6        $c_i \leftarrow \min(c_i + 1, b_i)$ ;
7     while  $f > 0$  do
8        $\text{pattern}' \leftarrow [i \times c_i \mid c_i > 0, \text{ for all } i]$ ;                // build the pattern
9        $f' \leftarrow \min(f, \min\{b_i \mid i \in \text{pattern}'\})$ ;          // how many times we can use pattern'
10       $L'.\text{append}((f', \text{pattern}'))$ ;                                // add the pattern to the list of patterns
11      foreach  $i \in \text{pattern}'$  do
12         $b_i \leftarrow b_i - f'$ ;
13         $c_i \leftarrow \min(c_i, b_i)$ ;
14       $f \leftarrow f - f'$ ;
15 return  $L'$ ;

```

3.5 Breaking symmetry

In the next section, we will present a three-step graph compression method whose first step consists of breaking the symmetry. Let us consider a cardinality constrained bin packing instance with bins of capacity $W = 9$, cardinality limit 3 and items of sizes 4, 3, 2 with demands 1, 3, 1, respectively. Figure 3.9 shows the graph produced by the graph construction algorithm without the final loss arcs. This graph contains symmetry. For instance, the paths $(s, (4,1), i=1) \rightarrow ((4,1), (7,2), i=2) \rightarrow ((7,2), (9,3), i=3)$ and $(s, (4,1), i=1) \rightarrow ((4,1), (6,2), i=3) \rightarrow ((6,2), (9,3), i=2)$ correspond to the same pattern with one item of each size, but the second one does not respect the order defined in Section 3.3.

Figure 3.9: Initial graph/Step-1 graph (with symmetry).



Graph corresponding to a cardinality constrained bin packing instance with bins of capacity $W = 9$, cardinality limit 3 and items of sizes 4, 3, 2 with demands 1, 3, 1, respectively.

An easy way to break symmetry is to divide the graph into levels, one level for each

item size. We introduce in each node a third dimension that indicates the level where it belongs. Nodes (a', b') are transformed into sets of nodes $\{(a', b', i'), (a', b', i''), \dots\}$. Each set has at most one node per level, which we connect using loss arcs. Arcs $((a', b'), (a'', b''), i)$ are transformed into arcs $((a', b', i), (a'', b'', i), i)$. On level i , we have only arcs associated with the i -th item. If we only connect a node (a', b', i') to a node (a', b', i'') in case $i' < i''$, we ensure that every path will respect the order and thus there is no symmetry. Recall that the initial graph must contain every valid packing pattern (respecting the order) represented as a path from S to T .

Algorithm 17 shows how to compute the graph division by levels (Step-2 graph). It receives as input the Step-1 graph produced by Algorithm 13 and creates a Step-2 graph. The algorithm starts by transforming every arc $((a', b'), (a'', b''), i)$ into $((a', b', i), (a'', b'', i), i)$, in lines 4-8. During this process, in line 8, the algorithm stores for each node (a', b') , of the initial graph, the set of new nodes in which the original node is subdivided. In lines 9-11, after processing all the arcs, the algorithm connects the nodes from different levels that were created from the same node. For each node (a', b') in the input graph, it sorts in lexicographical order the list of nodes $(V'_{a'b'})$ in which the original node was subdivided and it connects the pairs of nodes that appear consecutively in the list.

Algorithm 17: Break symmetry with levels

input : V - set of vertices; A - set of arcs; m - number of different item sizes

output: V' - set of vertices with levels; A' - set of arcs with levels

```

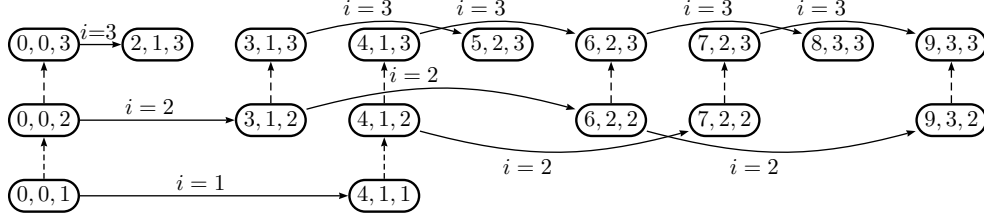
1 function breakSymmetry( $G = (V, A), m$ ):
2    $A' \leftarrow \{ \}$ ;
3    $V'_i \leftarrow \{ \}$ , for all  $i$ ;
4   foreach  $((a', b'), (a'', b''), i) \in A$  do                                     // for each arc in the original graph
5      $u \leftarrow (a', b', i)$ ;  $v \leftarrow (a'', b'', i)$ ;                         // the labels of the new nodes
6      $A' \leftarrow A' \cup \{(u, v, i)\}$ ;                                         // add the arc to the new graph
7      $V' \leftarrow V' \cup \{u, v\}$ ;                                             // add the new vertices to the new graph
8      $V'_{a'b'} \leftarrow V'_{a'b'} \cup \{u, v\}$ ;
9   foreach  $(a', b') \in V$  do // connect the nodes from different levels associated with each original node  $(a', b')$ 
10     $\text{lst} \leftarrow \text{sort}(V'_{a'b'})$ ;                                           // sort the set of nodes in lexicographical order
11    for  $i = 2$  to  $|\text{lst}|$  do  $A' \leftarrow A' \cup \{(\text{lst}[i-1], \text{lst}[i], 0)\}$ ; // connect consecutive nodes
12  return  $G' = (V', A')$ ;

```

Figure 3.10 shows the graph with levels that results from applying Algorithm 17 to the graph in Figure 3.9. Although there is no symmetry, there are still patterns that use some items more than their demand. To avoid this, other alternatives to break

symmetry could be used; however this method is appropriate for the sake of simplicity and speed.

Figure 3.10: Graph with levels/Step-2 graph (without symmetry).



All the patterns respect the order since there are no arcs from higher levels to lower levels. Moreover, it is also easy to check that no valid pattern was removed. In this graph, we consider $s = (0, 0, 1)$ since it is the only node without arcs incident to it. The target (τ) and the loss arcs connecting every internal node to it can be added using Algorithm 14.

Property 14 *No valid pattern (respecting the order) is removed by breaking symmetry with levels if the original graph contains every valid packing pattern (respecting the order) represented as a path from s to τ .*

Proof For every valid packing pattern (respecting the order) in the initial graph, there is a path in the Step-2 graph corresponding to the same pattern. Note that every path can be seen as a sequence of consecutive arcs. Let (v_1, v_2, i_1) and (v_2, v_3, i_2) be a pair of consecutive arcs in any valid packing pattern in the Step-1 graph. In the Step-2 graph, these arcs appear as $((v_1, i_1), (v_2, i_1), i_1)$ and $((v_2, i_2), (v_3, i_2), i_2)$. If $i_1 = i_2$, the pair of consecutive arcs appear connected at the same level. If not, and given that (v_2, i_1) and (v_2, i_2) were created from v_2 , a set of loss arcs appear between the head of the first arc and the tail of the second and hence there is again a sequence of arcs for that part of the pattern. \square

3.6 Graph compression

Symmetry may be helpful as long as it leads to large reductions in the graph size. In this section we show how to reduce the graph by taking advantage of common sub-patterns that can be represented by a single sub-graph. This method may increase symmetry, but it usually helps by reducing dramatically the graph size. The graph compression method is composed by three steps, the first of which was presented in the previous section.

In the graphs we have seen so far, a node label (a', b') means that every sub-pattern from the source to the node uses a' space in the first dimension and b' in the second. This means that a' and b' correspond to the length of the longest path from the source to the node in each dimension. Similarly, the longest path to the target can also be used as a label and nodes with the same label can be combined into one single node. This usually allows large reductions in the graph size. This reduction can be improved by breaking symmetry first (as described in the previous section), which allows us to consider only paths to the target with a specific order.

Algorithm 18 presents the main compression algorithm that creates the Step-3 graph. This algorithm receives as input the Step-2 graph produced by Algorithm 17. As we said before, we will use the longest paths to the target in each dimension to relabel the nodes. Let $(\varphi(u), \psi(u))$ be the label of node u in the first and second dimensions, respectively. We define $\varphi(u)$ and $\psi(u)$ as follows:

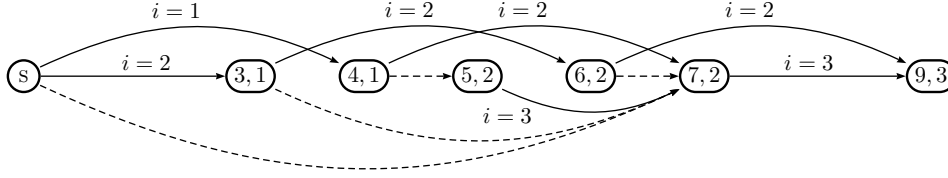
$$\varphi(u) = \begin{cases} 0 & \text{if } u = s, \\ X & \text{if } u = t, \\ \min_{(u,v,i) \in A} \{\varphi(v) - x_i\} & \text{otherwise.} \end{cases} \quad (3.11)$$

$$\psi(u) = \begin{cases} 0 & \text{if } u = s, \\ Y & \text{if } u = t, \\ \min_{(u,v,i) \in A} \{\psi(v) - y_i\} & \text{otherwise.} \end{cases} \quad (3.12)$$

For the sake of simplicity, we define loss arcs as items with no weight in both dimensions, $x_0 = y_0 = 0$. In the paths from s to t in Step-2 graph usually there is some float. In this process, we are moving this float as much as possible to the beginning of the path. The label in each dimension of every node u (except s) corresponds to the highest position where the sub-patterns from u to t can start in each dimension so that capacity constraints are not violated. By using these labels we are allowing arcs to be longer than the items to which they are associated. We use dynamic programming to compute these labels iteratively. In line 2, we obtain an adjacency list of the graph (including loss arcs connecting every internal node to the target). A reverse topological order, which is needed for dynamic programming, is computed in line 3; the way the nodes are labeled in Step-2 graph allows us to obtain a reverse topological order just by sorting the nodes in decreasing lexicographical order. In lines 4-10, we compute the new label of each node using dynamic programming. The Step-3 graph is produced in lines 11-12. This algorithm is linear in the graph size. Figure 3.11 shows the graph from Figure 3.10 after the main compression step. Even in this small instance, a few nodes and arcs were removed comparing to the initial graph of Figure 3.9. In large

instances, the reduction rate tends to be much greater.

Figure 3.11: Step-3 graph (after the main compression step).



The Step-3 graph has 8 nodes and 17 arcs (considering also the final loss arcs connecting internal nodes to T).

Algorithm 18: Main Compression Step

input : V - set of vertices; A - set of arcs; X, Y - capacity limits

output: V' - set of vertices; A' - set of arcs

```

1 function mainCompression( $G = (V, A), X, Y$ ):
2    $\text{adj}_u \leftarrow \{(v, i) \mid (u, v, i) \in A\} \cup \{(T, 0)\}$ , for all  $u \notin \{S, T\}$ ; // adjacency list of the graph
3    $\text{order} \leftarrow \text{reverse}(\text{sort}(V \setminus \{S, T\}))$ ; // reverse topological order of the graph
4    $\varphi(T) \leftarrow X$ ;  $\psi(T) \leftarrow Y$ ;
5    $\text{label}[T] \leftarrow (\varphi(T), \psi(T))$ ;
6   foreach  $u \in \text{order}$  do
7      $\varphi(u) \leftarrow \min_{(v, i) \in \text{adj}_u} \{\varphi(v) - x_i\}$ ;
8      $\psi(u) \leftarrow \min_{(v, i) \in \text{adj}_u} \{\psi(v) - y_i\}$ ;
9      $\text{label}[u] \leftarrow (\varphi(u), \psi(u))$ ;
10   $\text{label}[S] \leftarrow (0, 0)$ ;
11   $A' \leftarrow \{(\text{label}[u], \text{label}[v], i) \mid (u, v, i) \in A, v \neq T\}$ ;
12   $V' \leftarrow \{u \mid (u, v, i) \in A'\} \cup \{v \mid (u, v, i) \in A'\}$ ;
13  return  $G' = (V', A')$ ;
```

Finally, Algorithm 19 relabels once more the graph using the longest paths from the source in each dimension in order to try to reduce the graph size even more by combining nodes that were separated in the previous steps and that did not join any other node. Let $(\varphi'(v), \psi'(v))$ be the label of node v in the first and second dimensions, respectively. We define $\varphi'(v)$ and $\psi'(v)$ as follows:

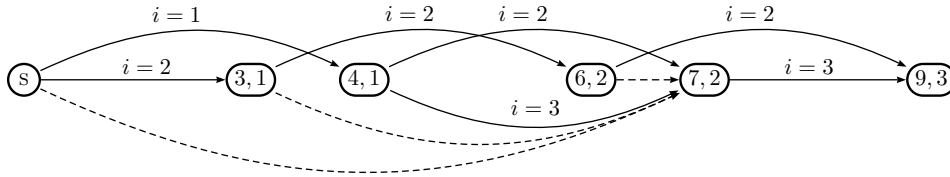
$$\varphi'(v) = \begin{cases} 0 & \text{if } v = S, \\ \max_{(u, v, i) \in A} \{\varphi'(u) + x_i\} & \text{otherwise.} \end{cases} \quad (3.13)$$

$$\psi'(v) = \begin{cases} 0 & \text{if } v = S, \\ \max_{(u, v, i) \in A} \{\psi'(u) + y_i\} & \text{otherwise.} \end{cases} \quad (3.14)$$

The final compression algorithm uses dynamic programming to compute these labels, and it is very similar to Algorithm 18; the main difference is the use of the transpose

graph, which results from reversing the orientation of the arcs in the input graph. The adjacency list of the transpose graph is required for dynamic programming since we need to know which arcs enter in each node. In this case, the reverse topological order of the transpose graph is simply the lexicographical order of the nodes. This algorithm is also linear in the graph size. Figure 3.12 shows the Step-4 graph without the final loss arcs. This final compression step is not as important as the main compression step, but it is easy to compute and usually removes many nodes and arcs.

Figure 3.12: Step-4 graph (after the final compression step).



The Step-4 graph has 7 nodes and 15 arcs (considering also the final loss arcs connecting internal nodes to T). In this case, the only difference from the Step-3 graph is the node (5, 2) that collapsed with the node (4, 1). The initial Step-1 graph had 9 nodes and 18 arcs.

Algorithm 19: Final Compression Step

input : V - set of vertices; A - set of arcs

output: V' - set of vertices; A' - set of arcs

```

1 function finalCompression( $G = (V, A)$ ):
2    $\text{adj}_v^T \leftarrow \{(u, i) \mid (u, v, i) \in A\}$ , for all  $v \notin \{s, T\}$ ;           // adjacency list of the transpose graph
3    $\text{order} \leftarrow \text{sort}(V \setminus \{s, T\})$ ;                                   // reverse topological order of the transpose graph
4    $\varphi'(s) \leftarrow 0$ ;  $\psi'(s) \leftarrow 0$ ;
5    $\text{label}[s] \leftarrow (\varphi(s), \psi(s))$ ;
6   foreach  $v \in \text{order}$  do
7      $\varphi'(v) \leftarrow \max_{(u, i) \in \text{adj}_v^T} \{\varphi'(u) + x_i\}$ ;
8      $\psi'(v) \leftarrow \max_{(u, i) \in \text{adj}_v^T} \{\psi'(u) + y_i\}$ ;
9      $\text{label}[v] \leftarrow (\varphi'(v), \psi'(v))$ ;
10   $A' \leftarrow \{(\text{label}[u], \text{label}[v], i) \mid (u, v, i) \in A\}$ ;
11   $V' \leftarrow \{u \mid (u, v, i) \in A'\} \cup \{v \mid (u, v, i) \in A'\}$ ;
12  return  $G' = (V', A')$ ;
```

Note that, in this case, the initial Step-1 graph had some symmetry and the final Step-4 graph does not contain any symmetry. Graph compression may increase symmetry in some situations, but this is not a problem as long as it leads to large reductions in the graph size. Since we are dealing with a very small instance, the improvement is not as substantial as in large instances. For instance, in the standard BPP instance HARD4 from Scholl et al. (1997) with 200 items of 198 different sizes and bins of

capacity 100,000, we obtained reductions of 97% in the number of vertices and 95% in the number of the arcs. The resulting model was solved in a few seconds. Without graph compression it would be much harder to solve this kind of instances using the arc flow formulation.

Property 15 (Graph compression 1) *None of the non-redundant patterns in the initial graph is removed by graph compression.*

Proof Any pattern in the initial graph is represented by a path between s and t . Consider a path $\pi = v_1 v_2 \dots v_n$. Graph compression will reduce the graph size by relabeling nodes. Therefore, the path $\pi' = \phi(v_1) \phi(v_2) \dots \phi(v_n)$, where ϕ is the map between the initial and final labels, will represent exactly the same pattern as the one represented by the path π . \square

Property 16 (Graph compression 2) *Graph compression will not introduce any invalid pattern.*

Proof An invalid pattern consists of a set of items whose sum of weights exceeds the bin capacity. Patterns are formed from paths in the graph and the total weight is the length of the path. The main compression step just relabels every node u (except s) in each dimension with the highest position where the sub-patterns from u to t can start in each dimension so that capacity constraints are not violated. The node t is labeled with (X, Y) and no label will be smaller than $(0, 0)$ since all the patterns in the input graph are required to be valid. However, we could have invalid patterns, even with all the nodes labeled between $(0, 0)$ and (X, Y) , if any arc had length smaller than the item it represents, but this is not possible. The label of every internal node u is given by $(\varphi(u), \psi(u))$, where $\varphi(u) = \min\{\varphi(v) - x_i \mid (u, v, i) \in A\}$ and $\psi(u) = \min\{\psi(v) - y_i \mid (u, v, i) \in A\}$; for every node v such that there is an arc between u and v , the difference between their labels is at least the weight of the item associated with the arc in each dimension. Therefore, no invalid patterns are introduced. An analogous proof can be derived for the final compression step. \square

3.7 Integrality gap and heuristic solutions

There have been many studies (see, e.g., Scheithauer and Terno 1995, Scheithauer and Terno 1997) about the integrality gap for BPP, many of them about the integrality

gap using Gilmore-Gomory's model. Our proposed arc flow formulation is equivalent to Gilmore-Gomory's model and hence the lower bounds are the same when the same set of patterns is considered. Therefore, the results found on these studies, which we summarize next, are also valid for the arc flow formulation.

Definition 1 (Integer Property) *A linear integer optimization problem P has the integer property (IP) if*

$$z_{ip}^*(E) = z_{lp}^*(E) \text{ for every instance } E \in P$$

Definition 2 (Integer Round-Up Property) *A linear integer optimization problem P has the integer round-up property (IRUP) if*

$$z_{ip}^*(E) = \lceil z_{lp}^*(E) \rceil \text{ for every instance } E \in P$$

Definition 3 (Modified Integer Round-Up Property) *A linear integer optimization problem P has the modified integer round-up property (MIRUP) if*

$$z_{ip}^*(E) = \lceil z_{lp}^*(E) \rceil + 1 \text{ for every instance } E \in P$$

Rietz et al. (2002a) describe families of instances of the one-dimensional cutting stock problem without the integer round-up property. One of the families is the so-called divisible case, where every item size w_i is a factor of the bin capacity W , which was firstly proposed by Nica (1994). Since the method we propose usually solves bin packing problems quickly, it was used to solve millions of instances from this family keeping track of the largest gap found, which was $1.0378\dots$, the same gap as the one found by Scheithauer and Terno (1997).

Gau (1994) presents an instance with a gap of 1.0666 . The largest gap known so far is $7/6$ and it was found by Rietz et al. (2002b). Scheithauer and Terno (1997) conjecture that the general one-dimensional cutting stock problem has the modified integer round-up property (MIRUP). Moreover, instances for BPP and CSP usually have the integer round-up property (IRUP). Concerning the results obtained using the arc flow formulation in cardinality constrained BPP and two-constraint BPP, most of the instances of these problems also have the IRUP, and no instance violated the MIRUP. The largest gap we found in all the instances from the benchmark test data sets was 1.0067 .

The lower bound provided by the linear relaxation of the proposed arc flow formulation is usually very tight in every problem we considered; hence, the branch-and-bound process usually finds the optimal solution quickly (see Section 3.8). Moreover, very good solutions are usually found when rounding the linear programming (LP) solution. Rounding up the fractional variables of the LP solution of Gilmore and Gomory’s model guarantees a heuristic solution of value at most $z_{lp}^* + m$, where z_{lp}^* is the optimum value of the linear relaxation, since we need to round up at most one variable for each different item size in order to obtain a valid integer solution. Rounding up fractional flow paths of the linear relaxation of the arc flow formulation gives the same guarantee. Wäscher and Gau (1996) present more elaborate rounding heuristics for Gilmore and Gomory’s model that usually lead to the optimal solution in cutting stock instances. Note that these rounding heuristics usually work well in cutting stock instances where the demands are large, but they may have a poor performance in bin packing instances where the values of variables are often a fraction of unity. In our model we try to overcome the problems introduced by low demands with upper bounds on the variable values, and by requiring the demand of items with demand one to be satisfied exactly.

3.8 Results

CPU times were obtained using a computer with two Quad-Core Intel Xeon at 2.66GHz, running Mac OS X 10.8.0, with 16 GBytes of memory. The amount of memory required to solve most of the instances was much less than the 16 GBytes of memory available; only a few of the hardest two-constraint BPP instances required more than 4 GBytes of memory. All the algorithms were implemented in C++, and Gurobi 5.0.0, a state-of-the-art mixed integer programming solver, was used to solve the generated arc flow model. The parameters used in Gurobi were Threads = 1 (single thread), Presolve = 1 (conservative), Method = 2 (Interior point methods), MIPFocus = 1 (feasible solutions), Heuristics = 1, MIPGap = 0, MIPGapAbs = 1-1e-5 and the remaining parameters were Gurobi’s default values. The branch-and-cut solver used in Gurobi uses a series of cuts; in our models the most frequently used were Gomory, Zero half and MIR. Table 3.1 presents the meaning of each column in subsequent tables. In some of the following tables we present average results per class. For more detailed results please consult the appendix or the web-page <http://www.dcc.fc.up.pt/~fdabrandao/research/arcflow/results/>, which contains all the results and Gurobi’s logs.

Table 3.1: Meaning of the data displayed in subsequent tables.

Label	Description
W	bin capacity
X	capacity of the first dimension
Y	capacity of the second dimension
C	cardinality limit
n	number of items
m	number of different item sizes
w^{avg}	average weight
w^{min}	weight of the smallest item
w^{max}	weight of the largest item
z^*	optimum objective value
lb^{lp}	arc flow linear relaxation lower bound
lb^{lp1}	arc flow linear relaxation lower bound in the first dimension
lb^{lp2}	arc flow linear relaxation lower bound in the second dimension
lb^{sp}	space lower bound
lb^{crd}	cardinality lower bound
lb^{sp1}	space lower bound in the first dimension
lb^{sp2}	space lower bound in the second dimension
n^{bb}	number of nodes explored in the branch-and-bound procedure
t^{pp}	time spent in pre/post-processing in seconds
t^{lp}	time spent in the linear relaxation of the root node in seconds
t^{lp}	time spent in the branch-and-bound procedure in seconds
t^{tot}	total run time in seconds
$\#v$	number of vertices in the final graph
$\#a$	number of arcs in the final graph
$\%v$	ratio between the number of vertices after compression and the initial number of vertices
$\%a$	ratio between the number of arcs after compression and the initial number of arcs
$\#op$	number of open instances solved

3.8.1 Standard bin packing and cutting stock

We used the arc flow formulation to solve every instance from the OR-LIBRARY (2012) bin packing test data set that was proposed by Falkenauer (1996). This test has two classes of instances: uniform instances (uNNN), where items have randomly generated weights, and the harder triplets instances (tNNN), where the optimal solution for each bin is completely filled with three items. Each of these is further divided into subclasses of varying sizes. Each subclass contains 20 instances. Valério de Carvalho (1999) solved this data set using his arc flow formulation. In the following tables, we present average values over 20 instances for each class u120, u250, ..., t501. In Table 3.2, we have the data set characteristics. In Table 3.4, we have the results of our method in this data set, and in Table 3.3 we report the results using Valério de Carvalho's formulation. The average run time is less than 2 seconds using Valério de Carvalho's formulation and less than 1 second using the proposed arc flow formulation with graph compression.

We generated cutting stock instances from the OR-LIBRARY (2012) bin packing test data set by multiplying the demand of each item by one million. Table 3.5 reports the results obtained. Since the graph remains very small, even with extremely large

Table 3.2: Bin packing test data set.

class	W	n	m	w^{\min}	w^{\max}	w^{avg}
u120	150	120	63.20	20	100	60.57
u250	150	250	77.25	20	100	60.64
u500	150	500	80.80	20	100	60.19
u1000	150	1,000	81.00	20	100	60.00
t60	1,000	60	49.95	250	499	333.33
t120	1,000	120	86.15	250	499	333.33
t249	1,000	249	140.10	250	499	333.33
t501	1,000	501	194.25	250	499	333.33

Table 3.3: Bin packing results using Valério de Carvalho's graph.

class	z^*	lb^{LP}	lb^{SP}	$\#v$	$\#a$	n^{bb}	t^{PP}	t^{LP}	t^{IP}	t^{tot}
u120	49.05	48.50	48.46	131.7	2,312.2	0.00	0.02	0.03	0.26	0.30
u250	101.60	101.09	101.07	131.9	3,103.3	0.00	0.03	0.04	0.42	0.49
u500	201.20	200.64	200.64	131.9	3,333.8	0.15	0.03	0.05	0.47	0.55
u1000	400.55	400.01	400.01	132.0	3,347.0	0.00	0.03	0.05	0.29	0.36
t60	20.00	20.00	20.00	751.8	5,480.1	0.00	0.04	0.05	0.41	0.49
t120	40.00	40.00	40.00	752.0	10,753.5	0.00	0.06	0.10	1.41	1.57
t249	83.00	83.00	83.00	752.0	19,359.0	0.00	0.11	0.24	2.41	2.76
t501	167.00	167.00	167.00	752.0	29,928.4	0.00	0.17	0.49	4.00	4.66

Table 3.4: Bin packing results.

class	z^*	lb^{LP}	lb^{SP}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{PP}	t^{LP}	t^{IP}	t^{tot}
u120	49.05	48.50	48.46	100.2	1,968.8	78.3	85.3	0.00	0.03	0.03	0.10	0.16
u250	101.60	101.09	101.07	109.5	2,727.2	83.1	87.9	0.00	0.04	0.05	0.20	0.29
u500	201.20	200.64	200.64	111.9	2,940.9	84.2	88.2	0.00	0.04	0.06	0.20	0.30
u1000	400.55	400.01	400.01	112.0	2,956.0	84.2	88.3	0.00	0.04	0.05	0.24	0.33
t60	20.00	20.00	20.00	54.4	690.7	9.9	13.1	0.00	0.04	0.01	0.03	0.08
t120	40.00	40.00	40.00	95.6	1,971.8	16.2	18.6	0.80	0.08	0.03	0.19	0.30
t249	83.00	83.00	83.00	150.3	5,600.5	23.4	29.1	0.00	0.16	0.11	0.77	1.04
t501	167.00	167.00	167.00	202.2	11,827.3	29.0	39.6	0.00	0.27	0.27	1.68	2.22

demands, the problem does not become harder to solve. Moreover, rounding heuristics tend to work better when demands are high (see Section 3.7). Note that in the class u1000 there are one thousand million items in each instance, and the average run time is less than 1 second.

Table 3.5: Cutting stock results.

class	z^*	lb^{LP}	n	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{PP}	t^{LP}	t^{IP}	t^{tot}
u120	1.2e+08	4.8e+07	4.8e+07	101.1	1,996.3	79.0	85.5	0.00	0.03	0.04	0.16	0.23
u250	2.5e+08	1e+08	1e+08	109.7	2,735.6	83.2	87.9	0.00	0.04	0.05	0.29	0.38
u500	5e+08	2e+08	2e+08	112.0	2,941.8	84.2	88.2	0.00	0.05	0.07	0.28	0.39
u1000	1e+09	4e+08	4e+08	112.0	2,956.0	84.2	88.3	0.00	0.04	0.06	0.24	0.35
t60	6e+07	2e+07	2e+07	53.5	690.6	9.8	12.5	0.00	0.04	0.01	0.02	0.07
t120	1.2e+08	4e+07	4e+07	90.5	1,964.2	15.4	18.1	0.00	0.08	0.03	0.10	0.21
t249	2.5e+08	8.3e+07	8.3e+07	144.8	5,610.6	22.5	28.9	0.00	0.16	0.12	0.49	0.77
t501	5e+08	1.7e+08	1.7e+08	199.4	11,863.4	28.6	39.5	0.00	0.28	0.29	1.75	2.31

Scholl (2012) provides three data sets that were generated for Scholl et al. (1997). The first is composed of randomly generated instances with $n \in \{50, 100, 200, 500\}$, $W \in \{100, 120, 150\}$ and weights uniformly chosen from the intervals $[1, 100]$, $[20, 100]$ and $[30, 100]$. The expected number of items per bin is not larger than 3. The second test data set is composed of instances with $n \in \{50, 100, 200, 500\}$, $W = 1,000$ and weights chosen such that the expected average number of items per bin is 3, 5, 7 or 9. Finally, the third test data set is composed of 10 difficult instances with $n = 200$, $W = 100,000$ and weights randomly chosen from the interval $[20,000, 35,000]$. Scholl et al. (1997) solved many of these instances, Schoenfield (2002) solved the first two test data sets entirely, and Schwerin and Wäscher (1997) solved the third one. Table 3.6 shows the results for these three test data sets. Note that the initial graph of the instances from the third data set is usually really large because of the large bin capacities; the graph compression reduces dramatically the graph sizes and these instances are solved in a few seconds using the proposed method. The average run time is less than 20 seconds in these test data sets.

Table 3.6: Scholl results.

class	z^*	lb^P	W	n	m	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{tot}
bin1	108.86	108.49	123.3	212.5	63.8	78.3	1,485.6	74.9	85.6	0.00	0.15
bin2	42.18	41.66	1,000.0	212.5	97.6	632.9	33,624.0	75.2	85.5	0.21	46.19
bin3	56.20	55.58	100,000.0	200.0	199.0	1,846.2	83,015.7	3.8	5.4	0.00	30.65

Umetani (2012) provides two large test data sets for cutting stock problems. The first data set is composed by 1800 randomly generated instances of 18 classes. These instances were generated using the problem generator proposed by Gau and Wäscher (1995). Table 3.7 shows the results for this data set grouped by class. The second data set was taken from a real application in a chemical fiber company in Japan. There are 40 instances with the number of different piece lengths ranging from 6 to 29, the length of stock rolls being 9080 or 5180, the demands for each piece length ranging from 2 to 264, and the length of each piece ranging from 500 to 2,000. Table 3.8 shows the results for each instance in this data set. The average run time is less than 2 seconds in the first data set and less than 1 second in the second.

Schoenfield (2002) provides the hard28 test data set. This data set is composed of instances selected from a huge testing. Among these 28 instances, 5 are non-IRUP, so the integer programming solver had to use branch-and-cut to reduce the gap and prove optimality. The remaining instances are IRUP, yet very hard for heuristics. Table 3.9 shows the results for this test data set. The average run time in this data set is less than 2 minutes.

Table 3.7: Cutgen results.

type	z^*	lb^{lp}	W	n	m	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{tot}
1	11.48	10.98	1,000	100.0	9.8	570.4	2,917.8	67.2	68.5	0.24	0.47
2	110.25	109.73	1,000	1,000.0	9.8	585.2	3,021.4	68.3	69.1	0.00	0.46
3	22.13	21.57	1,000	200.0	19.0	822.9	9,138.6	88.6	91.1	0.00	2.94
4	215.93	215.40	1,000	2,000.0	19.0	826.9	9,239.9	88.8	91.2	0.00	2.89
5	42.95	42.49	1,000	400.1	36.4	915.7	21,234.3	94.9	97.6	0.00	11.28
6	424.68	424.21	1,000	4,000.0	36.4	917.1	21,331.5	95.0	97.6	0.00	9.98
7	50.24	49.95	1,000	100.0	9.9	43.9	160.8	15.6	24.5	0.00	0.02
8	499.62	499.33	1,000	1,000.0	9.9	53.9	185.0	16.9	24.4	0.00	0.02
9	93.65	93.34	1,000	200.0	19.8	199.7	944.1	31.8	41.9	0.00	0.07
10	932.26	931.95	1,000	2,000.0	19.8	217.0	996.6	33.2	42.2	0.00	0.08
11	176.90	176.59	1,000	400.1	39.0	578.5	4,534.8	68.8	75.3	0.00	0.48
12	1,763.45	1,763.18	1,000	4,000.0	39.0	588.8	4,600.5	69.3	75.4	0.00	0.45
13	63.47	63.23	1,000	100.0	9.9	8.6	31.4	17.1	31.5	0.00	0.01
14	632.36	632.09	1,000	1,000.0	9.9	8.8	32.1	17.0	31.3	0.00	0.01
15	119.59	119.35	1,000	200.0	19.7	16.2	86.4	9.6	22.9	0.00	0.01
16	1,192.00	1,191.75	1,000	2,000.0	19.7	16.4	87.0	9.5	22.6	0.00	0.01
17	224.85	224.56	1,000	400.1	38.8	34.5	279.9	9.1	21.6	0.00	0.03
18	2,242.59	2,242.30	1,000	4,000.0	38.8	34.8	281.1	9.1	21.5	0.00	0.03

Table 3.8: Fiber results.

name	z^*	lb^{lp}	lb^{sp}	W	n	m	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{tot}
fiber06_5180	33	32.83	32.32	5,180	198	6	29	108	7.8	14.2	0	0.02
fiber06_9080	19	18.45	18.44	9,080	198	6	121	449	6.4	9.8	0	0.06
fiber07_5180	33	32.60	31.43	5,180	183	4	11	35	8.0	11.7	0	0.02
fiber07_9080	19	18.11	17.93	9,080	183	4	27	88	5.7	8.1	0	0.02
fiber08_5180	86	85.76	83.28	5,180	455	4	25	79	21.9	30.3	0	0.02
fiber08_9080	48	47.89	47.51	9,080	455	4	80	262	18.9	25.2	0	0.03
fiber09_5180	53	52.96	49.42	5,180	269	7	39	169	50.0	57.9	0	0.02
fiber09_9080	29	28.32	28.19	9,080	269	7	123	564	78.8	79.8	0	0.05
fiber10_5180	69	68.88	67.18	5,180	349	6	21	92	7.9	14.5	0	0.02
fiber10_9080	39	38.49	38.32	9,080	349	6	106	416	10.7	14.0	0	0.06
fiber11_5180	67	66.57	65.06	5,180	354	7	34	140	15.4	24.1	0	0.02
fiber11_9080	38	37.30	37.12	9,080	354	7	114	484	18.7	22.4	0	0.04
fiber13a_5180	56	55.80	54.68	5,180	279	8	24	108	3.8	7.4	0	0.02
fiber13a_9080	32	31.22	31.19	9,080	279	8	137	537	3.8	5.5	0	0.07
fiber13b_5180	28	27.52	27.21	5,180	155	9	139	612	36.7	33.6	0	0.04
fiber13b_9080	16	15.56	15.52	9,080	155	9	524	2,362	68.1	53.7	0	0.22
fiber14_5180	47	46.88	46.47	5,180	223	10	107	504	8.3	12.9	0	0.05
fiber14_9080	27	26.51	26.51	9,080	223	10	649	3,015	13.2	14.2	0	0.39
fiber15_5180	57	56.80	55.37	5,180	330	7	40	166	15.1	22.2	0	0.02
fiber15_9080	32	31.81	31.59	9,080	330	7	161	706	24.6	28.3	0	0.09
fiber16_5180	82	81.88	81.72	5,180	419	13	147	886	8.3	13.7	0	0.10
fiber16_9080	47	46.62	46.62	9,080	419	13	1,137	6,219	20.2	18.7	0	1.07
fiber17_5180	83	82.80	81.54	5,180	528	12	96	520	7.5	11.8	0	0.05
fiber17_9080	47	46.52	46.52	9,080	528	12	595	3,194	12.1	14.1	0	0.41
fiber18_5180	96	95.31	94.20	5,180	494	12	117	650	19.3	25.9	0	0.05
fiber18_9080	54	53.74	53.74	9,080	494	12	588	3,432	42.4	39.7	0	0.48
fiber19_5180	133	132.17	126.69	5,180	667	12	76	428	6.2	11.0	0	0.05
fiber19_9080	73	72.93	72.27	9,080	667	12	619	3,183	12.8	14.8	0	0.43
fiber20_5180	32	31.74	31.74	5,180	177	17	147	1,038	22.8	28.0	0	0.09
fiber20_9080	19	18.11	18.11	9,080	177	17	684	5,028	48.0	40.8	0	0.76
fiber23_5180	141	140.94	140.03	5,180	719	15	106	701	29.5	33.9	0	0.08
fiber23_9080	80	79.88	79.88	9,080	719	15	529	4,049	70.6	62.6	0	0.69
fiber26_5180	190	189.03	185.70	5,180	1,121	15	121	803	18.7	26.4	0	0.06
fiber26_9080	107	106.43	105.94	9,080	1,121	15	724	4,856	50.7	46.6	0	0.65
fiber28a_5180	83	82.98	82.95	5,180	500	17	247	1,801	61.9	60.7	0	0.16
fiber28a_9080	48	47.32	47.32	9,080	500	17	654	6,367	82.9	79.5	0	1.27
fiber28b_5180	117	116.95	116.38	5,180	731	19	138	1,042	21.2	27.8	0	0.15
fiber29_5180	62	61.25	61.25	5,180	381	20	207	1,850	52.1	56.1	0	0.18
fiber29_9080	35	34.94	34.94	9,080	381	20	627	7,408	79.7	79.0	0	1.54

Table 3.9: Hard28 results.

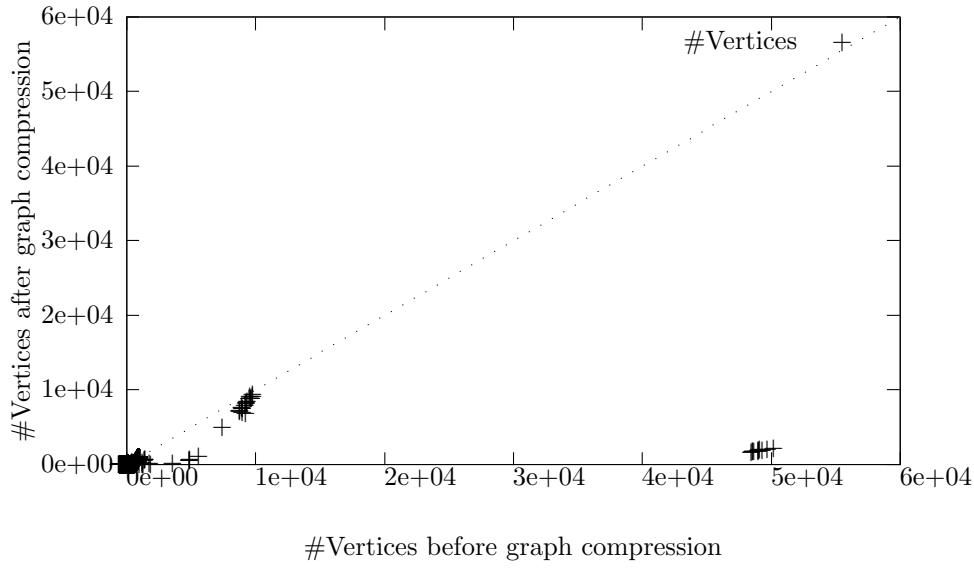
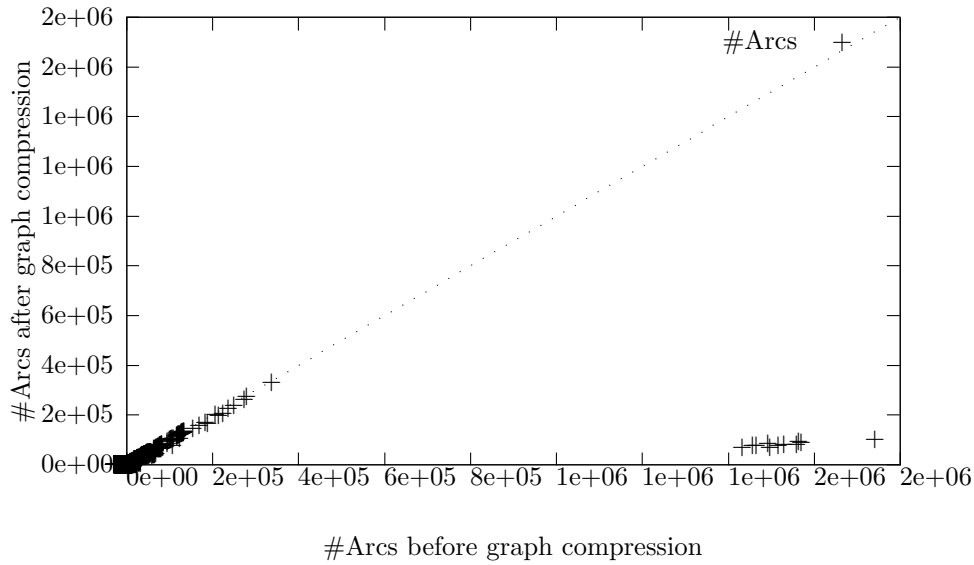
inst.	z^*	lb^{lp}	lb^{sp}	W	n	m	$\#v$	$\#a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
13	67	67.00	66.96	1,000	180	161	941	38,960	12	0.43	3.07	7.94	11.44
14	62	61.00	60.96	1,000	160	136	944	30,353	279	0.32	2.74	4.34	7.40
40	59	59.00	58.98	1,000	160	144	917	39,279	75	0.43	2.73	11.50	14.66
47	71	71.00	70.89	1,000	180	158	940	38,022	0	0.41	2.49	17.08	19.98
60	63	63.00	62.94	1,000	160	144	744	25,581	97	0.29	1.58	3.41	5.28
119	77	76.00	75.98	1,000	200	173	928	39,640	0	0.44	3.41	4.88	8.73
144	73	73.00	73.00	1,000	200	173	945	40,018	505	0.46	3.48	19.25	23.19
175	84	83.00	82.92	1,000	200	185	951	38,732	195	0.42	2.35	38.13	40.90
178	80	80.00	79.93	1,000	200	178	964	39,947	24	0.44	3.08	9.25	12.77
181	72	72.00	71.94	1,000	180	157	934	33,687	0	0.36	3.16	3.06	6.58
195	64	64.00	63.99	1,000	180	161	980	41,288	564	0.46	3.18	2,053.18	2,056.82
359	76	75.00	74.85	1,000	180	164	884	30,377	19	0.33	2.07	2.79	5.19
360	62	62.00	61.92	1,000	160	148	921	33,560	0	0.37	2.31	8.35	11.03
419	80	80.00	79.96	1,000	200	189	957	42,302	0	0.45	2.72	6.29	9.46
485	71	71.00	70.95	1,000	180	163	990	38,456	40	0.41	2.49	7.06	9.96
531	83	83.00	82.84	1,000	200	175	954	35,378	0	0.37	2.31	5.09	7.77
561	72	72.00	71.99	1,000	200	177	951	43,580	32	0.53	3.57	204.36	208.46
640	74	74.00	73.90	1,000	180	165	918	34,963	0	0.39	2.32	22.90	25.61
645	58	58.00	57.99	1,000	160	141	984	36,755	81	0.42	3.34	111.79	115.55
709	67	67.00	66.99	1,000	180	160	912	33,232	77	0.36	2.50	4.26	7.12
716	76	75.00	74.84	1,000	180	158	948	29,178	5	0.32	1.61	10.43	12.36
742	64	64.00	63.94	1,000	160	148	900	30,929	0	0.34	2.00	9.74	12.08
766	62	62.00	61.96	1,000	160	143	894	30,458	4	0.32	2.45	3.40	6.17
781	71	71.00	71.00	1,000	200	174	980	46,105	20	0.52	5.02	191.24	196.78
785	68	67.99	67.98	1,000	180	163	940	37,476	0	0.41	3.06	44.17	47.64
814	81	81.00	80.82	1,000	200	179	966	42,427	0	0.45	2.94	7.12	10.51
832	60	60.00	59.98	1,000	160	139	889	27,201	125	0.32	2.72	52.30	55.34
900	75	75.00	74.98	1,000	200	173	980	41,622	67	0.47	3.33	98.98	102.78

Finally, ESICUP (2012) provides four test data sets collected from Schwerin and Wäscher (1997), and Wäscher and Gau (1996). Table 3.10 shows average results for each data set. The average run time is less than 10 minutes in these test data sets.

Table 3.10: Schwerin/Wäscher/Gau results.

class	z^*	lb^{lp}	W	n	m	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{tot}
SCH.WAE1	18.00	17.71	1,000	100.0	44.0	202.9	3,518.2	29.4	30.2	0.00	0.61
SCH.WAE2	21.92	21.32	1,000	120.0	46.2	208.8	3,771.8	30.1	30.3	0.00	0.56
WAE.GAU1	16.42	16.25	10,000	133.5	49.6	7,930.4	170,172.9	86.4	92.9	12.25	5,469.32
WAE.GAU2	19.80	19.79	10,000	119.6	49.8	8,173.6	146,461.0	88.2	94.7	11.80	8,991.44

Figures 3.13 and 3.14 show the relation between the number of vertices and arcs before and after graph compression. The graph compression algorithm is remarkably effective in the Scholl's bin3 data set where it removes on average more than 96% of the vertices and 94% of the arcs. Without graph compression it would take much longer to solve these instances.

Figure 3.13: Graph size reduction (vertices) in standard bin packing instances.**Figure 3.14:** Graph size reduction (arcs) in standard bin packing instances.

3.8.2 Cardinality constrained bin packing

In order to test the extended capabilities of our method, as explained in Section 3.2.1, we added cardinality constraints to the instances of OR-LIBRARY. We tested every instance with every cardinality limit between 2 and the maximum number of items that fit into a single bin. Results for this problem are presented in Table 3.11 for uniform classes, and in Table 3.12 for triplets classes. The rows highlighted in bold correspond to the smallest cardinality limit that allowed the optimum objective value

to be the same as the optimum without cardinality constraints.

Table 3.11: Cardinality constrained BPP results in uniform classes.

class	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{tot}
u120	7	49.05	48.50	48.46	17.14	97.0	1,981.7	25.0	53.1	0.00	0.18
	6	49.05	48.50	48.46	20.00	96.1	1,976.2	24.8	53.0	0.00	0.18
	5	49.05	48.50	48.46	24.00	96.0	1,917.5	26.0	52.1	0.00	0.18
	4	49.05	48.50	48.46	30.00	93.8	1,742.8	28.7	50.1	0.00	0.17
	3	49.05	48.50	48.46	40.00	68.3	1,032.8	26.1	34.8	0.00	0.12
	2	60.00	60.00	48.46	60.00	19.9	140.2	11.4	7.6	0.00	0.03
u250	7	101.60	101.09	101.07	35.71	108.4	2,743.2	25.4	53.2	0.00	0.29
	6	101.60	101.09	101.07	41.67	106.0	2,720.1	25.1	52.9	0.00	0.29
	5	101.60	101.09	101.07	50.00	109.5	2,645.5	27.7	52.4	0.00	0.29
	4	101.60	101.09	101.07	62.50	107.3	2,428.4	30.8	51.0	0.00	0.26
	3	101.60	101.09	101.07	83.33	81.8	1,462.0	29.3	35.7	0.00	0.18
	2	125.00	125.00	101.07	125.00	26.6	177.6	14.0	6.6	0.00	0.03
u500	7	201.20	200.64	200.64	71.43	111.9	2,945.7	25.1	52.2	0.00	0.32
	6	201.20	200.64	200.64	83.33	109.2	2,911.2	25.1	51.8	0.00	0.33
	5	201.20	200.64	200.64	100.00	114.5	2,841.1	28.2	51.7	0.00	0.32
	4	201.20	200.64	200.64	125.00	110.7	2,608.0	31.2	50.5	0.00	0.28
	3	201.20	200.64	200.64	166.67	84.8	1,574.7	29.8	35.5	0.00	0.20
	2	250.00	250.00	200.64	250.00	28.9	188.3	14.9	6.4	0.00	0.03
u1000	7	400.55	400.01	400.01	142.86	112.0	2,956.2	24.9	52.0	0.00	0.33
	6	400.55	400.01	400.01	166.67	109.8	2,922.4	25.1	51.6	0.00	0.34
	5	400.55	400.01	400.01	200.00	116.0	2,857.0	28.5	51.6	0.00	0.30
	4	400.55	400.01	400.01	250.00	111.0	2,621.0	31.2	50.4	0.00	0.26
	3	400.55	400.01	400.01	333.33	85.0	1,581.0	29.8	35.4	0.00	0.20
	2	500.00	500.00	400.01	500.00	29.0	189.0	14.9	6.4	0.00	0.03

Table 3.12: Cardinality constrained BPP results in triplets classes.

class	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{tot}
t60	4	20.00	20.00	20.00	15.00	54.1	690.3	7.7	12.7	0.00	0.09
	3	20.00	20.00	20.00	20.00	54.1	690.3	7.7	12.7	0.00	0.09
	2	30.00	30.00	20.00	30.00	4.0	101.0	0.9	5.8	0.00	0.02
t120	4	40.00	40.00	40.00	30.00	95.1	1,970.8	11.8	18.2	0.80	0.31
	3	40.00	40.00	40.00	40.00	94.5	1,969.6	11.7	18.2	0.80	0.31
	2	60.00	60.00	40.00	60.00	4.0	173.5	0.7	4.0	0.00	0.04
t249	4	83.00	83.00	83.00	62.25	150.3	5,600.5	17.0	28.7	0.00	1.05
	3	83.00	83.00	83.00	83.00	148.6	5,596.9	16.8	28.7	0.00	1.01
	2	125.00	124.50	83.00	124.50	4.0	281.2	0.6	2.7	0.00	0.08
t501	4	167.00	167.00	167.00	125.25	202.2	11,827.3	21.4	39.3	0.00	2.21
	3	167.00	167.00	167.00	167.00	200.2	11,823.3	21.2	39.3	0.05	2.65
	2	251.00	250.50	167.00	250.50	4.0	389.9	0.6	2.0	0.00	0.15

For the remaining test data sets we solved every instance with cardinalities between 2 and the minimum cardinality limit that allowed the optimum objective value to be the same as the optimum without cardinality constraints. For more detailed results please consult <http://www.dcc.fc.up.pt/~fdabrandao/research/arcflow/results/>. In Table 3.13, we show the results obtained in the hard28 test data set. Note that, in this data set, the instances became much easier to solve after adding cardinality constraints. The average run time is less than 2 seconds.

The proposed formulation proved to work very well in cardinality constrained bin

Table 3.13: Cardinality constrained BPP results in Hard28 instances.

inst.	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
13	3	67	67.00	66.96	60.00	166	3,478	8.1	7.6	0	0.36	0.08	0.41	0.85
	2	90	90.00	66.96	90.00	32	323	2.9	2.9	0	0.09	0.00	0.01	0.10
14	3	62	61.00	60.96	53.33	140	2,993	6.9	8.6	171	0.28	0.07	0.89	1.24
	2	80	80.00	60.96	80.00	28	279	2.6	3.3	0	0.06	0.00	0.01	0.07
40	3	59	59.00	58.98	53.33	149	2,516	7.4	5.5	150	0.37	0.07	1.18	1.62
	2	80	80.00	58.98	80.00	28	294	2.6	3.2	0	0.07	0.00	0.01	0.08
47	3	71	71.00	70.89	60.00	163	2,996	7.9	6.7	0	0.36	0.08	0.36	0.80
	2	90	90.00	70.89	90.00	33	318	3.0	3.1	0	0.08	0.00	0.01	0.09
60	3	63	63.00	62.94	53.33	150	3,344	8.3	10.4	83	0.26	0.07	0.83	1.16
	2	80	80.00	62.94	80.00	30	293	3.0	3.3	0	0.07	0.00	0.01	0.08
119	3	77	76.00	75.98	66.67	177	4,636	8.7	9.8	0	0.40	0.11	0.33	0.84
	2	100	100.00	75.98	100.00	34	354	3.1	2.7	0	0.10	0.00	0.01	0.11
144	3	73	73.00	73.00	66.67	185	4,814	8.9	10.2	0	0.39	0.14	1.06	1.59
	2	100	100.00	73.00	100.00	31	349	2.8	2.6	0	0.43	0.00	0.01	0.44
175	3	84	83.00	82.92	66.67	184	4,722	8.7	10.1	15	0.38	0.13	1.81	2.32
	2	100	100.00	82.92	100.00	43	379	3.7	2.9	0	0.10	0.00	0.01	0.11
178	3	80	80.00	79.93	66.67	180	4,443	8.6	9.3	0	0.39	0.12	0.96	1.47
	2	100	100.00	79.93	100.00	40	365	3.5	2.8	0	0.10	0.00	0.01	0.11
181	3	72	72.00	71.94	60.00	163	3,540	7.9	8.9	1	0.31	0.08	0.54	0.93
	2	90	90.00	71.94	90.00	40	331	3.6	3.3	0	0.08	0.00	0.01	0.09
195	3	64	64.00	63.99	60.00	170	3,774	8.0	7.8	32	0.39	0.10	3.35	3.84
	2	90	90.00	63.99	90.00	32	332	2.8	2.8	0	0.09	0.00	0.01	0.10
359	3	76	75.00	74.85	60.00	164	4,001	8.2	11.0	7	0.29	0.08	0.51	0.88
	2	90	90.00	74.85	90.00	42	344	3.9	3.3	0	0.08	0.00	0.01	0.09
360	3	62	62.00	61.92	53.33	154	2,717	7.5	6.7	0	0.32	0.07	0.18	0.57
	2	80	80.00	61.92	80.00	31	293	2.8	3.2	0	0.07	0.00	0.01	0.08
419	3	80	80.00	79.96	66.67	192	4,930	9.1	9.7	607	0.41	0.13	2.37	2.91
	2	100	100.00	79.96	100.00	46	394	4.0	2.8	0	0.11	0.00	0.01	0.12
485	3	71	71.00	70.95	60.00	167	3,522	7.8	7.8	40	0.35	0.08	0.54	0.97
	2	90	90.00	70.95	90.00	38	335	3.3	3.1	0	0.08	0.00	0.01	0.09
531	3	83	83.00	82.84	66.67	181	4,199	8.6	9.9	0	0.34	0.11	0.20	0.65
	2	100	100.00	82.84	100.00	41	355	3.6	3.0	0	0.09	0.00	0.01	0.10
561	3	72	72.00	71.99	66.67	183	4,692	8.7	9.1	136	0.42	0.11	5.59	6.12
	2	100	100.00	71.99	100.00	34	363	3.0	2.6	0	0.11	0.00	0.01	0.12
640	3	74	74.00	73.90	60.00	170	3,753	8.3	8.9	0	0.34	0.12	0.60	1.06
	2	90	90.00	73.90	90.00	39	337	3.5	3.1	0	0.08	0.00	0.01	0.09
645	3	58	58.00	57.99	53.33	146	2,707	6.9	6.4	344	0.33	0.06	1.74	2.13
	2	80	80.00	57.99	80.00	29	289	2.6	3.2	0	0.07	0.00	0.01	0.08
709	3	67	67.00	66.99	60.00	165	4,411	8.1	11.3	41	0.31	0.08	0.81	1.20
	2	90	90.00	66.99	90.00	35	331	3.2	2.9	0	0.09	0.00	0.01	0.10
716	3	76	75.00	74.84	60.00	161	3,318	7.8	9.4	0	0.27	0.06	0.12	0.45
	2	90	90.00	74.84	90.00	38	319	3.4	3.4	0	0.07	0.00	0.01	0.08
742	3	64	64.00	63.94	53.33	154	2,948	7.6	7.7	1	0.30	0.07	1.62	1.99
	2	80	80.00	63.94	80.00	37	311	3.4	3.4	0	0.07	0.00	0.01	0.08
766	3	62	62.00	61.96	53.33	148	3,008	7.4	8.5	4	0.27	0.07	0.40	0.74
	2	80	80.00	61.96	80.00	35	304	3.3	3.5	0	0.07	0.00	0.01	0.08
781	3	71	71.00	71.00	66.67	177	4,034	8.2	7.3	0	0.45	0.11	2.48	3.04
	2	100	100.00	71.00	100.00	32	345	2.8	2.5	0	0.10	0.00	0.01	0.11
785	3	68	67.99	67.98	60.00	168	3,818	8.1	8.7	46	0.35	0.09	1.93	2.37
	2	90	90.00	67.98	90.00	35	331	3.1	2.9	0	0.09	0.00	0.01	0.10
814	3	81	81.00	80.82	66.67	183	4,171	8.6	8.3	0	0.40	0.14	0.23	0.77
	2	100	100.00	80.82	100.00	39	360	3.4	2.8	0	0.10	0.00	0.01	0.11
832	3	60	60.00	59.98	53.33	144	3,497	7.3	10.9	0	0.25	0.07	0.99	1.31
	2	80	80.00	59.98	80.00	27	287	2.6	3.2	0	0.06	0.00	0.01	0.07
900	3	75	75.00	74.98	66.67	182	4,573	8.6	9.2	213	0.40	0.13	3.75	4.28
	2	100	100.00	74.98	100.00	30	344	2.6	2.6	0	0.10	0.00	0.01	0.11

packing problems, for all values of cardinality. We used this approach to solve every instance from many different data sets. Graph compression reduces substantially the graph sizes in cardinality constrained bin packing problems. Figures 3.15 and 3.16

show the relation between the number of vertices and arcs before and after graph compression. The final graph usually has a size comparable to the graph without cardinality constraints, even when cardinality is not excluding any pattern. For some of the instances we knew, by construction, that there would be a solution with at most three items in each bin, but we were not aware of any good method in the literature for solving the cardinality constrained BPP in general. The proposed method allowed us to solve cardinality constrained BPP almost as easily as standard BPP.

Figure 3.15: Graph size reduction (vertices) in cardinality constrained BPP instances.

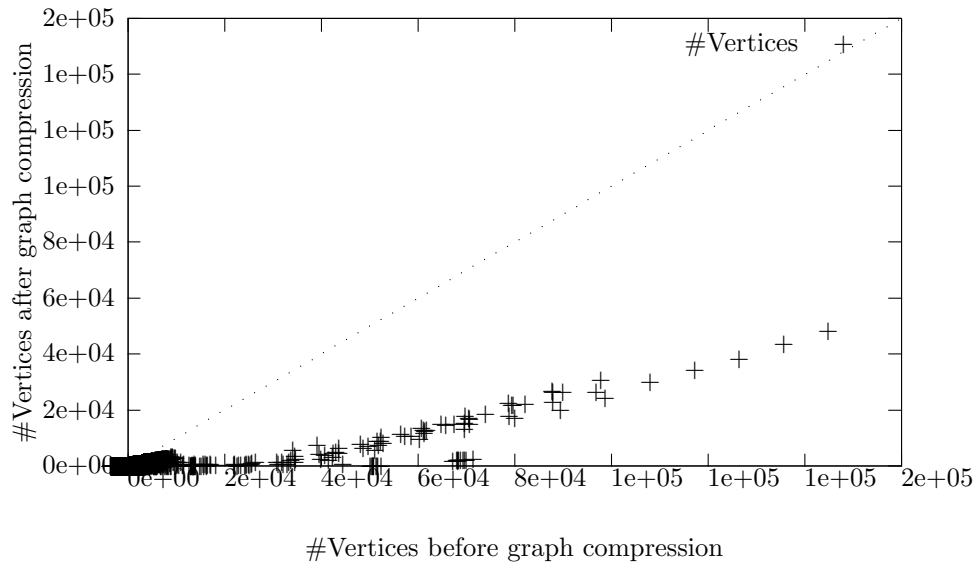
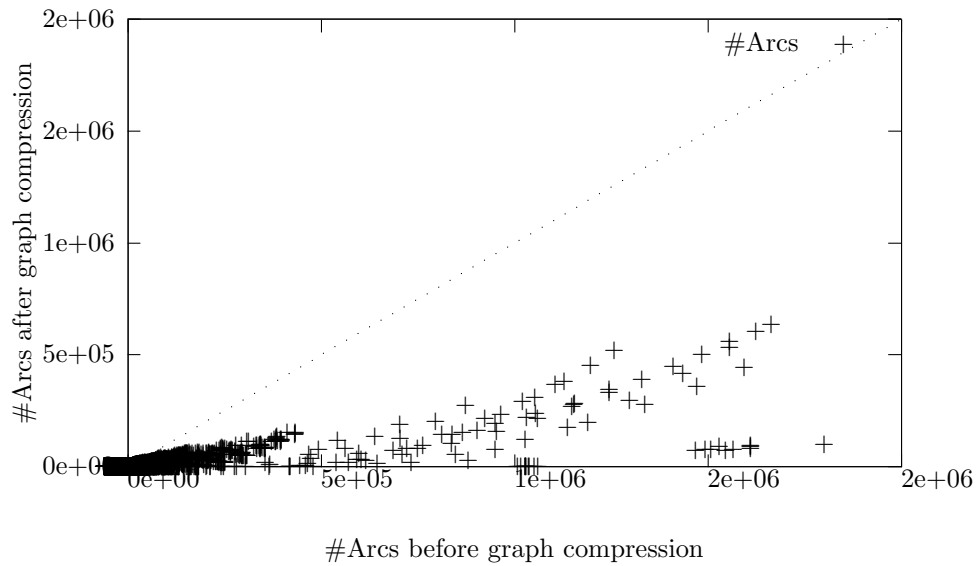


Figure 3.16: Graph size reduction (arcs) in cardinality constrained BPP instances.



3.8.3 Two-constraint bin packing

We used the proposed arc flow formulation to solve 320 of the 400 instances from the DEIS - Operations Research Group Library of Instances (2012) two-constraint bin packing test data set, which was proposed by Caprara and Toth (2001). Table 3.14 summarizes the characteristics of this test data set, and the average run times in seconds for the extended arc flow formulation with graph compression. This data set has several sizes for each class, each pair (class, size) having 10 instances; we report average results in these 10 instances. Tables 3.15, 3.16, 3.17 and 3.18 present more detailed results for $n = 25, 50, 100$ and 200 , respectively. Table 3.19 presents results in the class 10. The rows highlighted in bold correspond to classes for which there were no previously known solutions and that we completely solved using the proposed method.

Table 3.14: Two-constraint bin packing test data set characteristics and results.

class	n	X	Y	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{tot}	$\#op$
1	25	1,000.0	1,000.0	287.5	1,832.2	2.6	8.2	0.00	0.21	0
	50	1,000.0	1,000.0	1,613.0	16,412.6	2.0	7.9	0.00	3.25	0
	100	1,000.0	1,000.0	10,044.2	163,687.2	4.6	11.5	1.10	131.86	5
	200	1,000.0	1,000.0	53,508.6	1,507,591.2	15.8	22.8	0.00	8,275.81	7
2	25	1,000.0	1,000.0	52.1	204.1	8.2	16.2	0.00	0.02	10
	50	1,000.0	1,000.0	137.3	668.4	4.9	11.9	0.00	0.06	10
	100	1,000.0	1,000.0	655.7	4,018.2	3.5	10.4	0.00	0.36	10
	200	1,000.0	1,000.0	9,554.8	79,578.8	5.2	13.9	0.00	36.29	10
3	25	1,000.0	1,000.0	24.5	105.5	12.6	27.4	0.00	0.02	10
	50	1,000.0	1,000.0	42.7	269.6	7.9	25.1	0.00	0.02	10
	100	1,000.0	1,000.0	106.3	918.2	5.3	22.8	0.00	0.06	10
	200	1,000.0	1,000.0	257.8	3,426.3	2.8	17.9	0.00	0.24	10
4	25	1,000.0	1,000.0	16,636.6	108,195.2	7.7	12.3	0.00	425.56	10
6	25	150.0	150.0	49.8	286.2	7.0	19.7	0.00	0.03	0
	50	150.0	150.0	113.3	1,034.5	5.1	19.9	0.00	0.09	1
	100	150.0	150.0	342.7	4,578.2	6.6	22.9	0.00	0.49	5
	200	150.0	150.0	1,171.1	22,566.7	14.4	29.0	0.00	5.11	8
7	25	150.0	150.0	82.4	519.8	6.6	17.0	0.00	0.05	0
	50	150.0	150.0	280.9	2,564.9	10.5	21.4	0.00	0.24	1
	100	150.0	150.0	818.9	11,395.6	20.6	31.0	0.00	1.93	7
	200	150.0	150.0	1,783.9	50,902.3	33.8	48.2	0.00	11.73	3
8	25	150.0	150.0	17.7	132.6	8.1	29.5	0.00	0.02	10
	50	150.0	150.0	29.1	400.6	4.6	26.5	0.00	0.03	10
	100	150.0	150.0	55.8	1,406.3	4.6	31.0	0.00	0.08	10
	200	150.0	150.0	96.1	4,831.8	6.3	36.5	0.00	0.26	10
9	25	915.8	923.9	237.4	1,442.4	3.2	9.7	0.00	0.16	0
	50	890.6	893.2	974.4	9,325.0	2.3	9.5	0.00	1.25	1
	100	960.9	961.4	7,533.6	123,954.7	4.1	11.7	0.00	48.55	10
10	24	100.0	100.0	55.7	376.5	7.1	21.7	0.00	0.03	0
	51	100.0	100.0	193.4	1,972.1	9.0	24.8	0.00	0.13	0
	99	100.0	100.0	895.8	11,956.2	24.6	42.8	0.00	0.95	0
	201	100.0	100.0	2,417.6	61,213.2	46.4	67.8	23.30	70.73	0

The classes 4 and 5 are really hard, even for our method, because of the large number of items that fit in a single bin. Most of the instances that remain open belong to

these two classes. The 8 subclasses that we did not solve contain at least one instance that takes more than 12 hours to solve exactly.

In this problem, the use of interior point methods to solve the linear relaxation at the root node is very important, as the graphs (and the corresponding linear programs) become very large in this case. The average run time is less than 300 seconds.

Table 3.15: Results for two-constraint bin packing results with $n = 25$.

class	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	6.9	6.07	5.76	5.67	5.76	5.67	0.00	0.11	0.03	0.07	0.21
2	14.2	14.00	12.03	12.38	10.80	10.99	0.00	0.02	0.00	0.00	0.02
3	14.2	14.00	12.47	12.80	11.47	11.59	0.00	0.02	0.00	0.00	0.02
4	3.3	2.97	2.87	2.90	2.87	2.90	0.00	6.44	101.84	317.27	425.56
6	10.1	9.70	9.11	9.22	9.07	9.17	0.00	0.02	0.00	0.01	0.03
7	9.6	9.18	9.11	8.89	9.07	8.86	0.00	0.02	0.01	0.02	0.05
8	13.0	12.50	9.11	10.19	9.07	9.92	0.00	0.01	0.00	0.00	0.02
9	7.3	6.38	6.30	6.30	6.30	6.30	0.00	0.07	0.02	0.07	0.16

Table 3.16: Results for two-constraint bin packing with $n = 50$.

class	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	13.5	12.94	12.22	11.86	12.22	11.86	0.00	1.32	0.66	1.27	3.25
2	31.5	31.30	25.74	25.99	23.35	23.51	0.00	0.03	0.01	0.02	0.06
3	31.5	31.30	26.93	27.18	23.99	24.09	0.00	0.01	0.00	0.01	0.02
6	21.5	20.86	19.13	19.22	19.09	19.18	0.00	0.03	0.01	0.04	0.09
7	19.7	19.31	19.13	18.89	19.09	18.88	0.00	0.08	0.05	0.11	0.24
8	25.0	25.00	19.13	19.05	19.09	18.98	0.00	0.02	0.00	0.01	0.03
9	14.5	13.56	13.49	13.49	13.49	13.49	0.00	0.56	0.23	0.46	1.25

Table 3.17: Results for two-constraint bin packing with $n = 100$.

class	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	25.5	25.03	24.67	24.69	24.67	24.69	1.10	14.44	44.25	73.17	131.86
2	57.4	57.40	51.77	52.77	48.94	49.80	0.00	0.20	0.06	0.11	0.36
3	56.9	56.85	51.95	52.00	49.35	49.38	0.00	0.03	0.01	0.01	0.06
6	41.0	40.47	39.42	39.45	39.40	39.43	0.00	0.14	0.07	0.28	0.49
7	40.2	39.64	39.42	39.27	39.40	39.25	0.00	0.29	0.32	1.33	1.93
8	50.0	50.00	39.42	36.89	39.40	36.89	0.00	0.04	0.01	0.03	0.08
9	26.7	25.71	25.69	25.69	25.69	25.69	0.00	10.42	24.31	13.82	48.55

Table 3.18: Results for two-constraint bin packing with $n = 200$.

class	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	50.3	49.83	49.62	49.68	49.62	49.68	0.00	90.03	3,434.73	4,751.05	8,275.81
2	113.5	113.30	101.42	101.67	98.84	99.05	0.00	4.31	16.10	15.88	36.29
3	113.5	113.30	102.38	102.58	99.24	99.37	0.00	0.12	0.03	0.08	0.24
6	81.1	80.61	79.31	79.42	79.31	79.42	0.00	0.72	0.60	3.79	5.11
7	80.1	79.53	79.31	79.17	79.31	79.17	0.00	1.07	3.34	7.33	11.73
8	100.0	100.00	79.31	73.30	79.31	73.30	0.00	0.13	0.04	0.10	0.26

Note that among the instances presented in Table 3.14 there were 178 instances with no previously known optimum. We solved to optimality 320 instances out of the

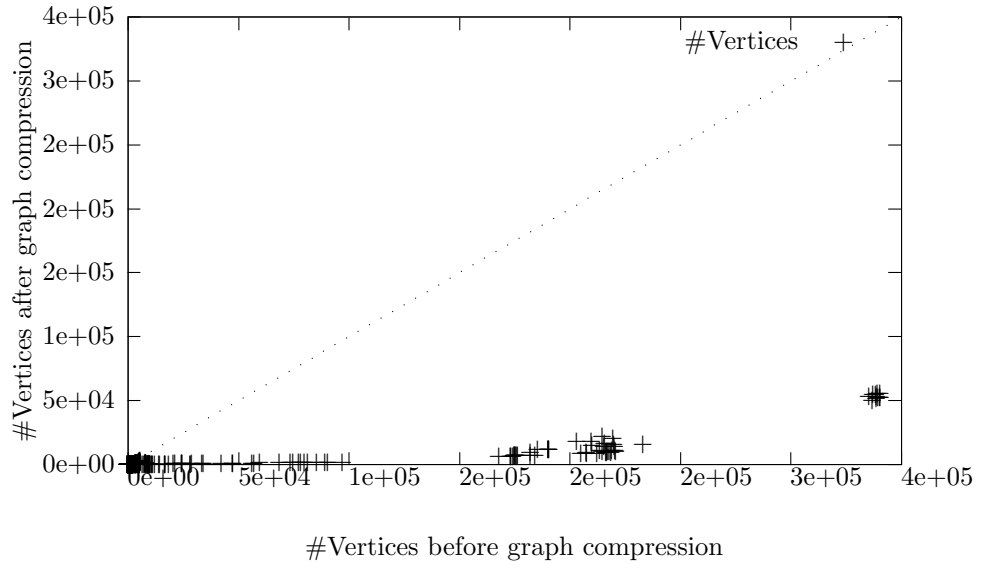
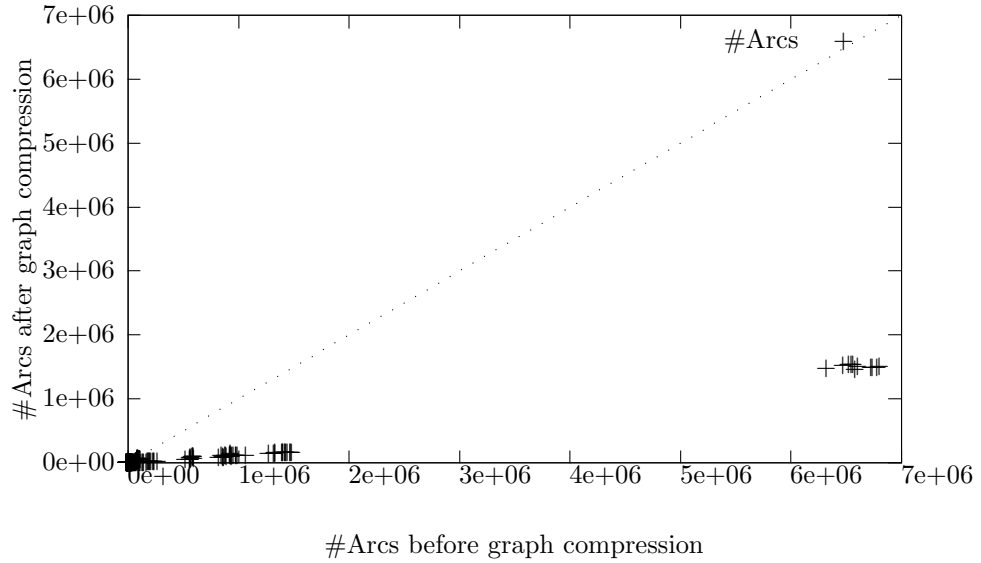
Table 3.19: Results for two-constraint bin packing in the class 10.

n	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
24	8.0	8.00	8.00	8.00	8.00	8.00	0.00	0.02	0.00	0.01	0.03
51	17.0	17.00	17.00	17.00	17.00	17.00	0.00	0.06	0.02	0.05	0.13
99	33.0	33.00	33.00	33.00	33.00	33.00	0.00	0.25	0.30	0.40	0.95
201	67.0	67.00	66.84	67.00	66.84	67.00	23.30	1.04	7.29	62.40	70.73

400 instances and there are still 80 open instances. For more detailed results and Gurobi's logs please consult the web-page <http://www.dcc.fc.up.pt/~fdabrandao/research/arcflow/results/>.

One may ask why so many of these instances could not be solved before. A reasonable explanation may be the fact that, for example, in instances from classes 2, 3 and 8, the lower bound provided by assignment-based formulations is rather loose. Also, in some of these instances, the average number of items per bin in the optimal solution is reasonably large, which leads to an extremely large number of possible patterns. Caprara and Toth (2001) were not able to compute the linear relaxation of many of these instances using Gilmore-Gomory's model and column generation within 100,000 seconds (more than 27 hours). Note that the computer they used is much slower than the one we used. Nevertheless, this shows how hard it can be to compute a linear relaxation of Gilmore-Gomory's model as the number of patterns increases and the subproblems become harder to solve. With the proposed method, graph compression leads to very large reductions in the graph size and allows us to represent all these patterns in reasonably small graphs. None of the instances from these classes has been solved before.

Figures 3.17 and 3.18 show the relation between the number of vertices and arcs before and after graph compression. The graph compression algorithm is remarkably effective in every two-constraint bin packing instance. Without graph compression it would be almost impossible to solve many of these instances within a reasonable amount of time. Moreover, some of these graphs without compression occupy a huge amount of memory.

Figure 3.17: Graph size reduction (vertices) in two-constraint bin packing instances.**Figure 3.18:** Graph size reduction (arcs) in two-constraint bin packing instances.

3.9 Run time analysis

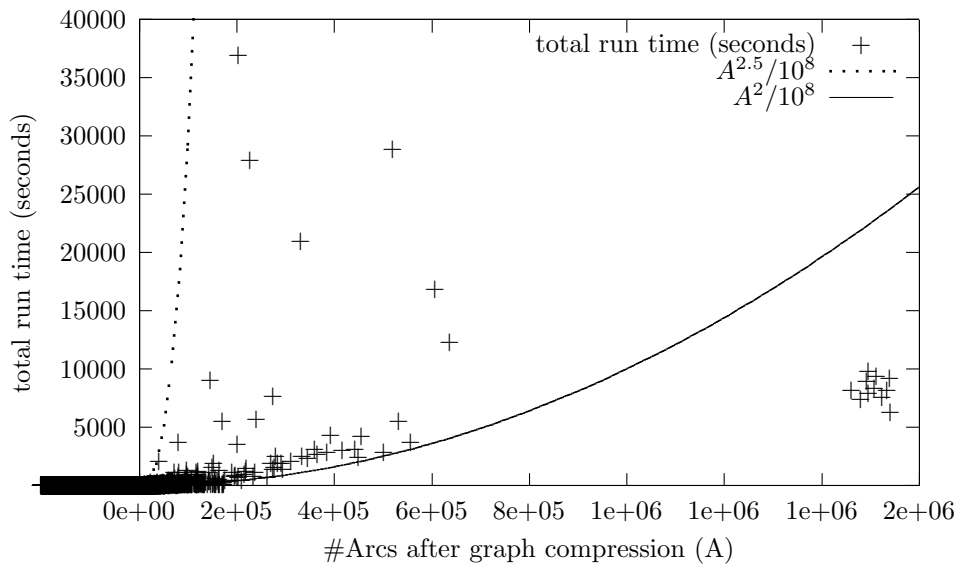
Using the proposed method, we solved sequentially 17,487 benchmark instances in 5 days, spending less than 25 seconds per instance, on average. These benchmark instances belong to four different problems and were solved using the same method. The four problems we considered are: bin packing, cutting stock, cardinality constrained bin packing and two-constraint bin packing. The same method was used to solve all

the problems without any modification based on the problem.

CPU times were obtained using a computer with two Quad-Core Intel Xeon at 2.66GHz, running Mac OS X 10.8.0, with 16 GBytes of memory. All the algorithms were implemented in C++, and Gurobi 5.0.0 was used to solve the generated arc flow model. The parameters used in Gurobi were Threads = 1 (single thread), Presolve = 1 (conservative), Method = 2 (Interior point methods), MIPFocus = 1 (feasible solutions), Heuristics = 1, MIPGap = 0, MIPGapAbs = 1-1e-5 and the remaining parameters were Gurobi's default values.

Figure 3.19 shows the relation between the number of arcs after graph compression and the total run time. The two curves $A^2/10^8$ and $A^{2.5}/10^8$ show an approximation of the run time (in seconds) of algorithms with complexities $\Theta(n^2)$ and $\Theta(n^{2.5})$, with very low constant factors, on the computer we used. The large majority of the observed run times appear between these two curves. Many of them are very close to the quadratic run time, which is impressive since solving the arc flow model is NP-hard. The few instances that lead to run times far from quadratic are mainly instances where the number of items that fit in each bin is large (e.g., more than 10) and hence the total number of patterns is huge. Using heuristics, very good solutions are usually found for these instances easily, since the waste tends to be small. Nevertheless, it may be really hard to find the optimal solution.

Figure 3.19: Run time analysis.



3.10 Conclusions

The method presented in Section 3.2 proved to be very powerful for solving bin packing problems, including cardinality constrained and two-constraint variants. The proposed formulation is simple, but provides a very strong lower bound in every problem considered.

The presented graph compression method is simple and proved to be very effective in many instances. It may increase the symmetry sometimes, but this is not an issue as long as it leads to large reductions in the graph size.

Using the proposed method and **Gurobi**, 17,487 benchmark instances were solved within 5 days, spending on average less than 25 seconds per instance. Among these instances, thousands are not known to have been previously solved. Large cutting stock instances with millions of items were solved quickly. Cardinality constrained bin packing problems were solved almost as easily as standard bin packing problems. Furthermore, many previously open two-constraint bin packing instances were also solved exactly.

In order to test the limits of our method, we also generated bin packing and cutting stock instances with up to one thousand different item sizes. Near-optimal solutions were usually found in a few minutes and many of these instances were solved exactly in a few hours. These results are even more impressive considering that today's hardest instances contain no more than two hundred different item sizes.

Our method creates models that can be solved using any general-purpose mixed-integer programming solver. Using **GLPK**, an open-source solver, instead of **Gurobi**, it has been possible to solve exactly most of the instances used in our experiments. However, some instances take a very long time to solve. Nevertheless, these results have shown that even with a non-commercial mixed-integer programming solver it is possible to solve many benchmark instances easily using the proposed model.

Chapter 4

Conclusions

We proposed novel exact and approximate methods for solving bin packing and related problems. The proposed methods show a large advantage in practice when compared with traditional ones. For the cutting stock problem, we presented pattern-based heuristics that allow finding solutions for extremely large cutting stock instances, with billions of items, in a very short amount of time. For bin packing, cutting stock, cardinality constrained bin packing and two-constraint bin packing, we presented a very simple and powerful method that allows solving exactly many thousands of benchmark instances in a few seconds, on average.

The conventional assignment-based first/best fit decreasing algorithms (FFD/BFD) are not polynomial in the cutting stock input size in its most common format. Therefore, even for small cutting stock instances with large demands, they may be unable to compute FFD/BFD solutions. We presented pattern-based FFD and BFD algorithms for cutting stock problems. The pattern-based algorithms do not make assignments piece-by-piece to rolls and, therefore, they can be faster than linear in the number of items. The efficient pattern-based FFD/BFD implementations run in polynomial time in the cutting stock input size. These algorithms are as fast as the best known FFD/BFD implementations in BPP instances and they are much faster than any conventional assignment-based algorithm in cutting stock instances. We also presented a reasonably efficient algorithm for the sum-of-squares decreasing algorithm, an off-line variant of the on-line sum-of-squares heuristics, which was compared with the two well-known off-line heuristics FFD/BFD.

Conventional formulations for bin packing problems are usually highly symmetric and provide very weak lower bounds. We proposed an exact method, based on an arc flow

formulation with side constraints, which solves bin packing problems — including two-constraint variants — by simply representing all the patterns in a very compact graph. The formulation is simple, but provides a very strong lower bound in every problem we considered. We also presented a graph compression algorithm that usually reduces substantially the size of the underlying graph without weakening the bounds. The proposed exact method allowed us to solve many previously open instances. Actually, using the proposed formulation and a state-of-the-art mixed-integer programming solver, current benchmark instances for standard bin packing, cutting stock and cardinality constrained bin packing are not very challenging anymore. Our method solved every data set we found in the literature for bin packing and cutting stock with and without cardinality constraints. We solved 17,487 benchmark instances in 5 days, spending less than 25 seconds per instance, on average. Among these instances there are thousands of previously open instances. For the two-constraint bin packing problem, we were able to solve 178 previously open instances, but there are still instances that could not be solved within a reasonable time.

Our exact method can also be easily extended for solving p -dimensional vector packing by generalizing the idea that we used to solve two-constraint bin packing problems. Beyond that, it remains unknown whether there are more applications of this method in other problems.

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Appendix A

Detailed results

Table A.1: Bin packing results.

instance	z^*	lb^{LP}	lb^{SP}	New graph					VC's graph				
				n^{bb}	t^{PP}	t^{LP}	t^{IP}	t^{tot}	n^{bb}	t^{PP}	t^{LP}	t^{IP}	t^{tot}
u120.00	48	47.27	47.19	0	0.03	0.03	0.10	0.16	0	0.02	0.02	0.42	0.46
u120.01	49	48.05	48.03	0	0.03	0.03	0.07	0.13	0	0.02	0.02	0.06	0.10
u120.02	46	45.29	45.29	0	0.03	0.04	0.14	0.21	0	0.02	0.04	0.20	0.26
u120.03	49	48.63	48.57	0	0.03	0.03	0.10	0.16	0	0.02	0.02	0.28	0.32
u120.04	50	49.09	49.03	0	0.03	0.03	0.09	0.15	0	0.03	0.02	0.07	0.12
u120.05	48	47.49	47.48	0	0.03	0.03	0.09	0.15	0	0.03	0.03	0.06	0.12
u120.06	48	47.58	47.58	0	0.04	0.04	0.13	0.21	0	0.02	0.04	0.70	0.76
u120.07	49	48.66	48.63	0	0.03	0.03	0.11	0.17	0	0.02	0.02	0.34	0.38
u120.08	50	49.91	49.85	0	0.03	0.03	0.10	0.16	0	0.03	0.03	0.20	0.26
u120.09	46	45.80	45.80	0	0.03	0.04	0.22	0.29	0	0.02	0.04	0.67	0.73
u120.10	52	51.28	51.20	0	0.03	0.03	0.07	0.13	0	0.02	0.02	0.41	0.45
u120.11	49	48.39	48.31	0	0.03	0.03	0.07	0.13	0	0.03	0.02	0.14	0.19
u120.12	48	47.87	47.87	0	0.03	0.04	0.14	0.21	0	0.02	0.04	0.73	0.79
u120.13	49	48.01	48.01	0	0.03	0.03	0.10	0.16	0	0.03	0.03	0.09	0.15
u120.14	50	49.17	49.15	0	0.03	0.03	0.08	0.14	0	0.02	0.02	0.20	0.24
u120.15	48	47.38	47.35	0	0.03	0.03	0.09	0.15	0	0.02	0.03	0.07	0.12
u120.16	52	51.33	51.25	0	0.03	0.03	0.06	0.12	0	0.02	0.02	0.29	0.33
u120.17	52	51.50	51.35	0	0.03	0.02	0.07	0.12	0	0.02	0.02	0.04	0.08
u120.18	49	48.38	48.37	0	0.03	0.03	0.09	0.15	0	0.03	0.03	0.04	0.10
u120.19	49	48.86	48.81	0	0.04	0.04	0.09	0.17	0	0.02	0.03	0.10	0.15
u250.00	99	98.55	98.55	0	0.03	0.04	0.18	0.25	0	0.03	0.04	0.10	0.17
u250.01	100	99.03	99.03	0	0.04	0.06	0.18	0.28	0	0.03	0.05	0.17	0.25
u250.02	102	101.42	101.42	0	0.03	0.04	0.14	0.21	0	0.03	0.03	0.10	0.16
u250.03	100	99.43	99.43	0	0.04	0.05	0.20	0.29	0	0.03	0.04	0.52	0.59
u250.04	101	100.61	100.61	0	0.03	0.05	0.22	0.30	0	0.03	0.04	0.97	1.04
u250.05	101	100.83	100.83	0	0.04	0.06	0.38	0.48	0	0.02	0.04	0.49	0.55
u250.06	102	101.03	101.03	0	0.04	0.05	0.21	0.30	0	0.03	0.05	0.16	0.24
u250.07	103	102.89	102.79	0	0.04	0.04	0.18	0.26	0	0.03	0.03	0.47	0.53
u250.08	105	104.92	104.91	0	0.04	0.04	0.13	0.21	0	0.03	0.04	0.49	0.56
u250.09	101	100.20	100.20	0	0.04	0.05	0.12	0.21	0	0.03	0.04	0.56	0.63
u250.10	105	104.39	104.37	0	0.04	0.04	0.11	0.19	0	0.03	0.04	0.69	0.76
u250.11	101	100.71	100.71	0	0.04	0.06	0.18	0.28	0	0.03	0.06	0.25	0.34
u250.12	105	104.98	104.93	0	0.04	0.04	0.49	0.57	0	0.03	0.03	0.70	0.76
u250.13	103	102.04	101.96	0	0.04	0.04	0.11	0.19	0	0.03	0.03	0.09	0.15
u250.14	100	99.17	99.17	0	0.04	0.05	0.17	0.26	0	0.03	0.05	0.12	0.20
u250.15	105	104.86	104.81	0	0.04	0.04	0.15	0.23	0	0.03	0.03	0.96	1.02
u250.16	97	96.51	96.51	0	0.04	0.05	0.20	0.29	0	0.03	0.05	0.31	0.39
u250.17	100	99.17	99.17	0	0.04	0.06	0.27	0.37	0	0.03	0.05	0.63	0.71
u250.18	100	99.70	99.70	0	0.04	0.05	0.21	0.30	0	0.02	0.05	0.38	0.45
u250.19	102	101.36	101.36	0	0.04	0.06	0.16	0.26	0	0.03	0.05	0.28	0.36

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instance	z^*	lb^lP	lb^{SP}	New graph					VC's graph				
				n^{bb}	t^{PP}	t^{lP}	t^{iP}	t^{tot}	n^{bb}	t^{PP}	t^{lP}	t^{iP}	t^{tot}
u500_00	198	197.58	197.58	0	0.04	0.06	0.18	0.28	0	0.03	0.05	0.47	0.55
u500_01	201	200.85	200.85	0	0.05	0.06	0.21	0.32	0	0.03	0.05	0.39	0.47
u500_02	202	201.44	201.44	0	0.05	0.05	0.22	0.32	0	0.03	0.05	0.41	0.49
u500_03	204	203.81	203.81	0	0.04	0.06	0.27	0.37	0	0.03	0.05	0.40	0.48
u500_04	206	205.11	205.11	0	0.04	0.05	0.21	0.30	0	0.03	0.05	0.15	0.23
u500_05	206	205.09	205.09	0	0.04	0.06	0.20	0.30	0	0.03	0.05	0.13	0.21
u500_06	207	206.91	206.89	0	0.05	0.05	0.17	0.27	0	0.03	0.04	0.38	0.45
u500_07	204	203.98	203.98	0	0.04	0.06	0.23	0.33	3	0.03	0.04	2.35	2.42
u500_08	196	195.68	195.68	0	0.05	0.05	0.21	0.31	0	0.03	0.05	1.13	1.21
u500_09	202	201.06	201.06	0	0.04	0.06	0.17	0.27	0	0.03	0.05	0.43	0.51
u500_10	200	199.07	199.07	0	0.05	0.06	0.16	0.27	0	0.03	0.05	0.17	0.25
u500_11	200	199.43	199.43	0	0.04	0.05	0.15	0.24	0	0.03	0.05	0.15	0.23
u500_12	199	198.62	198.62	0	0.05	0.05	0.16	0.26	0	0.03	0.05	0.91	0.99
u500_13	196	195.59	195.59	0	0.04	0.05	0.24	0.33	0	0.03	0.05	0.11	0.19
u500_14	204	203.03	203.03	0	0.05	0.06	0.15	0.26	0	0.03	0.05	0.12	0.20
u500_15	201	200.13	200.13	0	0.04	0.06	0.22	0.32	0	0.03	0.05	0.32	0.40
u500_16	202	201.01	201.01	0	0.04	0.06	0.18	0.28	0	0.03	0.05	0.44	0.52
u500_17	198	197.43	197.43	0	0.04	0.06	0.20	0.30	0	0.03	0.05	0.20	0.28
u500_18	202	201.29	201.29	0	0.05	0.06	0.28	0.39	0	0.02	0.04	0.16	0.22
u500_19	196	195.63	195.63	0	0.04	0.06	0.23	0.33	0	0.03	0.05	0.57	0.65
u1000_00	399	398.43	398.43	0	0.04	0.05	0.26	0.35	0	0.03	0.05	0.35	0.43
u1000_01	406	405.25	405.25	0	0.04	0.05	0.21	0.30	0	0.03	0.05	0.37	0.45
u1000_02	411	410.20	410.20	0	0.04	0.05	0.20	0.29	0	0.03	0.05	0.11	0.19
u1000_03	411	410.87	410.87	0	0.04	0.06	0.30	0.40	0	0.03	0.05	0.39	0.47
u1000_04	397	396.74	396.74	0	0.05	0.06	0.26	0.37	0	0.03	0.04	0.39	0.46
u1000_05	399	398.49	398.49	0	0.04	0.05	0.29	0.38	0	0.03	0.05	0.36	0.44
u1000_06	395	394.21	394.21	0	0.04	0.05	0.23	0.32	0	0.03	0.05	0.40	0.48
u1000_07	404	403.16	403.16	0	0.05	0.05	0.27	0.37	0	0.03	0.05	0.13	0.21
u1000_08	399	398.43	398.43	0	0.04	0.05	0.17	0.26	0	0.03	0.05	0.13	0.21
u1000_09	397	396.93	396.93	0	0.04	0.05	0.26	0.35	0	0.05	0.05	0.46	0.56
u1000_10	400	399.34	399.34	0	0.04	0.05	0.28	0.37	0	0.02	0.04	0.33	0.39
u1000_11	401	400.52	400.52	0	0.04	0.05	0.29	0.38	0	0.03	0.05	0.30	0.38
u1000_12	393	392.24	392.24	0	0.04	0.05	0.20	0.29	0	0.03	0.04	0.16	0.23
u1000_13	396	395.27	395.27	0	0.04	0.05	0.24	0.33	0	0.03	0.04	0.16	0.23
u1000_14	394	393.89	393.89	0	0.04	0.05	0.21	0.30	0	0.04	0.04	0.36	0.44
u1000_15	402	401.81	401.81	0	0.04	0.05	0.17	0.26	0	0.03	0.05	0.47	0.55
u1000_16	404	403.03	403.03	0	0.04	0.05	0.15	0.24	0	0.03	0.05	0.15	0.23
u1000_17	404	403.80	403.80	0	0.04	0.06	0.24	0.34	0	0.03	0.05	0.40	0.48
u1000_18	399	398.19	398.19	0	0.04	0.05	0.27	0.36	0	0.03	0.04	0.16	0.23
u1000_19	400	399.33	399.33	0	0.04	0.05	0.24	0.33	0	0.03	0.04	0.13	0.20
t60_00	20	20.00	20.00	0	0.04	0.01	0.06	0.11	0	0.04	0.05	0.10	0.19
t60_01	20	20.00	20.00	0	0.05	0.01	0.06	0.12	0	0.04	0.06	0.90	1.00
t60_02	20	20.00	20.00	0	0.04	0.01	0.02	0.07	0	0.03	0.05	0.49	0.57
t60_03	20	20.00	20.00	0	0.04	0.01	0.03	0.08	0	0.03	0.04	0.44	0.51
t60_04	20	20.00	20.00	0	0.04	0.01	0.02	0.07	0	0.04	0.04	0.27	0.35
t60_05	20	20.00	20.00	0	0.04	0.01	0.03	0.08	0	0.04	0.05	0.09	0.18
t60_06	20	20.00	20.00	0	0.04	0.01	0.04	0.09	0	0.04	0.05	0.65	0.74
t60_07	20	20.00	20.00	0	0.04	0.01	0.03	0.08	0	0.04	0.05	0.47	0.56
t60_08	20	20.00	20.00	0	0.04	0.01	0.02	0.07	0	0.04	0.05	0.43	0.52
t60_09	20	20.00	20.00	0	0.04	0.01	0.02	0.07	0	0.04	0.05	0.42	0.51
t60_10	20	20.00	20.00	0	0.04	0.01	0.08	0.13	0	0.04	0.06	0.94	1.04
t60_11	20	20.00	20.00	0	0.04	0.01	0.02	0.07	0	0.03	0.04	0.39	0.46
t60_12	20	20.00	20.00	0	0.05	0.01	0.03	0.09	0	0.04	0.05	0.56	0.65
t60_13	20	20.00	20.00	0	0.04	0.01	0.03	0.08	0	0.05	0.05	0.23	0.33
t60_14	20	20.00	20.00	0	0.04	0.01	0.03	0.08	0	0.04	0.05	0.10	0.19
t60_15	20	20.00	20.00	0	0.05	0.01	0.03	0.09	0	0.04	0.05	0.51	0.60
t60_16	20	20.00	20.00	0	0.04	0.01	0.02	0.07	0	0.03	0.04	0.44	0.51
t60_17	20	20.00	20.00	0	0.04	0.01	0.02	0.07	0	0.04	0.04	0.48	0.56
t60_18	20	20.00	20.00	0	0.04	0.01	0.03	0.08	0	0.04	0.04	0.10	0.18
t60_19	20	20.00	20.00	0	0.04	0.01	0.03	0.08	0	0.04	0.05	0.12	0.21
t120_00	40	40.00	40.00	0	0.08	0.03	0.16	0.27	0	0.06	0.11	0.35	0.52

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instance	z^*	lb^{lp}	lb^{sp}	New graph					VC's graph				
				n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t120.01	40	40.00	40.00	0	0.07	0.03	0.16	0.26	0	0.06	0.09	0.97	1.12
t120.02	40	40.00	40.00	0	0.09	0.03	0.23	0.35	0	0.07	0.11	1.57	1.75
t120.03	40	40.00	40.00	0	0.08	0.03	0.07	0.18	0	0.07	0.12	1.09	1.28
t120.04	40	40.00	40.00	0	0.09	0.03	0.26	0.38	0	0.07	0.12	0.28	0.47
t120.05	40	40.00	40.00	0	0.09	0.03	0.10	0.22	0	0.07	0.11	1.73	1.91
t120.06	40	40.00	40.00	0	0.08	0.03	0.17	0.28	0	0.06	0.10	2.17	2.33
t120.07	40	40.00	40.00	0	0.09	0.03	0.27	0.39	0	0.06	0.10	1.20	1.36
t120.08	40	40.00	40.00	0	0.08	0.03	0.41	0.52	0	0.07	0.10	1.13	1.30
t120.09	40	40.00	40.00	0	0.08	0.03	0.08	0.19	0	0.06	0.10	1.89	2.05
t120.10	40	40.00	40.00	0	0.08	0.02	0.08	0.18	0	0.06	0.10	1.64	1.80
t120.11	40	40.00	40.00	0	0.08	0.03	0.22	0.33	0	0.06	0.11	2.44	2.61
t120.12	40	40.00	40.00	0	0.08	0.03	0.14	0.25	0	0.06	0.09	1.03	1.18
t120.13	40	40.00	40.00	0	0.08	0.03	0.11	0.22	0	0.07	0.11	1.16	1.34
t120.14	40	40.00	40.00	0	0.08	0.03	0.08	0.19	0	0.06	0.10	1.13	1.29
t120.15	40	40.00	40.00	0	0.07	0.03	0.07	0.17	0	0.06	0.10	2.17	2.33
t120.16	40	40.00	40.00	0	0.08	0.03	0.19	0.30	0	0.06	0.10	1.52	1.68
t120.17	40	40.00	40.00	16	0.09	0.03	0.77	0.89	0	0.07	0.11	1.72	1.90
t120.18	40	40.00	40.00	0	0.08	0.03	0.08	0.19	0	0.06	0.10	1.06	1.22
t120.19	40	40.00	40.00	0	0.09	0.04	0.07	0.20	0	0.07	0.10	1.90	2.07
t249.00	83	83.00	83.00	0	0.15	0.10	0.46	0.71	0	0.11	0.21	2.01	2.33
t249.01	83	83.00	83.00	0	0.16	0.10	0.70	0.96	0	0.10	0.23	2.60	2.93
t249.02	83	83.00	83.00	0	0.15	0.11	0.57	0.83	0	0.11	0.26	3.89	4.26
t249.03	83	83.00	83.00	0	0.16	0.12	1.05	1.33	0	0.11	0.24	2.64	2.99
t249.04	83	83.00	83.00	0	0.15	0.09	0.21	0.45	0	0.11	0.22	0.46	0.79
t249.05	83	83.00	83.00	0	0.17	0.13	0.63	0.93	0	0.11	0.26	4.42	4.79
t249.06	83	83.00	83.00	0	0.15	0.10	0.48	0.73	0	0.11	0.24	3.46	3.81
t249.07	83	83.00	83.00	0	0.15	0.11	0.78	1.04	0	0.10	0.21	0.81	1.12
t249.08	83	83.00	83.00	0	0.16	0.11	0.32	0.59	0	0.10	0.23	2.15	2.48
t249.09	83	83.00	83.00	0	0.15	0.10	0.32	0.57	0	0.10	0.26	0.50	0.86
t249.10	83	83.00	83.00	0	0.16	0.11	0.52	0.79	0	0.11	0.24	2.10	2.45
t249.11	83	83.00	83.00	0	0.16	0.14	0.89	1.19	0	0.11	0.23	4.56	4.90
t249.12	83	83.00	83.00	0	0.15	0.11	0.75	1.01	0	0.11	0.24	2.93	3.28
t249.13	83	83.00	83.00	0	0.16	0.12	1.42	1.70	0	0.11	0.24	1.66	2.01
t249.14	83	83.00	83.00	0	0.17	0.11	2.01	2.29	0	0.12	0.28	2.80	3.20
t249.15	83	83.00	83.00	0	0.17	0.10	1.10	1.37	0	0.11	0.26	3.60	3.97
t249.16	83	83.00	83.00	0	0.17	0.12	0.60	0.89	0	0.12	0.26	2.28	2.66
t249.17	83	83.00	83.00	0	0.16	0.12	1.13	1.41	0	0.11	0.25	2.22	2.58
t249.18	83	83.00	83.00	0	0.15	0.10	0.96	1.21	0	0.10	0.23	2.49	2.82
t249.19	83	83.00	83.00	0	0.16	0.09	0.50	0.75	0	0.11	0.21	0.55	0.87
t501.00	167	167.00	167.00	0	0.26	0.27	1.05	1.58	0	0.17	0.42	4.66	5.25
t501.01	167	167.00	167.00	0	0.26	0.27	1.48	2.01	0	0.18	0.51	1.50	2.19
t501.02	167	167.00	167.00	0	0.27	0.25	1.54	2.06	0	0.17	0.51	5.35	6.03
t501.03	167	167.00	167.00	0	0.29	0.30	2.67	3.26	0	0.18	0.56	5.87	6.61
t501.04	167	167.00	167.00	0	0.27	0.26	1.85	2.38	0	0.17	0.52	3.98	4.67
t501.05	167	167.00	167.00	0	0.28	0.29	1.37	1.94	0	0.17	0.48	1.96	2.61
t501.06	167	167.00	167.00	0	0.28	0.27	1.90	2.45	0	0.17	0.51	2.12	2.80
t501.07	167	167.00	167.00	0	0.27	0.27	1.49	2.03	0	0.17	0.48	4.78	5.43
t501.08	167	167.00	167.00	0	0.28	0.27	1.95	2.50	0	0.17	0.50	5.49	6.16
t501.09	167	167.00	167.00	0	0.26	0.26	0.64	1.16	0	0.17	0.45	2.96	3.58
t501.10	167	167.00	167.00	0	0.26	0.23	2.90	3.39	0	0.17	0.46	1.83	2.46
t501.11	167	167.00	167.00	0	0.27	0.28	1.34	1.89	0	0.17	0.47	2.59	3.23
t501.12	167	167.00	167.00	0	0.26	0.25	2.89	3.40	0	0.16	0.43	3.47	4.06
t501.13	167	167.00	167.00	0	0.27	0.30	1.83	2.40	0	0.18	0.50	5.55	6.23
t501.14	167	167.00	167.00	0	0.29	0.27	1.85	2.41	0	0.18	0.53	6.33	7.04
t501.15	167	167.00	167.00	0	0.27	0.28	1.88	2.43	0	0.18	0.49	4.83	5.50
t501.16	167	167.00	167.00	0	0.27	0.29	1.32	1.88	0	0.18	0.50	4.15	4.83
t501.17	167	167.00	167.00	0	0.28	0.27	1.31	1.86	0	0.18	0.47	3.04	3.69
t501.18	167	167.00	167.00	0	0.27	0.26	0.75	1.28	0	0.17	0.46	5.23	5.86
t501.19	167	167.00	167.00	0	0.26	0.25	1.54	2.05	0	0.16	0.47	4.35	4.98

Table A.2: Cutting stock results.

instance	z^*	lb^b	lb^{sp}	n	$\#v$	$\#a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120.00	47,265,958	4.7e+07	4.7e+07	120,000,000	102	1,834	0	0.03	0.04	0.12	0.19
u120.01	48,048,612	4.8e+07	4.8e+07	120,000,000	100	1,878	0	0.03	0.03	0.10	0.16
u120.02	45,293,334	4.5e+07	4.5e+07	120,000,000	109	2,136	0	0.03	0.04	0.34	0.41
u120.03	48,623,077	4.9e+07	4.9e+07	120,000,000	105	2,244	0	0.03	0.04	0.26	0.33
u120.04	49,085,035	4.9e+07	4.9e+07	120,000,000	100	1,914	0	0.03	0.04	0.17	0.24
u120.05	47,486,395	4.7e+07	4.7e+07	120,000,000	101	1,932	0	0.03	0.04	0.11	0.18
u120.06	47,580,000	4.8e+07	4.8e+07	120,000,000	106	2,174	0	0.03	0.05	0.06	0.14
u120.07	48,656,463	4.9e+07	4.9e+07	120,000,000	94	1,908	0	0.03	0.03	0.08	0.14
u120.08	49,911,565	5e+07	5e+07	120,000,000	105	2,237	0	0.04	0.04	0.24	0.32
u120.09	45,800,000	4.6e+07	4.6e+07	120,000,000	105	2,185	0	0.03	0.04	0.05	0.12
u120.10	51,280,317	5.1e+07	5.1e+07	120,000,000	98	1,918	0	0.03	0.03	0.18	0.24
u120.11	48,392,858	4.8e+07	4.8e+07	120,000,000	97	1,702	0	0.03	0.03	0.06	0.12
u120.12	47,866,667	4.8e+07	4.8e+07	120,000,000	104	2,186	0	0.03	0.04	0.25	0.32
u120.13	48,013,334	4.8e+07	4.8e+07	120,000,000	102	1,954	0	0.03	0.04	0.13	0.20
u120.14	49,166,667	4.9e+07	4.9e+07	120,000,000	97	1,792	0	0.03	0.03	0.20	0.26
u120.15	47,384,058	4.7e+07	4.7e+07	120,000,000	102	1,974	0	0.03	0.03	0.46	0.52
u120.16	51,333,334	5.1e+07	5.1e+07	120,000,000	93	1,812	0	0.03	0.03	0.05	0.11
u120.17	51,500,000	5.2e+07	5.1e+07	120,000,000	93	1,785	0	0.03	0.03	0.04	0.10
u120.18	48,381,503	4.8e+07	4.8e+07	120,000,000	100	2,084	0	0.03	0.03	0.20	0.26
u120.19	48,860,545	4.9e+07	4.9e+07	120,000,000	109	2,277	0	0.04	0.04	0.18	0.26
u250.00	98,553,334	9.9e+07	9.9e+07	250,000,000	106	2,305	0	0.04	0.04	0.18	0.26
u250.01	99,026,667	9.9e+07	9.9e+07	250,000,000	112	2,834	0	0.04	0.06	0.34	0.44
u250.02	101,421,769	1e+08	1e+08	250,000,000	108	2,737	0	0.04	0.06	0.65	0.75
u250.03	99,426,667	9.9e+07	9.9e+07	250,000,000	110	2,821	0	0.04	0.05	0.21	0.30
u250.04	100,613,334	1e+08	1e+08	250,000,000	110	2,653	0	0.04	0.05	0.30	0.39
u250.05	100,826,667	1e+08	1e+08	250,000,000	110	2,760	0	0.04	0.06	0.28	0.38
u250.06	101,026,667	1e+08	1e+08	250,000,000	112	2,857	0	0.04	0.06	0.52	0.62
u250.07	102,885,186	1e+08	1e+08	250,000,000	108	2,688	0	0.05	0.05	0.14	0.24
u250.08	104,918,368	1e+08	1e+08	250,000,000	109	2,730	0	0.04	0.04	0.16	0.24
u250.09	100,201,389	1e+08	1e+08	250,000,000	111	2,806	0	0.05	0.05	0.24	0.34
u250.10	104,391,157	1e+08	1e+08	250,000,000	108	2,694	0	0.04	0.05	0.44	0.53
u250.11	100,713,334	1e+08	1e+08	250,000,000	110	2,862	0	0.04	0.06	0.14	0.24
u250.12	104,977,163	1e+08	1e+08	250,000,000	105	2,586	0	0.04	0.05	0.31	0.40
u250.13	102,036,586	1e+08	1e+08	250,000,000	110	2,645	0	0.03	0.04	0.17	0.24
u250.14	99,166,667	9.9e+07	9.9e+07	250,000,000	112	2,839	0	0.04	0.06	0.32	0.42
u250.15	104,861,112	1e+08	1e+08	250,000,000	108	2,760	0	0.04	0.05	0.12	0.21
u250.16	96,513,334	9.7e+07	9.7e+07	250,000,000	112	2,763	0	0.04	0.06	0.73	0.83
u250.17	99,166,667	9.9e+07	9.9e+07	250,000,000	110	2,723	0	0.04	0.05	0.30	0.39
u250.18	99,700,000	1e+08	1e+08	250,000,000	111	2,692	0	0.03	0.05	0.11	0.19
u250.19	101,360,000	1e+08	1e+08	250,000,000	112	2,956	0	0.04	0.06	0.11	0.21
u500.00	197,580,000	2e+08	2e+08	500,000,000	112	2,956	0	0.05	0.07	0.13	0.25
u500.01	200,846,667	2e+08	2e+08	500,000,000	112	2,956	0	0.04	0.06	0.41	0.51
u500.02	201,440,000	2e+08	2e+08	500,000,000	112	2,884	0	0.05	0.06	0.09	0.20
u500.03	203,813,334	2e+08	2e+08	500,000,000	112	2,956	0	0.05	0.07	0.25	0.37
u500.04	205,113,334	2.1e+08	2.1e+08	500,000,000	111	2,806	0	0.04	0.06	0.40	0.50
u500.05	205,086,667	2.1e+08	2.1e+08	500,000,000	112	2,956	0	0.04	0.07	0.31	0.42
u500.06	206,905,798	2.1e+08	2.1e+08	500,000,000	112	2,956	0	0.04	0.07	0.22	0.33
u500.07	203,980,000	2e+08	2e+08	500,000,000	112	2,904	0	0.05	0.07	0.13	0.25
u500.08	195,680,000	2e+08	2e+08	500,000,000	112	2,956	0	0.04	0.07	0.09	0.20
u500.09	201,060,000	2e+08	2e+08	500,000,000	112	2,956	0	0.09	0.06	0.40	0.55
u500.10	199,066,667	2e+08	2e+08	500,000,000	112	2,956	0	0.04	0.07	0.33	0.44
u500.11	199,426,667	2e+08	2e+08	500,000,000	112	2,956	0	0.04	0.06	0.33	0.43
u500.12	198,620,000	2e+08	2e+08	500,000,000	112	2,956	0	0.05	0.07	0.18	0.30
u500.13	195,586,667	2e+08	2e+08	500,000,000	112	2,956	0	0.04	0.06	0.22	0.32
u500.14	203,026,667	2e+08	2e+08	500,000,000	112	2,956	0	0.04	0.07	0.40	0.51
u500.15	200,133,334	2e+08	2e+08	500,000,000	112	2,946	0	0.05	0.07	0.30	0.42
u500.16	201,006,667	2e+08	2e+08	500,000,000	112	2,956	0	0.04	0.06	0.29	0.39
u500.17	197,426,667	2e+08	2e+08	500,000,000	112	2,956	0	0.05	0.06	0.36	0.47
u500.18	201,293,334	2e+08	2e+08	500,000,000	112	2,956	0	0.04	0.06	0.30	0.40
u500.19	195,633,334	2e+08	2e+08	500,000,000	112	2,956	0	0.04	0.07	0.43	0.54
u1000.00	398,426,667	4e+08	4e+08	1,000,000,000	112	2,956	0	0.05	0.07	0.51	0.63
u1000.01	405,253,334	4.1e+08	4.1e+08	1,000,000,000	112	2,956	0	0.05	0.06	0.41	0.52

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instance	z^*	lb^{LP}	lb^{SP}	n	$\#v$	$\#a$	n^{bb}	t^{PP}	t^{LP}	t^{IP}	t^{tot}
u1000_02	410,200,000	4.1e+08	4.1e+08	1,000,000,000	112	2,956	0	0.04	0.07	0.27	0.38
u1000_03	410,866,667	4.1e+08	4.1e+08	1,000,000,000	112	2,956	0	0.05	0.07	0.23	0.35
u1000_04	396,740,000	4e+08	4e+08	1,000,000,000	112	2,956	0	0.04	0.06	0.10	0.20
u1000_05	398,493,334	4e+08	4e+08	1,000,000,000	112	2,956	0	0.04	0.07	0.20	0.31
u1000_06	394,206,667	3.9e+08	3.9e+08	1,000,000,000	112	2,956	0	0.04	0.07	0.20	0.31
u1000_07	403,160,000	4e+08	4e+08	1,000,000,000	112	2,956	0	0.04	0.07	0.15	0.26
u1000_08	398,433,334	4e+08	4e+08	1,000,000,000	112	2,956	0	0.04	0.06	0.32	0.42
u1000_09	396,926,667	4e+08	4e+08	1,000,000,000	112	2,956	0	0.05	0.06	0.32	0.43
u1000_10	399,340,000	4e+08	4e+08	1,000,000,000	112	2,956	0	0.04	0.06	0.10	0.20
u1000_11	400,520,000	4e+08	4e+08	1,000,000,000	112	2,956	0	0.04	0.06	0.11	0.21
u1000_12	392,240,000	3.9e+08	3.9e+08	1,000,000,000	112	2,956	0	0.05	0.06	0.08	0.19
u1000_13	395,273,334	4e+08	4e+08	1,000,000,000	112	2,956	0	0.04	0.06	0.24	0.34
u1000_14	393,886,667	3.9e+08	3.9e+08	1,000,000,000	112	2,956	0	0.05	0.06	0.26	0.37
u1000_15	401,806,667	4e+08	4e+08	1,000,000,000	112	2,956	0	0.04	0.06	0.24	0.34
u1000_16	403,026,667	4e+08	4e+08	1,000,000,000	112	2,956	0	0.05	0.06	0.39	0.50
u1000_17	403,800,000	4e+08	4e+08	1,000,000,000	112	2,956	0	0.04	0.06	0.12	0.22
u1000_18	398,193,334	4e+08	4e+08	1,000,000,000	112	2,956	0	0.04	0.06	0.34	0.44
u1000_19	399,333,334	4e+08	4e+08	1,000,000,000	112	2,956	0	0.05	0.07	0.30	0.42
t60_00	20,000,000	2e+07	2e+07	60,000,000	52	665	0	0.05	0.01	0.04	0.10
t60_01	20,000,000	2e+07	2e+07	60,000,000	57	767	0	0.05	0.01	0.02	0.08
t60_02	20,000,000	2e+07	2e+07	60,000,000	52	624	0	0.04	0.01	0.02	0.07
t60_03	20,000,000	2e+07	2e+07	60,000,000	53	700	0	0.04	0.01	0.02	0.07
t60_04	20,000,000	2e+07	2e+07	60,000,000	52	687	0	0.04	0.01	0.02	0.07
t60_05	20,000,000	2e+07	2e+07	60,000,000	53	689	0	0.05	0.01	0.01	0.07
t60_06	20,000,000	2e+07	2e+07	60,000,000	55	735	0	0.04	0.01	0.02	0.07
t60_07	20,000,000	2e+07	2e+07	60,000,000	52	693	0	0.04	0.01	0.02	0.07
t60_08	20,000,000	2e+07	2e+07	60,000,000	51	658	0	0.04	0.01	0.01	0.06
t60_09	20,000,000	2e+07	2e+07	60,000,000	52	668	0	0.04	0.01	0.01	0.06
t60_10	20,000,000	2e+07	2e+07	60,000,000	58	785	0	0.04	0.01	0.02	0.07
t60_11	20,000,000	2e+07	2e+07	60,000,000	49	627	0	0.04	0.01	0.01	0.06
t60_12	20,000,000	2e+07	2e+07	60,000,000	57	748	0	0.04	0.01	0.02	0.07
t60_13	20,000,000	2e+07	2e+07	60,000,000	56	741	0	0.04	0.01	0.02	0.07
t60_14	20,000,000	2e+07	2e+07	60,000,000	53	663	0	0.04	0.01	0.02	0.07
t60_15	20,000,000	2e+07	2e+07	60,000,000	56	685	0	0.04	0.01	0.02	0.07
t60_16	20,000,000	2e+07	2e+07	60,000,000	49	623	0	0.04	0.01	0.01	0.06
t60_17	20,000,000	2e+07	2e+07	60,000,000	53	660	0	0.04	0.01	0.02	0.07
t60_18	20,000,000	2e+07	2e+07	60,000,000	54	729	0	0.04	0.01	0.02	0.07
t60_19	20,000,000	2e+07	2e+07	60,000,000	56	666	0	0.05	0.01	0.01	0.07
t120_00	40,000,000	4e+07	4e+07	120,000,000	92	1,911	0	0.09	0.03	0.05	0.17
t120_01	40,000,000	4e+07	4e+07	120,000,000	90	1,983	0	0.07	0.03	0.08	0.18
t120_02	40,000,000	4e+07	4e+07	120,000,000	94	2,107	0	0.10	0.04	0.07	0.21
t120_03	40,000,000	4e+07	4e+07	120,000,000	91	1,884	0	0.08	0.03	0.07	0.18
t120_04	40,000,000	4e+07	4e+07	120,000,000	94	2,207	0	0.10	0.04	0.27	0.41
t120_05	40,000,000	4e+07	4e+07	120,000,000	93	2,000	0	0.09	0.03	0.05	0.17
t120_06	40,000,000	4e+07	4e+07	120,000,000	90	1,949	0	0.08	0.03	0.07	0.18
t120_07	40,000,000	4e+07	4e+07	120,000,000	91	2,053	0	0.09	0.03	0.06	0.18
t120_08	40,000,000	4e+07	4e+07	120,000,000	89	1,993	0	0.08	0.03	0.06	0.17
t120_09	40,000,000	4e+07	4e+07	120,000,000	89	1,895	0	0.08	0.03	0.06	0.17
t120_10	40,000,000	4e+07	4e+07	120,000,000	88	1,799	0	0.09	0.03	0.26	0.38
t120_11	40,000,000	4e+07	4e+07	120,000,000	92	1,891	0	0.08	0.03	0.06	0.17
t120_12	40,000,000	4e+07	4e+07	120,000,000	85	1,892	0	0.08	0.03	0.20	0.31
t120_13	40,000,000	4e+07	4e+07	120,000,000	92	1,982	0	0.08	0.03	0.06	0.17
t120_14	40,000,000	4e+07	4e+07	120,000,000	87	1,732	0	0.08	0.02	0.05	0.15
t120_15	40,000,000	4e+07	4e+07	120,000,000	86	1,676	0	0.08	0.03	0.17	0.28
t120_16	40,000,000	4e+07	4e+07	120,000,000	90	2,036	0	0.08	0.03	0.07	0.18
t120_17	40,000,000	4e+07	4e+07	120,000,000	94	2,177	0	0.09	0.04	0.07	0.20
t120_18	40,000,000	4e+07	4e+07	120,000,000	89	1,989	0	0.08	0.03	0.06	0.17
t120_19	40,000,000	4e+07	4e+07	120,000,000	93	2,128	0	0.08	0.03	0.08	0.19
t249_00	83,000,000	8.3e+07	8.3e+07	249,000,000	139	5,155	0	0.15	0.10	0.30	0.55
t249_01	83,000,000	8.3e+07	8.3e+07	249,000,000	145	5,611	0	0.16	0.10	0.22	0.48
t249_02	83,000,000	8.3e+07	8.3e+07	249,000,000	145	5,382	0	0.16	0.11	0.66	0.93
t249_03	83,000,000	8.3e+07	8.3e+07	249,000,000	147	5,966	0	0.17	0.13	0.64	0.94
t249_04	83,000,000	8.3e+07	8.3e+07	249,000,000	138	5,145	0	0.16	0.11	0.17	0.44

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instance	z^*	lb^{lp}	lb^{sp}	n	$\#v$	$\#a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t249_05	83,000,000	8.3e+07	8.3e+07	249,000,000	150	5,941	0	0.17	0.12	0.20	0.49
t249_06	83,000,000	8.3e+07	8.3e+07	249,000,000	141	5,679	0	0.16	0.12	0.21	0.49
t249_07	83,000,000	8.3e+07	8.3e+07	249,000,000	142	5,487	0	0.16	0.11	1.05	1.32
t249_08	83,000,000	8.3e+07	8.3e+07	249,000,000	142	5,477	0	0.16	0.12	0.16	0.44
t249_09	83,000,000	8.3e+07	8.3e+07	249,000,000	147	5,668	0	0.16	0.12	1.71	1.99
t249_10	83,000,000	8.3e+07	8.3e+07	249,000,000	144	5,657	0	0.17	0.13	0.40	0.70
t249_11	83,000,000	8.3e+07	8.3e+07	249,000,000	146	5,722	0	0.16	0.11	0.54	0.81
t249_12	83,000,000	8.3e+07	8.3e+07	249,000,000	146	5,590	0	0.16	0.12	0.51	0.79
t249_13	83,000,000	8.3e+07	8.3e+07	249,000,000	145	5,795	0	0.16	0.11	0.29	0.56
t249_14	83,000,000	8.3e+07	8.3e+07	249,000,000	149	5,708	0	0.17	0.13	0.82	1.12
t249_15	83,000,000	8.3e+07	8.3e+07	249,000,000	146	5,578	0	0.17	0.11	0.38	0.66
t249_16	83,000,000	8.3e+07	8.3e+07	249,000,000	150	5,933	0	0.18	0.11	0.20	0.49
t249_17	83,000,000	8.3e+07	8.3e+07	249,000,000	151	5,868	0	0.17	0.13	0.30	0.60
t249_18	83,000,000	8.3e+07	8.3e+07	249,000,000	143	5,355	0	0.17	0.12	0.41	0.70
t249_19	83,000,000	8.3e+07	8.3e+07	249,000,000	141	5,495	0	0.16	0.11	0.58	0.85
t501_00	167,000,000	1.7e+08	1.7e+08	501,000,000	195	11,575	0	0.27	0.26	2.26	2.79
t501_01	167,000,000	1.7e+08	1.7e+08	501,000,000	197	11,518	0	0.28	0.28	1.39	1.95
t501_02	167,000,000	1.7e+08	1.7e+08	501,000,000	196	10,920	0	0.27	0.26	1.35	1.88
t501_03	167,000,000	1.7e+08	1.7e+08	501,000,000	204	12,186	0	0.29	0.33	1.70	2.32
t501_04	167,000,000	1.7e+08	1.7e+08	501,000,000	201	11,881	0	0.27	0.32	2.41	3.00
t501_05	167,000,000	1.7e+08	1.7e+08	501,000,000	200	11,953	0	0.28	0.25	0.64	1.17
t501_06	167,000,000	1.7e+08	1.7e+08	501,000,000	201	12,092	0	0.27	0.27	0.77	1.31
t501_07	167,000,000	1.7e+08	1.7e+08	501,000,000	198	11,560	0	0.28	0.30	3.88	4.46
t501_08	167,000,000	1.7e+08	1.7e+08	501,000,000	200	12,190	0	0.29	0.27	4.56	5.12
t501_09	167,000,000	1.7e+08	1.7e+08	501,000,000	195	11,089	0	0.27	0.27	1.35	1.89
t501_10	167,000,000	1.7e+08	1.7e+08	501,000,000	195	11,332	0	0.26	0.25	2.34	2.85
t501_11	167,000,000	1.7e+08	1.7e+08	501,000,000	200	12,054	0	0.27	0.27	0.68	1.22
t501_12	167,000,000	1.7e+08	1.7e+08	501,000,000	194	11,271	0	0.27	0.24	0.47	0.98
t501_13	167,000,000	1.7e+08	1.7e+08	501,000,000	204	12,436	0	0.28	0.31	0.66	1.25
t501_14	167,000,000	1.7e+08	1.7e+08	501,000,000	208	12,940	0	0.30	0.38	2.04	2.72
t501_15	167,000,000	1.7e+08	1.7e+08	501,000,000	202	12,069	0	0.29	0.32	0.52	1.13
t501_16	167,000,000	1.7e+08	1.7e+08	501,000,000	203	12,376	0	0.28	0.30	2.48	3.06
t501_17	167,000,000	1.7e+08	1.7e+08	501,000,000	201	12,218	0	0.29	0.34	1.63	2.26
t501_18	167,000,000	1.7e+08	1.7e+08	501,000,000	198	11,788	0	0.28	0.27	1.90	2.45
t501_19	167,000,000	1.7e+08	1.7e+08	501,000,000	197	11,820	0	0.27	0.28	1.91	2.46

Table A.3: Cutgen results.

type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	1	13	12.35	12.35	472	2,509	55.1	56.6	0	0.04	0.11	0.15	0.30
	2	10	9.73	9.73	625	3,444	73.4	68.0	0	0.04	0.19	0.34	0.57
	3	9	8.39	8.39	622	2,771	69.7	66.0	0	0.04	0.11	0.17	0.32
	4	11	10.19	10.19	793	3,945	84.7	90.4	0	0.05	0.21	0.32	0.58
	5	14	13.76	13.76	398	1,906	51.6	50.2	0	0.03	0.06	0.09	0.18
	6	16	15.76	15.76	396	1,879	51.6	59.3	0	0.03	0.07	0.10	0.20
	7	11	10.29	10.29	658	3,236	76.4	74.4	0	0.04	0.16	0.25	0.45
	8	10	9.62	9.62	762	3,985	84.0	85.2	0	0.05	0.18	0.40	0.63
	9	8	7.49	7.49	829	4,900	87.9	91.1	0	0.05	0.26	0.49	0.80
	10	12	11.80	11.80	658	3,362	74.8	78.5	0	0.04	0.21	0.34	0.59
	11	12	11.02	11.02	652	3,282	77.1	73.3	0	0.04	0.18	0.32	0.54
	12	16	15.29	15.29	298	1,368	37.1	41.9	0	0.03	0.04	0.08	0.15
	13	8	7.11	7.11	808	4,797	85.7	90.5	0	0.05	0.27	0.65	0.97
	14	13	12.38	12.38	399	1,984	50.1	46.7	0	0.04	0.07	0.12	0.23
	15	12	11.67	11.67	443	2,159	54.4	54.6	0	0.04	0.07	0.16	0.27
	16	9	8.39	8.39	641	3,111	71.1	60.1	0	0.05	0.15	0.25	0.45
	17	11	10.40	10.40	651	3,325	77.1	78.0	0	0.04	0.15	0.27	0.46
	18	11	11.00	11.00	549	2,773	64.5	64.1	0	0.04	0.12	0.19	0.35
	19	13	12.57	12.57	488	2,238	59.4	60.0	0	0.04	0.08	0.14	0.26
	20	7	6.20	6.20	737	4,148	82.7	76.6	0	0.05	0.27	0.56	0.88
	21	13	12.31	12.30	688	3,459	77.1	77.1	0	0.04	0.16	0.38	0.58
	22	10	9.75	9.75	751	3,892	83.4	82.4	0	0.04	0.19	0.50	0.73
	23	13	12.24	12.24	405	1,942	51.3	48.3	0	0.04	0.06	0.10	0.20

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	24	13	12.21	12.21	527	2,348	61.3	65.9	0	0.03	0.10	0.17	0.30
	25	10	9.98	9.98	678	3,684	76.7	77.1	0	0.04	0.23	1.35	1.62
	26	14	13.20	13.20	504	2,471	60.0	63.6	0	0.03	0.12	0.14	0.29
	27	9	8.78	8.78	740	4,305	82.4	80.1	0	0.05	0.22	0.57	0.84
	28	11	10.19	10.19	701	3,600	79.0	79.3	0	0.05	0.16	0.34	0.55
	29	11	10.77	10.77	502	2,221	59.8	59.7	0	0.03	0.08	0.10	0.21
	30	10	9.38	9.38	358	2,005	81.5	75.6	0	0.03	0.06	0.15	0.24
	31	13	12.14	12.14	405	1,943	51.2	52.0	0	0.04	0.07	0.07	0.18
	32	10	9.64	9.64	701	3,377	79.0	75.8	0	0.04	0.15	0.28	0.47
	33	11	10.79	10.79	559	2,501	66.5	61.9	0	0.04	0.11	0.19	0.34
	34	16	15.53	15.53	449	2,344	52.9	61.4	0	0.04	0.11	0.08	0.23
	35	14	13.13	13.13	490	2,441	59.8	59.8	0	0.04	0.10	0.18	0.32
	36	13	12.23	12.23	489	2,383	57.8	62.3	0	0.04	0.09	0.16	0.29
	37	12	11.98	11.98	415	1,997	52.7	49.5	0	0.03	0.05	0.55	0.63
	38	10	9.96	9.96	449	2,139	52.8	50.1	0	0.04	0.08	0.17	0.29
	39	13	12.04	12.04	546	2,630	63.8	68.1	0	0.04	0.11	0.14	0.29
	40	11	10.43	10.43	817	4,238	87.5	89.9	0	0.05	0.22	0.48	0.75
	41	9	8.82	8.82	829	5,031	86.8	92.7	0	0.06	0.27	0.86	1.19
	42	9	8.22	8.22	475	1,807	53.5	58.5	0	0.03	0.04	0.07	0.14
	43	13	12.97	12.97	327	1,417	41.6	43.7	24	0.03	0.04	0.50	0.57
	44	9	8.96	8.96	815	4,931	89.2	89.9	0	0.05	0.32	1.00	1.37
	45	10	9.43	9.43	522	2,745	60.0	60.2	0	0.04	0.10	0.17	0.31
	46	12	11.97	11.97	555	3,015	64.3	72.4	0	0.04	0.15	0.32	0.51
	47	13	12.02	12.02	564	2,437	66.4	65.6	0	0.04	0.09	0.14	0.27
	48	13	12.71	12.70	515	2,421	61.7	64.2	0	0.04	0.11	0.12	0.27
	49	9	8.90	8.90	738	3,928	82.8	76.6	0	0.05	0.21	0.40	0.66
	50	11	11.00	11.00	528	2,718	61.5	67.5	0	0.04	0.11	0.29	0.44
	51	11	10.85	10.85	661	3,232	74.8	72.5	0	0.04	0.14	0.25	0.43
	52	10	9.55	9.55	673	3,420	75.6	70.6	0	0.04	0.22	0.29	0.55
	53	6	5.59	5.59	902	5,705	93.9	94.7	0	0.06	0.25	0.65	0.96
	54	12	11.59	11.59	507	2,270	60.4	60.3	0	0.03	0.08	0.18	0.29
	55	11	10.98	10.98	469	2,347	57.5	60.9	0	0.04	0.09	0.18	0.31
	56	13	12.47	12.47	357	1,751	46.0	50.4	0	0.03	0.04	0.07	0.14
	57	12	11.57	11.57	477	2,353	59.0	55.8	0	0.03	0.08	0.17	0.28
	58	11	10.54	10.54	512	2,585	62.2	56.8	0	0.04	0.12	0.22	0.38
	59	11	10.89	10.89	661	3,508	75.8	71.2	0	0.04	0.18	0.37	0.59
	60	11	10.12	10.12	741	4,214	82.0	86.2	0	0.05	0.26	0.59	0.90
	61	10	9.80	9.80	656	3,226	74.9	78.0	0	0.04	0.15	0.24	0.43
	62	14	13.23	13.23	595	2,706	71.0	75.1	0	0.03	0.13	0.23	0.39
	63	11	10.06	10.06	484	2,025	58.0	59.0	0	0.03	0.07	0.10	0.20
	64	13	12.40	12.40	404	2,024	50.5	51.6	0	0.03	0.06	0.13	0.22
	65	8	7.55	7.55	804	5,340	86.6	90.5	0	0.06	0.35	0.92	1.33
	66	11	10.73	10.73	804	4,351	89.3	92.9	0	0.05	0.31	0.71	1.07
	67	10	9.02	9.02	508	2,768	60.9	57.8	0	0.04	0.11	0.20	0.35
	68	9	8.12	8.12	835	5,334	90.9	88.3	0	0.06	0.35	0.83	1.24
	69	13	12.31	12.31	397	1,935	50.3	48.2	0	0.03	0.07	0.10	0.20
	70	12	11.91	11.91	437	2,148	54.4	48.9	0	0.04	0.08	0.25	0.37
	71	9	8.26	8.26	822	4,481	87.5	88.9	0	0.05	0.18	0.64	0.87
	72	14	13.55	13.55	356	1,561	47.6	50.2	0	0.03	0.05	0.06	0.14
	73	13	12.11	12.11	508	2,116	60.1	62.8	0	0.03	0.08	0.15	0.26
	74	11	10.74	10.74	313	1,479	40.9	38.9	0	0.03	0.04	0.08	0.15
	75	16	15.18	15.18	135	680	20.5	23.7	0	0.03	0.01	0.02	0.06
	76	12	11.87	11.87	612	2,685	71.3	77.2	0	0.03	0.12	0.18	0.33
	77	11	10.26	10.26	792	4,665	86.7	86.5	0	0.05	0.27	0.53	0.85
	78	14	13.38	13.38	225	1,058	30.9	32.9	0	0.03	0.03	0.04	0.10
	79	12	11.96	11.96	527	2,690	62.7	62.7	0	0.04	0.13	0.44	0.61
	80	10	9.62	9.62	469	2,269	56.1	52.1	0	0.03	0.07	0.13	0.23
	81	13	12.35	12.35	585	2,564	68.5	64.5	0	0.03	0.13	0.15	0.31
	82	11	10.25	10.25	621	2,924	70.3	65.4	0	0.04	0.13	0.21	0.38
	83	7	6.56	6.56	755	4,111	82.4	75.9	0	0.05	0.22	0.58	0.85
	84	13	12.01	12.01	621	3,153	72.0	69.0	0	0.04	0.16	0.24	0.44
	85	12	11.37	11.37	611	2,793	70.6	70.5	0	0.04	0.12	0.18	0.34
	86	12	11.12	11.12	499	2,166	58.9	59.3	0	0.03	0.06	0.11	0.20
	87	14	13.32	13.32	491	2,477	58.3	67.4	0	0.04	0.13	0.13	0.30

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	88	12	11.51	11.51	770	4,223	82.9	88.9	0	0.05	0.23	0.39	0.67
	89	12	11.73	11.73	590	2,930	68.7	72.0	0	0.04	0.14	0.34	0.52
	90	11	10.61	10.61	234	1,055	31.6	34.3	0	0.03	0.02	0.07	0.12
	91	12	11.46	11.46	908	4,921	95.1	96.3	0	0.05	0.26	0.72	1.03
	92	9	8.63	8.63	655	3,557	73.8	68.2	0	0.05	0.18	0.35	0.58
	93	8	7.47	7.47	663	3,163	75.0	68.2	0	0.04	0.13	0.22	0.39
	94	16	15.21	15.21	177	874	25.4	28.6	0	0.03	0.02	0.03	0.08
	95	15	14.10	14.09	442	2,250	52.7	54.9	0	0.03	0.09	0.20	0.32
	96	12	11.68	11.68	625	3,001	71.2	78.8	0	0.04	0.15	0.17	0.36
	97	14	13.28	13.28	484	2,431	61.4	62.9	0	0.04	0.10	0.15	0.29
	98	11	10.90	10.90	713	3,692	80.6	82.2	0	0.05	0.17	0.37	0.59
	99	9	8.35	8.35	630	3,191	70.4	65.3	0	0.04	0.15	0.26	0.45
	100	13	12.47	12.47	406	1,888	50.1	54.2	0	0.03	0.07	0.10	0.20
1	1	123	122.07	122.07	484	2,525	55.4	56.3	0	0.04	0.11	0.18	0.33
	2	98	97.33	97.33	631	3,461	74.1	68.1	0	0.05	0.20	0.31	0.56
	3	84	83.89	83.89	646	2,898	72.3	67.8	0	0.04	0.10	0.26	0.40
	4	105	104.10	104.10	793	4,067	84.7	90.7	0	0.05	0.23	0.33	0.61
	5	137	136.83	136.83	403	1,918	50.6	49.2	0	0.03	0.07	0.15	0.25
	6	159	158.41	158.38	447	2,126	54.2	59.9	0	0.03	0.08	0.09	0.20
	7	104	103.87	103.87	668	3,270	76.6	74.4	0	0.05	0.19	0.25	0.49
	8	96	95.68	95.68	769	3,999	84.7	85.4	0	0.05	0.20	0.44	0.69
	9	76	75.20	75.20	829	4,900	87.9	91.1	0	0.05	0.25	0.73	1.03
	10	118	117.55	117.55	724	3,680	79.3	80.2	0	0.04	0.23	0.25	0.52
	11	110	109.64	109.64	653	3,343	74.8	71.9	0	0.04	0.20	0.34	0.58
	12	155	154.87	154.87	309	1,382	37.9	41.9	0	0.03	0.04	0.08	0.15
	13	72	71.72	71.72	809	4,981	85.8	90.9	0	0.05	0.31	0.54	0.90
	14	123	122.58	122.58	452	2,157	56.6	49.2	0	0.04	0.08	0.15	0.27
	15	118	117.34	117.34	443	2,159	54.4	54.6	0	0.04	0.07	0.14	0.25
	16	86	85.05	85.05	646	3,185	71.6	60.8	0	0.04	0.17	0.22	0.43
	17	105	104.85	104.85	695	3,726	80.2	80.1	0	0.05	0.20	0.31	0.56
	18	112	111.06	111.06	568	2,868	66.7	65.4	0	0.04	0.13	0.23	0.40
	19	127	126.01	126.01	506	2,380	61.6	61.5	0	0.04	0.08	0.09	0.21
	20	63	62.82	62.82	740	4,373	83.1	77.4	0	0.05	0.24	0.62	0.91
	21	123	122.35	122.32	754	3,945	84.5	81.8	0	0.05	0.19	0.40	0.64
	22	100	99.06	99.06	751	3,902	83.4	82.4	0	0.04	0.20	0.42	0.66
	23	123	122.05	122.05	406	1,952	51.3	48.4	0	0.03	0.06	0.09	0.18
	24	122	121.95	121.95	532	2,404	61.3	66.1	0	0.04	0.10	0.16	0.30
	25	99	98.66	98.66	681	3,724	77.0	77.3	0	0.05	0.23	0.28	0.56
	26	133	132.32	132.32	532	2,514	60.5	62.2	0	0.04	0.12	0.16	0.32
	27	90	89.70	89.70	740	4,327	82.4	80.2	0	0.05	0.24	0.63	0.92
	28	102	101.51	101.51	701	3,606	79.0	79.3	0	0.05	0.17	0.41	0.63
	29	108	107.89	107.89	510	2,312	60.8	60.8	0	0.04	0.09	0.13	0.26
	30	95	94.58	94.58	360	2,032	82.0	75.8	0	0.03	0.06	0.14	0.23
	31	121	120.90	120.90	423	2,008	52.4	52.6	0	0.03	0.07	0.09	0.19
	32	97	96.64	96.64	711	3,595	80.1	77.1	0	0.05	0.18	0.36	0.59
	33	109	108.03	108.03	563	2,561	67.0	61.9	0	0.04	0.11	0.17	0.32
	34	155	154.85	154.84	505	2,504	57.2	62.0	0	0.04	0.10	0.10	0.24
	35	132	131.05	131.05	516	2,519	61.7	60.6	0	0.04	0.11	0.11	0.26
	36	123	122.84	122.84	499	2,416	59.0	63.0	0	0.04	0.09	0.19	0.32
	37	120	119.99	119.99	415	1,998	52.7	49.5	0	0.04	0.06	0.15	0.25
	38	99	98.77	98.77	463	2,251	54.4	50.9	0	0.04	0.08	0.21	0.33
	39	121	120.03	120.03	586	2,847	68.2	70.8	0	0.03	0.13	0.21	0.37
	40	104	103.45	103.45	822	4,529	88.0	90.5	0	0.05	0.24	0.55	0.84
	41	89	88.87	88.87	845	5,215	88.5	93.6	0	0.06	0.31	0.60	0.97
	42	83	82.64	82.64	594	2,426	66.7	67.2	0	0.04	0.08	0.13	0.25
	43	130	129.92	129.92	328	1,420	41.7	43.8	0	0.03	0.04	0.06	0.13
	44	89	88.16	88.16	817	5,071	87.9	89.3	0	0.05	0.34	0.86	1.25
	45	93	92.55	92.55	558	2,865	64.1	61.9	0	0.04	0.10	0.26	0.40
	46	120	119.16	119.16	614	3,212	69.9	74.9	0	0.05	0.17	0.28	0.50
	47	120	119.86	119.86	593	2,651	69.4	67.8	0	0.03	0.12	0.19	0.34
	48	128	127.57	127.55	554	2,574	65.6	66.3	0	0.04	0.12	0.10	0.26
	49	89	88.09	88.09	742	4,032	83.3	77.1	0	0.05	0.23	0.36	0.64
	50	112	111.29	111.29	528	2,727	61.5	67.6	0	0.04	0.11	0.19	0.34
	51	110	109.31	109.31	697	3,369	78.7	74.3	0	0.04	0.16	0.23	0.43

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	52	96	95.38	95.38	683	3,791	76.4	72.5	0	0.05	0.22	0.41	0.68
	53	56	55.98	55.98	902	5,912	93.9	94.9	0	0.06	0.28	0.91	1.25
	54	116	115.46	115.46	508	2,288	60.5	60.5	0	0.03	0.08	0.10	0.21
	55	110	109.24	109.24	483	2,489	57.2	60.3	0	0.04	0.09	0.13	0.26
	56	126	125.58	125.58	357	1,810	46.0	50.6	0	0.03	0.04	0.08	0.15
	57	117	116.53	116.53	477	2,353	59.0	55.8	0	0.04	0.08	0.18	0.30
	58	105	104.93	104.93	534	2,677	64.9	57.8	0	0.05	0.13	0.18	0.36
	59	109	108.26	108.26	662	3,533	75.9	71.1	0	0.05	0.19	0.27	0.51
	60	101	100.72	100.72	741	4,274	81.3	86.0	0	0.05	0.23	0.61	0.89
	61	98	97.23	97.23	665	3,573	75.9	79.3	0	0.05	0.19	0.24	0.48
	62	131	130.19	130.19	626	2,784	70.7	74.2	0	0.04	0.13	0.25	0.42
	63	99	98.77	98.77	493	2,157	59.0	60.2	0	0.04	0.08	0.12	0.24
	64	125	124.26	124.26	409	2,046	50.7	51.7	0	0.03	0.07	0.10	0.20
	65	77	76.13	76.13	842	5,616	90.1	92.0	0	0.06	0.35	0.86	1.27
	66	108	107.11	107.11	808	4,365	87.4	91.6	0	0.05	0.31	0.75	1.11
	67	90	89.27	89.27	515	2,798	61.8	58.2	0	0.04	0.10	0.18	0.32
	68	81	80.45	80.45	838	5,391	89.8	87.9	0	0.06	0.29	0.89	1.24
	69	125	124.20	124.20	398	1,943	50.4	48.1	0	0.04	0.07	0.14	0.25
	70	119	118.94	118.94	454	2,209	56.5	49.9	0	0.03	0.08	0.29	0.40
	71	84	83.74	83.74	826	4,750	88.0	89.7	0	0.05	0.23	0.57	0.85
	72	137	136.44	136.44	360	1,561	45.6	48.6	0	0.03	0.05	0.06	0.14
	73	122	121.70	121.70	515	2,141	59.4	62.2	0	0.03	0.08	0.10	0.21
	74	107	106.11	106.11	329	1,575	43.0	40.3	0	0.03	0.04	0.07	0.14
	75	153	152.37	152.37	140	700	21.2	24.3	0	0.03	0.01	0.07	0.11
	76	119	118.02	118.02	618	2,870	71.7	77.0	0	0.03	0.13	0.17	0.33
	77	102	101.60	101.60	792	4,666	86.7	86.5	0	0.05	0.26	0.55	0.86
	78	136	135.27	135.27	229	1,091	31.5	33.0	0	0.03	0.03	0.04	0.10
	79	122	121.41	121.41	558	2,786	66.3	64.3	0	0.04	0.14	0.17	0.35
	80	97	96.56	96.56	476	2,299	56.9	52.6	0	0.03	0.07	0.15	0.25
	81	123	122.95	122.95	606	2,785	71.0	66.6	0	0.04	0.13	0.22	0.39
	82	103	102.48	102.48	627	3,018	70.1	65.7	0	0.04	0.15	0.23	0.42
	83	66	65.07	65.07	755	4,131	82.4	75.9	0	0.05	0.24	0.36	0.65
	84	122	121.00	121.00	639	3,216	74.0	69.8	0	0.05	0.15	0.23	0.43
	85	113	112.09	112.09	617	2,875	71.3	71.1	0	0.04	0.13	0.16	0.33
	86	111	110.38	110.38	499	2,166	58.9	59.3	0	0.04	0.06	0.11	0.21
	87	134	133.68	133.68	507	2,526	58.7	67.3	0	0.04	0.15	0.21	0.40
	88	115	114.47	114.47	833	4,463	88.4	91.8	0	0.06	0.24	0.39	0.69
	89	116	115.92	115.92	590	2,936	68.7	72.0	0	0.04	0.14	0.27	0.45
	90	108	107.20	107.20	285	1,251	38.3	36.9	0	0.02	0.03	0.05	0.10
	91	114	113.29	113.29	908	4,946	94.8	96.1	0	0.06	0.24	0.79	1.09
	92	88	87.16	87.16	657	3,576	74.0	68.3	0	0.05	0.20	0.38	0.63
	93	75	74.50	74.50	665	3,261	75.2	68.9	0	0.04	0.13	0.30	0.47
	94	152	151.27	151.26	193	938	27.5	29.2	0	0.02	0.02	0.04	0.08
	95	142	141.02	141.00	450	2,256	51.4	53.4	0	0.04	0.10	0.14	0.28
	96	116	115.99	115.99	629	3,034	71.2	78.8	0	0.04	0.16	0.22	0.42
	97	131	130.91	130.91	492	2,451	60.1	61.6	0	0.04	0.12	0.35	0.51
	98	109	108.71	108.71	714	3,695	80.7	82.2	0	0.04	0.18	0.37	0.59
	99	84	83.96	83.96	631	3,250	70.5	65.4	0	0.05	0.16	0.36	0.57
	100	125	124.49	124.49	430	2,013	52.7	55.6	0	0.03	0.07	0.11	0.21
1	1	23	22.27	22.27	744	7,821	82.7	86.1	0	0.09	0.59	1.76	2.44
	2	20	19.14	19.14	887	9,167	92.2	86.6	0	0.10	0.70	2.50	3.30
	3	30	29.35	29.35	775	8,559	87.2	96.2	0	0.09	0.80	1.72	2.61
	4	19	18.79	18.79	870	10,457	92.3	94.3	0	0.11	0.74	3.92	4.77
	5	24	23.04	23.04	824	9,612	87.6	95.1	0	0.09	0.99	2.10	3.18
	6	25	24.74	24.74	831	8,880	90.4	93.3	0	0.09	0.81	2.87	3.77
	7	21	20.48	20.48	923	11,560	95.1	98.5	0	0.12	1.02	3.46	4.60
	8	27	26.10	26.10	634	6,177	75.4	73.4	0	0.07	0.40	0.70	1.17
	9	22	21.08	21.08	914	11,325	95.0	96.9	0	0.12	0.81	3.34	4.27
	10	23	22.22	22.22	771	8,228	85.8	90.3	0	0.08	0.60	1.64	2.32
	11	22	21.49	21.49	879	9,865	92.8	92.5	0	0.10	0.64	2.68	3.42
	12	22	21.55	21.55	805	10,190	88.0	91.2	0	0.11	0.82	2.28	3.21
	13	23	22.86	22.86	908	10,663	93.8	98.5	0	0.10	0.95	1.75	2.80
	14	20	19.20	19.20	822	9,896	89.1	90.4	0	0.11	0.70	0.95	1.76
	15	23	22.30	22.30	796	8,752	87.0	92.5	0	0.09	0.71	1.61	2.41

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	16	21	20.76	20.76	809	9,306	88.2	92.6	0	0.10	0.89	1.86	2.85
	17	25	24.16	24.16	759	8,230	85.2	85.3	0	0.09	0.71	1.72	2.52
	18	23	22.53	22.53	813	8,707	88.9	91.1	0	0.09	0.63	1.98	2.70
	19	25	24.68	24.68	706	7,259	81.1	79.1	0	0.09	0.50	1.46	2.05
	20	25	24.85	24.85	810	8,283	87.1	94.4	0	0.08	0.74	1.98	2.80
	21	20	19.38	19.38	916	11,089	95.3	98.3	0	0.12	1.08	2.60	3.80
	22	23	22.73	22.73	693	6,667	80.0	80.1	0	0.07	0.49	1.19	1.75
	23	19	18.46	18.46	887	10,761	92.5	94.3	0	0.11	0.77	2.78	3.66
	24	24	23.14	23.14	861	9,990	91.5	95.6	0	0.10	0.96	2.11	3.17
	25	18	17.40	17.40	884	10,917	92.8	96.3	0	0.11	0.80	2.89	3.80
	26	19	18.20	18.20	861	10,649	92.0	94.6	0	0.11	0.89	1.18	2.18
	27	18	17.59	17.59	922	12,346	95.3	97.6	0	0.12	0.85	3.19	4.16
	28	20	19.67	19.67	880	10,468	92.5	95.9	0	0.11	1.01	3.36	4.48
	29	22	21.21	21.21	740	8,590	83.5	81.9	0	0.09	0.72	1.79	2.60
	30	16	15.69	15.69	880	11,160	93.1	94.6	0	0.12	0.72	3.01	3.85
	31	19	18.81	18.81	846	8,530	89.7	88.9	0	0.09	0.89	2.14	3.12
	32	27	26.63	26.63	740	6,757	80.8	85.3	0	0.07	0.56	0.89	1.52
	33	18	17.77	17.77	909	11,687	94.8	97.1	0	0.12	1.05	3.28	4.45
	34	20	19.55	19.55	838	10,746	90.7	91.9	0	0.11	0.74	2.27	3.12
	35	23	22.03	22.03	771	8,294	85.5	88.3	0	0.09	0.63	1.42	2.14
	36	23	22.24	22.24	871	9,317	93.1	95.1	0	0.10	0.68	1.93	2.71
	37	24	23.58	23.58	834	8,379	89.3	94.3	0	0.09	0.72	2.05	2.86
	38	31	30.18	30.18	567	4,624	68.4	68.1	0	0.07	0.30	0.62	0.99
	39	19	18.59	18.59	858	10,573	91.5	93.9	0	0.11	0.88	2.58	3.57
	40	23	22.36	22.36	716	8,182	80.2	80.2	0	0.09	0.60	1.66	2.35
	41	19	18.21	18.21	875	10,560	92.9	94.6	0	0.10	0.83	2.18	3.11
	42	23	22.82	22.82	788	8,015	85.7	85.8	0	0.09	0.71	1.79	2.59
	43	25	24.34	24.34	811	8,763	87.7	93.6	0	0.10	0.80	1.44	2.34
	44	24	23.90	23.90	768	9,023	83.0	93.9	0	0.10	0.86	2.10	3.06
	45	27	26.17	26.17	849	9,781	89.5	97.0	0	0.10	0.95	2.60	3.65
	46	19	18.68	18.68	938	10,590	95.8	94.9	0	0.10	0.91	3.09	4.10
	47	22	21.70	21.70	866	9,485	91.2	92.6	0	0.09	0.79	2.60	3.48
	48	21	20.36	20.36	835	9,696	88.0	87.2	0	0.11	0.89	2.37	3.37
	49	24	23.30	23.30	757	6,923	82.5	86.4	0	0.07	0.54	0.95	1.56
	50	21	20.34	20.34	823	9,055	87.5	76.5	0	0.10	0.79	2.11	3.00
	51	26	25.09	25.09	833	8,406	89.4	91.9	0	0.09	0.67	1.98	2.74
	52	18	17.54	17.54	828	9,077	88.7	89.9	0	0.10	0.81	2.14	3.05
	53	18	17.55	17.55	832	9,742	89.3	92.0	0	0.10	0.72	2.43	3.25
	54	22	21.42	21.42	828	8,534	88.4	89.4	0	0.09	0.65	1.57	2.31
	55	18	17.92	17.92	738	7,926	83.4	82.5	0	0.08	0.41	1.54	2.03
	56	16	15.46	15.46	907	10,646	94.4	97.4	0	0.11	0.95	3.07	4.13
	57	21	20.08	20.08	879	9,402	91.2	91.4	0	0.10	0.78	0.92	1.80
	58	23	22.92	22.92	717	7,837	81.1	81.0	0	0.09	0.63	1.52	2.24
	59	21	20.21	20.21	870	8,962	90.7	97.1	0	0.10	0.72	2.14	2.96
	60	20	19.74	19.74	874	9,545	90.9	89.9	0	0.10	0.80	2.64	3.54
	61	18	17.18	17.18	872	10,517	92.4	95.9	0	0.10	0.75	2.14	2.99
	62	22	21.77	21.77	840	8,923	89.8	93.3	0	0.09	0.69	2.30	3.08
	63	25	24.31	24.31	797	10,115	87.8	95.2	0	0.10	0.95	2.56	3.61
	64	22	21.80	21.80	811	9,426	88.2	90.1	0	0.10	0.87	2.47	3.44
	65	28	27.65	27.65	771	7,253	84.7	88.2	0	0.08	0.56	1.56	2.20
	66	26	25.89	25.89	797	8,336	88.2	95.4	0	0.08	0.70	2.24	3.02
	67	23	22.04	22.04	765	8,207	85.0	85.8	0	0.09	0.66	1.35	2.10
	68	24	23.78	23.78	833	8,829	88.8	95.3	0	0.09	0.83	2.48	3.40
	69	23	22.23	22.23	846	8,874	89.6	96.5	0	0.09	0.80	1.85	2.74
	70	24	23.04	23.04	771	7,797	84.9	89.7	0	0.08	0.55	1.44	2.07
	71	23	22.05	22.05	855	9,193	91.0	94.8	0	0.10	0.72	1.92	2.74
	72	24	23.68	23.68	897	10,821	93.0	97.6	0	0.11	0.80	3.02	3.93
	73	24	23.47	23.47	843	9,531	90.7	95.7	0	0.10	0.77	1.51	2.38
	74	22	21.20	21.20	847	9,585	90.0	95.1	0	0.10	0.67	2.26	3.03
	75	27	26.18	26.18	719	7,692	81.2	86.6	0	0.09	0.52	1.53	2.14
	76	18	17.89	17.89	886	9,459	92.3	97.4	0	0.09	0.76	2.54	3.39
	77	23	22.93	22.93	788	8,264	86.1	93.5	0	0.08	0.66	2.22	2.96
	78	26	25.73	25.73	792	8,847	86.5	91.9	0	0.10	0.69	2.00	2.79
	79	23	22.22	22.22	918	10,936	94.8	98.4	0	0.11	0.81	2.63	3.55

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	80	24	23.71	23.71	750	6,643	83.9	84.1	0	0.07	0.50	1.55	2.12
	81	19	18.38	18.38	913	11,074	93.7	92.6	0	0.11	0.91	3.19	4.21
	82	18	17.25	17.25	856	10,141	90.7	95.3	0	0.11	0.79	2.24	3.14
	83	21	20.60	20.60	878	9,397	92.7	94.6	0	0.09	0.80	2.94	3.83
	84	21	20.39	20.39	860	8,989	90.8	95.9	0	0.10	0.65	2.09	2.84
	85	16	15.06	15.06	904	11,312	93.2	93.6	0	0.12	0.80	2.61	3.53
	86	21	20.48	20.48	784	8,687	86.1	82.2	0	0.09	0.66	2.10	2.85
	87	22	21.31	21.31	844	8,373	89.2	84.5	0	0.10	0.65	1.98	2.73
	88	22	21.00	21.00	738	8,545	83.2	88.1	0	0.09	0.65	1.71	2.45
	89	23	22.43	22.43	747	8,222	83.8	81.7	0	0.09	0.68	1.56	2.33
	90	24	23.81	23.81	881	9,459	92.8	95.9	0	0.10	0.75	2.18	3.03
	91	19	18.21	18.21	822	8,498	89.3	91.5	0	0.08	0.63	2.17	2.88
	92	15	14.49	14.49	844	9,166	88.3	74.3	0	0.11	0.70	2.63	3.44
	93	22	21.78	21.78	887	9,480	92.4	89.7	0	0.10	0.74	2.63	3.47
	94	24	23.24	23.24	818	8,421	87.1	87.6	0	0.09	0.73	2.13	2.95
	95	25	24.26	24.26	848	8,852	90.2	94.5	0	0.10	0.85	1.91	2.86
	96	25	24.23	24.23	812	9,332	88.5	94.4	0	0.10	0.82	2.19	3.11
	97	17	16.03	16.03	819	8,231	89.3	90.6	0	0.09	0.45	1.62	2.16
	98	26	25.79	25.79	820	8,263	86.9	90.5	0	0.08	0.80	1.94	2.82
	99	19	18.16	18.16	887	10,985	93.2	95.9	0	0.11	0.69	2.90	3.70
	100	28	27.03	27.03	556	4,556	68.3	68.1	0	0.05	0.26	0.42	0.73
1	1	225	224.70	224.70	746	7,827	82.9	86.1	0	0.09	0.60	1.38	2.07
	2	193	192.10	192.10	889	9,311	92.4	86.8	0	0.10	0.67	1.43	2.20
	3	293	292.31	292.31	818	8,832	86.9	95.9	0	0.09	0.76	1.47	2.32
	4	188	187.10	187.10	870	10,533	92.3	94.3	0	0.10	0.72	2.53	3.35
	5	230	229.47	229.47	842	9,921	89.3	95.6	0	0.10	0.75	2.41	3.26
	6	246	245.61	245.61	831	8,879	89.6	93.0	0	0.09	0.72	2.62	3.43
	7	205	204.40	204.40	925	11,588	95.3	98.5	0	0.11	0.77	2.61	3.49
	8	262	261.96	261.96	636	6,408	75.6	74.0	0	0.08	0.41	1.00	1.49
	9	211	210.47	210.47	914	11,388	95.0	96.9	0	0.12	0.85	2.70	3.67
	10	224	223.09	223.09	771	8,245	85.8	90.3	0	0.09	0.60	1.66	2.35
	11	217	216.06	216.06	879	9,927	92.8	92.5	0	0.11	0.66	1.67	2.44
	12	215	214.97	214.97	805	10,236	88.0	91.2	0	0.10	0.64	2.76	3.50
	13	230	229.21	229.21	927	10,814	95.3	98.8	0	0.11	0.85	2.46	3.42
	14	191	190.92	190.92	822	9,957	89.1	90.5	0	0.11	0.68	1.39	2.18
	15	225	224.89	224.89	796	8,776	87.0	92.5	0	0.10	0.69	1.91	2.70
	16	206	205.78	205.78	809	9,335	88.2	92.6	0	0.10	0.73	2.65	3.48
	17	241	240.91	240.91	759	8,232	85.2	85.3	0	0.09	0.65	2.14	2.88
	18	225	224.84	224.84	813	8,741	88.9	91.2	0	0.10	0.63	2.21	2.94
	19	247	246.39	246.39	706	7,272	81.1	79.1	0	0.08	0.54	1.47	2.09
	20	247	246.79	246.79	811	8,363	87.2	94.5	0	0.09	0.67	1.85	2.61
	21	194	193.19	193.19	918	11,272	94.6	98.1	0	0.11	0.83	2.96	3.90
	22	229	228.41	228.41	695	6,778	80.3	80.4	0	0.08	0.46	1.40	1.94
	23	185	184.57	184.57	897	10,906	93.1	94.4	0	0.11	0.80	2.88	3.79
	24	234	233.15	233.15	863	10,063	91.7	95.6	0	0.10	0.67	1.79	2.56
	25	174	173.91	173.91	884	11,017	92.8	96.4	0	0.12	0.73	2.45	3.30
	26	182	181.86	181.86	861	10,866	92.0	94.7	0	0.12	0.80	2.34	3.26
	27	175	174.26	174.26	922	12,432	95.3	97.6	0	0.13	0.87	3.75	4.75
	28	198	197.16	197.16	880	10,650	92.5	96.0	0	0.11	0.75	2.60	3.46
	29	213	212.87	212.87	748	8,631	84.4	82.1	0	0.10	0.58	1.85	2.53
	30	157	156.47	156.47	880	11,260	93.1	94.6	0	0.11	0.73	2.11	2.95
	31	187	186.72	186.72	852	8,771	89.0	88.9	0	0.09	0.75	2.07	2.91
	32	266	265.95	265.95	751	6,801	81.5	85.4	0	0.07	0.55	1.83	2.45
	33	177	176.64	176.64	910	11,904	94.6	97.1	0	0.12	0.85	4.14	5.11
	34	195	194.11	194.11	838	10,779	90.5	91.8	0	0.11	0.82	2.50	3.43
	35	221	220.19	220.19	771	8,294	85.5	88.3	0	0.09	0.65	1.61	2.35
	36	224	223.21	223.21	871	9,372	92.1	94.8	0	0.10	0.73	2.23	3.06
	37	237	236.37	236.37	834	8,418	89.3	94.3	0	0.08	0.73	1.77	2.58
	38	302	301.55	301.55	581	4,782	70.0	68.9	0	0.07	0.27	0.46	0.80
	39	186	185.27	185.27	858	10,669	91.5	94.0	0	0.11	0.72	1.92	2.75
	40	225	224.69	224.69	728	8,345	81.5	80.7	0	0.09	0.56	1.78	2.43
	41	183	182.23	182.23	875	10,799	92.9	94.7	0	0.10	0.78	1.69	2.57
	42	229	228.75	228.75	795	8,089	86.4	86.0	0	0.09	0.64	1.79	2.52
	43	240	239.52	239.52	813	8,840	87.9	93.7	0	0.09	0.66	1.83	2.58

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	44	242	241.01	241.01	792	9,296	85.2	94.4	0	0.09	0.74	2.34	3.17
	45	262	261.85	261.85	874	9,929	91.3	97.4	0	0.11	0.85	3.40	4.36
	46	187	186.79	186.79	940	10,610	95.8	94.8	0	0.11	0.95	1.94	3.00
	47	219	218.32	218.32	872	9,532	91.7	92.7	0	0.10	0.69	1.68	2.47
	48	204	203.95	203.95	866	9,840	91.3	87.8	0	0.11	0.81	3.03	3.95
	49	235	234.06	234.06	762	6,935	83.0	86.5	0	0.08	0.51	1.39	1.98
	50	203	202.49	202.49	823	9,094	87.5	76.6	0	0.10	0.76	1.79	2.65
	51	248	247.32	247.32	834	8,428	88.7	91.7	0	0.09	0.67	1.55	2.31
	52	173	172.21	172.21	831	9,118	89.0	90.0	0	0.10	0.86	2.13	3.09
	53	177	176.56	176.56	832	9,766	89.3	92.1	0	0.10	0.76	2.22	3.08
	54	213	212.48	212.48	828	8,616	88.4	89.5	0	0.09	0.61	2.35	3.05
	55	179	178.84	178.84	738	7,951	83.4	82.6	0	0.09	0.41	2.20	2.70
	56	156	155.34	155.34	907	10,894	94.4	97.5	0	0.11	0.80	2.74	3.65
	57	200	199.76	199.76	879	9,402	91.2	91.4	0	0.10	0.80	1.99	2.89
	58	229	228.90	228.90	717	8,035	81.1	81.3	0	0.09	0.53	1.32	1.94
	59	202	201.22	201.22	870	9,011	90.7	97.1	0	0.09	0.76	1.88	2.73
	60	199	198.50	198.50	876	9,696	91.1	90.1	0	0.10	0.80	2.18	3.08
	61	173	172.22	172.22	872	10,552	92.4	95.9	0	0.10	0.76	2.54	3.40
	62	215	214.88	214.88	840	8,970	89.8	93.3	0	0.10	0.74	1.31	2.15
	63	243	242.93	242.93	806	10,187	87.2	95.0	0	0.11	1.00	3.27	4.38
	64	218	217.89	217.89	834	9,672	88.8	90.2	0	0.11	0.80	2.45	3.36
	65	276	275.11	275.11	802	7,558	87.9	89.3	0	0.08	0.61	0.75	1.44
	66	259	258.48	258.48	810	8,466	87.2	94.9	0	0.09	0.71	1.95	2.75
	67	220	219.26	219.26	768	8,330	85.2	86.0	0	0.09	0.59	1.54	2.22
	68	240	239.41	239.41	837	8,894	89.2	95.4	0	0.09	0.67	2.12	2.88
	69	218	217.22	217.22	846	9,083	89.6	96.6	0	0.09	0.65	2.11	2.85
	70	230	229.79	229.79	771	7,801	84.9	89.7	0	0.09	0.57	1.84	2.50
	71	220	219.18	219.18	855	9,271	91.0	94.8	0	0.10	0.74	2.13	2.97
	72	238	237.03	237.03	897	10,838	93.0	97.6	0	0.11	0.86	3.11	4.08
	73	232	231.14	231.14	844	9,723	90.0	95.4	0	0.10	0.73	1.93	2.76
	74	213	212.14	212.14	847	9,834	90.0	95.2	0	0.10	0.76	2.57	3.43
	75	259	258.12	258.12	722	7,701	81.5	86.6	0	0.09	0.55	0.93	1.57
	76	180	179.20	179.20	886	9,599	92.3	97.5	0	0.10	0.69	2.79	3.58
	77	229	228.66	228.66	794	8,296	86.2	93.4	0	0.09	0.61	1.93	2.63
	78	256	255.34	255.34	796	9,019	86.9	92.1	0	0.09	0.72	2.06	2.87
	79	226	225.21	225.21	918	10,968	94.8	98.4	0	0.11	0.84	2.99	3.94
	80	236	235.73	235.73	750	6,704	83.9	84.2	0	0.08	0.50	1.32	1.90
	81	185	184.30	184.30	916	11,167	94.0	92.7	0	0.12	0.89	2.15	3.16
	82	172	171.48	171.48	856	10,393	90.7	95.4	0	0.11	0.71	3.08	3.90
	83	208	207.50	207.50	889	9,485	93.5	94.8	0	0.10	0.75	2.22	3.07
	84	204	203.36	203.36	861	9,005	90.9	95.9	0	0.09	0.70	1.27	2.06
	85	152	151.15	151.15	904	11,417	93.2	93.7	0	0.11	0.85	3.15	4.11
	86	203	202.50	202.50	784	8,908	86.1	82.6	0	0.10	0.55	1.86	2.51
	87	213	212.89	212.89	848	8,433	89.6	84.6	0	0.10	0.59	2.11	2.80
	88	214	213.04	213.04	738	8,731	83.2	88.3	0	0.09	0.61	2.00	2.70
	89	225	224.07	224.07	748	8,242	84.0	81.7	0	0.09	0.64	1.28	2.01
	90	239	238.25	238.25	881	9,512	92.8	95.9	0	0.10	0.70	2.77	3.57
	91	180	179.88	179.88	822	8,578	89.3	91.6	0	0.09	0.62	2.13	2.84
	92	145	144.37	144.37	844	9,185	88.3	74.3	0	0.11	0.63	2.41	3.15
	93	220	219.49	219.49	888	9,504	92.5	89.7	0	0.10	0.72	1.88	2.70
	94	232	231.29	231.29	827	8,564	88.1	87.9	0	0.09	0.66	2.07	2.82
	95	243	242.69	242.69	852	9,010	90.3	94.5	0	0.10	0.72	2.37	3.19
	96	241	240.03	240.03	812	9,492	88.5	94.5	0	0.10	0.74	1.79	2.63
	97	160	159.50	159.50	819	8,410	89.3	90.8	0	0.09	0.55	1.70	2.34
	98	259	258.58	258.58	824	8,385	87.3	90.7	0	0.09	0.59	2.15	2.83
	99	184	183.16	183.16	887	11,039	93.2	95.9	0	0.11	0.76	3.18	4.05
	100	271	270.87	270.87	562	4,585	68.7	68.0	0	0.06	0.27	0.59	0.92
1	1	41	40.32	40.32	880	22,460	92.4	96.8	0	0.22	2.12	7.15	9.49
	2	48	47.29	47.29	945	19,845	96.7	99.2	0	0.20	1.56	6.61	8.37
	3	39	38.61	38.61	948	23,193	96.6	97.6	0	0.23	2.28	9.09	11.60
	4	41	40.75	40.75	939	21,406	96.0	96.1	0	0.21	1.89	8.95	11.05
	5	43	42.86	42.86	935	24,956	96.3	98.6	0	0.25	2.61	11.99	14.85
	6	42	41.74	41.74	908	20,553	94.5	96.4	0	0.21	1.73	8.67	10.61
	7	39	38.61	38.61	957	23,097	97.5	99.6	0	0.23	1.91	9.31	11.45

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	8	44	43.39	43.39	876	19,165	92.6	97.4	0	0.19	1.80	6.93	8.92
	9	44	43.48	43.48	884	20,188	93.1	94.2	0	0.21	1.62	12.70	14.53
	10	42	41.75	41.75	857	18,911	91.5	90.8	0	0.20	1.66	7.14	9.00
	11	39	38.97	38.97	957	22,584	97.4	99.4	0	0.22	1.94	10.23	12.39
	12	42	41.52	41.52	947	22,217	96.6	99.4	0	0.22	2.67	7.54	10.43
	13	41	40.58	40.58	917	23,379	95.1	98.8	0	0.23	2.58	9.41	12.22
	14	33	32.72	32.72	935	25,218	96.2	98.9	0	0.24	2.30	12.19	14.73
	15	39	38.30	38.30	880	20,731	92.3	98.3	0	0.21	1.60	6.19	8.00
	16	48	47.74	47.74	920	19,519	95.3	97.0	0	0.20	1.62	21.51	23.33
	17	41	40.75	40.75	954	22,641	97.3	99.4	0	0.22	1.89	10.94	13.05
	18	43	42.13	42.13	854	21,245	91.4	96.4	0	0.21	1.65	6.70	8.56
	19	50	49.72	49.72	911	18,993	93.8	95.5	0	0.19	1.81	7.21	9.21
	20	44	43.28	43.28	907	22,817	94.6	98.5	0	0.23	2.43	9.68	12.34
	21	37	36.73	36.73	913	21,971	94.9	98.4	0	0.22	2.13	13.57	15.92
	22	50	49.11	49.11	868	18,836	91.7	97.0	0	0.18	1.61	6.48	8.27
	23	39	38.26	38.26	947	21,433	96.8	97.4	0	0.21	1.96	7.40	9.57
	24	43	42.47	42.47	927	21,777	95.7	98.9	0	0.21	2.08	7.22	9.51
	25	47	46.09	46.09	898	18,930	93.3	98.3	0	0.19	1.52	6.28	7.99
	26	49	48.85	48.85	875	17,664	91.3	94.0	0	0.18	1.78	7.60	9.56
	27	42	41.09	41.09	875	21,137	92.4	96.1	0	0.21	1.94	7.24	9.39
	28	37	36.53	36.53	856	19,244	91.6	95.2	0	0.20	1.48	6.65	8.33
	29	46	45.56	45.56	919	19,406	94.5	94.9	0	0.20	1.90	6.43	8.53
	30	40	39.67	39.67	929	19,771	95.5	99.0	0	0.19	1.67	8.57	10.43
	31	38	37.13	37.13	922	20,431	95.3	98.7	0	0.20	1.61	6.53	8.34
	32	44	43.90	43.90	957	23,772	97.7	99.6	0	0.23	2.63	11.06	13.92
	33	46	45.55	45.55	949	20,986	97.0	99.2	0	0.20	2.52	7.86	10.58
	34	45	44.90	44.90	895	21,870	94.2	98.5	0	0.22	2.18	8.45	10.85
	35	45	44.88	44.88	928	17,541	95.7	98.9	0	0.18	1.85	5.33	7.36
	36	43	42.94	42.94	904	19,802	94.1	96.1	0	0.20	1.78	27.55	29.53
	37	44	43.69	43.69	933	20,062	96.1	96.8	0	0.20	1.74	7.46	9.40
	38	46	45.65	45.65	823	18,704	89.7	94.6	0	0.19	1.42	7.93	9.54
	39	47	46.35	46.35	913	21,766	94.3	99.0	0	0.22	2.33	9.67	12.22
	40	44	43.21	43.21	930	21,746	95.7	99.2	0	0.22	2.25	7.84	10.31
	41	40	39.35	39.35	927	22,670	95.1	96.3	0	0.23	2.11	8.74	11.08
	42	39	38.52	38.52	914	20,984	95.2	97.7	0	0.21	2.03	8.98	11.22
	43	41	40.84	40.84	939	22,825	96.0	96.9	0	0.23	2.14	22.19	24.56
	44	44	43.95	43.95	916	19,650	94.9	98.8	0	0.19	1.85	3.70	5.74
	45	47	46.03	46.03	870	19,839	92.5	96.3	0	0.20	1.90	5.96	8.06
	46	34	33.93	33.93	911	21,199	94.8	98.4	0	0.21	1.93	7.80	9.94
	47	44	43.42	43.42	920	21,659	94.9	93.3	0	0.22	2.37	8.35	10.94
	48	52	51.29	51.29	923	19,008	95.3	98.8	0	0.18	2.19	6.31	8.68
	49	41	40.60	40.60	909	22,123	94.6	96.3	0	0.23	2.28	7.77	10.28
	50	46	45.98	45.98	891	19,575	93.4	97.6	0	0.19	1.90	5.82	7.91
	51	40	39.55	39.55	947	20,527	96.8	99.3	0	0.20	1.95	8.60	10.75
	52	53	52.09	52.09	881	18,599	92.7	98.3	0	0.19	1.94	5.97	8.10
	53	40	39.24	39.24	913	22,879	94.8	98.7	0	0.22	1.71	9.37	11.30
	54	46	45.19	45.19	901	20,842	94.3	97.3	0	0.20	2.05	8.05	10.30
	55	52	51.44	51.44	952	19,479	96.8	97.2	0	0.19	1.87	8.47	10.53
	56	46	45.59	45.59	932	22,183	96.2	98.8	0	0.22	2.10	10.21	12.53
	57	47	46.71	46.71	890	17,847	93.1	89.9	0	0.19	1.56	7.16	8.91
	58	43	42.93	42.93	858	18,535	91.1	97.4	0	0.18	1.60	6.02	7.80
	59	46	45.32	45.32	924	18,964	95.0	98.7	0	0.19	2.18	5.50	7.87
	60	46	45.96	45.96	914	22,199	94.9	98.8	0	0.23	1.71	50.49	52.43
	61	36	35.44	35.44	942	22,764	96.2	95.6	0	0.23	2.12	9.11	11.46
	62	45	44.61	44.61	870	18,080	91.6	96.1	0	0.18	1.59	5.76	7.53
	63	45	44.03	44.03	946	20,953	96.8	99.2	0	0.20	2.26	7.76	10.22
	64	44	43.58	43.58	891	17,051	93.6	98.2	0	0.17	1.43	5.62	7.22
	65	39	38.22	38.22	894	23,273	94.0	97.5	0	0.23	2.08	9.29	11.60
	66	41	40.23	40.23	924	24,210	95.8	98.5	0	0.24	2.23	9.14	11.61
	67	40	39.85	39.85	943	24,261	96.4	97.7	0	0.25	2.43	10.44	13.12
	68	41	40.33	40.33	918	21,077	95.1	99.0	0	0.22	2.14	8.23	10.59
	69	46	45.55	45.55	889	17,895	93.1	97.6	0	0.18	1.51	5.71	7.40
	70	39	38.99	38.99	928	21,452	95.2	99.1	0	0.21	2.04	7.49	9.74
	71	45	44.70	44.70	914	21,435	94.4	99.0	0	0.21	1.70	7.73	9.64

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	72	45	44.57	44.57	879	22,512	92.9	97.3	0	0.23	1.74	8.77	10.74
	73	38	37.91	37.91	936	23,799	95.9	95.8	0	0.23	2.58	11.15	13.96
	74	41	40.36	40.36	959	23,180	97.7	99.6	0	0.23	2.39	11.43	14.05
	75	35	34.80	34.80	926	24,192	95.7	98.6	0	0.24	2.17	11.30	13.71
	76	43	42.57	42.57	873	21,260	92.6	97.4	0	0.21	1.92	8.93	11.06
	77	49	48.23	48.23	894	18,130	92.9	89.0	0	0.19	1.50	4.89	6.58
	78	38	37.94	37.94	919	23,126	95.2	99.0	0	0.22	2.19	10.49	12.90
	79	45	44.33	44.33	934	22,948	95.7	97.5	0	0.23	1.78	8.08	10.09
	80	40	39.86	39.86	934	20,990	96.0	96.7	0	0.21	2.15	8.27	10.63
	81	45	44.77	44.77	879	19,700	92.2	94.6	0	0.20	1.70	7.50	9.40
	82	46	45.12	45.12	933	19,495	95.9	99.1	0	0.19	2.07	5.55	7.81
	83	45	44.09	44.09	956	22,662	97.3	99.6	0	0.22	1.97	8.54	10.73
	84	40	39.07	39.07	949	24,438	96.9	99.4	0	0.24	2.48	10.34	13.06
	85	43	42.15	42.15	955	19,915	97.3	99.6	0	0.20	2.26	6.92	9.38
	86	41	40.72	40.72	936	23,299	96.3	99.0	0	0.23	2.09	9.77	12.09
	87	47	46.62	46.62	883	18,728	92.8	98.2	0	0.19	1.95	4.97	7.11
	88	42	41.05	41.05	943	20,569	96.5	99.2	0	0.21	1.96	6.86	9.03
	89	49	48.63	48.63	883	19,847	93.4	95.7	0	0.20	2.00	7.02	9.22
	90	47	46.06	46.06	899	20,458	93.9	97.4	0	0.21	2.03	6.72	8.96
	91	32	31.45	31.45	928	25,578	95.9	97.0	0	0.25	2.02	11.59	13.86
	92	45	44.38	44.38	913	21,875	94.6	98.8	0	0.22	2.01	7.31	9.54
	93	44	43.79	43.79	934	23,026	96.0	98.8	0	0.23	2.35	9.46	12.04
	94	45	44.78	44.78	962	21,699	97.8	99.5	0	0.21	2.35	9.16	11.72
	95	37	36.89	36.89	956	24,512	97.5	99.6	0	0.25	2.24	11.13	13.62
	96	39	38.84	38.84	930	22,511	95.7	96.9	0	0.22	1.71	8.43	10.36
	97	38	37.76	37.76	944	22,860	96.6	99.2	0	0.22	1.83	9.73	11.78
	98	41	40.32	40.32	949	24,919	96.9	99.5	0	0.25	2.56	9.12	11.93
	99	48	47.03	47.03	892	21,962	92.9	95.8	0	0.22	1.95	6.96	9.13
	100	44	43.96	43.96	927	21,240	95.7	96.2	0	0.22	2.00	8.59	10.81
1	1	406	405.99	405.99	880	22,570	92.4	96.8	0	0.23	1.84	4.42	6.49
	2	475	474.93	474.93	945	19,873	96.7	99.2	0	0.20	1.63	9.27	11.10
	3	388	387.04	387.04	948	23,251	96.6	97.6	0	0.23	2.09	9.39	11.71
	4	406	405.98	405.98	939	21,419	96.0	96.1	0	0.22	1.70	9.29	11.21
	5	429	428.67	428.67	935	25,000	96.3	98.6	0	0.25	2.34	16.72	19.31
	6	418	417.26	417.26	908	20,607	94.5	96.4	0	0.20	1.72	4.61	6.53
	7	386	385.17	385.17	957	23,141	97.5	99.6	0	0.22	1.83	7.91	9.96
	8	437	436.64	436.64	876	19,190	92.6	97.4	0	0.19	1.50	4.82	6.51
	9	436	435.42	435.42	884	20,306	93.1	94.2	0	0.21	1.64	7.42	9.27
	10	418	417.46	417.46	857	19,008	91.5	90.9	0	0.20	1.40	7.20	8.80
	11	391	390.46	390.46	957	22,638	97.4	99.4	0	0.22	1.90	6.06	8.18
	12	415	414.64	414.64	953	22,362	97.2	99.5	0	0.21	1.65	6.20	8.06
	13	403	402.33	402.33	918	23,415	95.0	98.8	0	0.23	1.76	9.02	11.01
	14	327	326.98	326.98	935	25,323	96.2	98.9	0	0.25	1.87	15.74	17.86
	15	384	383.81	383.81	881	20,760	92.3	98.3	0	0.21	1.65	8.88	10.74
	16	477	476.68	476.68	923	19,581	95.3	97.0	0	0.20	1.55	8.21	9.96
	17	406	405.14	405.14	954	22,648	97.3	99.4	0	0.23	1.72	7.82	9.77
	18	422	421.79	421.79	856	21,356	91.6	96.5	0	0.21	1.67	9.18	11.06
	19	496	495.47	495.47	911	19,014	93.8	95.5	0	0.20	1.90	7.03	9.13
	20	431	430.46	430.46	907	22,922	94.6	98.5	0	0.23	1.77	10.41	12.41
	21	369	368.53	368.53	920	22,013	95.3	98.4	0	0.22	1.98	7.58	9.78
	22	489	488.48	488.48	868	18,850	91.7	97.0	0	0.19	1.59	5.52	7.30
	23	383	382.63	382.63	947	21,587	96.8	97.4	0	0.21	2.15	8.39	10.75
	24	423	422.71	422.71	927	21,831	95.7	98.9	0	0.23	1.82	9.66	11.71
	25	460	459.40	459.40	898	18,968	93.3	98.3	0	0.19	1.57	6.55	8.31
	26	486	485.68	485.68	882	17,872	92.1	94.1	0	0.19	1.58	8.51	10.28
	27	409	408.67	408.67	875	21,338	92.4	96.2	0	0.22	1.72	8.53	10.47
	28	364	363.97	363.97	856	19,385	91.6	95.2	0	0.20	1.35	7.27	8.82
	29	457	456.34	456.34	919	19,440	94.5	94.9	0	0.19	1.48	6.54	8.21
	30	401	400.93	400.93	929	19,919	95.5	99.0	0	0.20	1.81	7.88	9.89
	31	372	371.57	371.57	922	20,485	95.3	98.7	0	0.20	1.56	6.48	8.24
	32	438	437.25	437.25	961	23,835	97.8	99.6	0	0.24	2.14	7.21	9.59
	33	451	450.31	450.31	949	21,085	97.0	99.2	0	0.21	1.68	7.14	9.03
	34	446	445.12	445.12	898	21,990	93.9	98.4	0	0.21	1.55	7.83	9.59
	35	450	449.02	449.02	930	17,621	95.7	98.9	0	0.17	1.44	5.63	7.24

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	36	432	431.57	431.57	904	20,057	94.1	96.2	0	0.20	1.71	8.15	10.06
	37	441	440.68	440.68	933	20,115	96.1	96.8	0	0.20	1.74	7.13	9.07
	38	456	456.00	456.00	823	18,953	89.7	94.6	0	0.20	1.25	8.36	9.81
	39	468	467.15	467.15	921	21,962	94.9	99.1	0	0.22	1.69	5.32	7.23
	40	431	430.17	430.17	930	21,803	95.7	99.2	0	0.21	1.66	8.21	10.08
	41	393	392.34	392.34	927	22,850	95.1	96.3	0	0.23	1.61	7.08	8.92
	42	385	384.60	384.60	914	21,080	95.2	97.7	0	0.21	1.56	7.16	8.93
	43	410	409.94	409.94	939	22,858	96.0	96.9	0	0.24	1.75	30.53	32.52
	44	440	439.13	439.13	916	19,738	94.9	98.9	0	0.19	1.39	7.02	8.60
	45	459	458.73	458.73	870	19,985	92.5	96.3	0	0.21	1.62	6.52	8.35
	46	336	335.72	335.72	911	21,388	94.8	98.4	0	0.22	1.95	7.31	9.48
	47	435	434.79	434.79	920	21,755	94.9	93.4	0	0.22	1.79	11.74	13.75
	48	514	513.70	513.70	923	19,082	95.3	98.8	0	0.19	1.56	7.63	9.38
	49	403	402.69	402.69	909	22,248	94.6	96.3	0	0.23	1.73	9.75	11.71
	50	456	455.89	455.89	898	19,618	93.9	97.7	0	0.21	1.62	8.34	10.17
	51	394	393.87	393.87	947	20,573	96.8	99.3	0	0.21	1.80	7.40	9.41
	52	522	521.50	521.50	890	18,688	93.1	98.3	0	0.19	1.51	4.54	6.24
	53	391	390.94	390.94	913	22,896	94.8	98.7	0	0.24	1.80	11.80	13.84
	54	450	449.84	449.84	901	20,875	94.3	97.3	0	0.21	1.71	7.56	9.48
	55	511	510.28	510.28	955	19,538	97.2	97.2	0	0.19	1.92	4.70	6.81
	56	454	453.52	453.52	932	22,234	96.2	98.8	0	0.22	1.75	7.49	9.46
	57	466	465.95	465.95	890	17,938	93.0	89.9	0	0.19	1.30	6.48	7.97
	58	433	432.35	432.35	858	18,764	91.1	97.4	0	0.19	1.29	5.76	7.24
	59	458	457.14	457.14	924	19,024	95.0	98.7	0	0.19	2.11	4.78	7.08
	60	461	460.70	460.70	914	22,203	94.9	98.8	0	0.24	1.74	5.26	7.24
	61	357	356.61	356.61	943	22,977	96.3	95.6	0	0.24	1.91	9.84	11.99
	62	445	444.46	444.46	871	18,125	91.7	96.1	0	0.18	1.50	5.95	7.63
	63	441	440.38	440.38	946	21,048	96.8	99.2	0	0.21	1.66	7.28	9.15
	64	435	434.31	434.31	891	17,175	93.6	98.2	0	0.17	1.23	5.86	7.26
	65	382	381.49	381.49	894	23,388	94.0	97.5	0	0.23	1.97	5.72	7.92
	66	401	400.20	400.20	924	24,238	95.8	98.5	0	0.24	1.92	6.47	8.63
	67	402	401.79	401.79	943	24,398	96.4	97.7	0	0.30	2.08	10.66	13.04
	68	407	406.02	406.02	918	21,115	94.9	99.0	0	0.21	1.65	7.64	9.50
	69	452	451.23	451.23	889	18,079	93.1	97.6	0	0.18	1.41	5.82	7.41
	70	391	390.28	390.28	940	21,871	96.2	99.3	0	0.22	1.69	8.60	10.51
	71	449	448.27	448.27	914	21,501	94.4	99.0	0	0.21	1.68	7.84	9.73
	72	447	446.56	446.56	881	22,547	93.0	97.3	0	0.24	1.57	11.74	13.55
	73	381	380.57	380.57	939	23,912	96.2	95.8	0	0.24	1.74	10.54	12.52
	74	404	403.75	403.75	959	23,225	97.7	99.6	0	0.24	2.15	9.15	11.54
	75	345	344.69	344.69	926	24,429	95.7	98.6	0	0.24	1.88	9.94	12.06
	76	426	425.65	425.65	873	21,318	92.6	97.4	0	0.22	1.49	7.23	8.94
	77	478	477.24	477.24	897	18,416	93.2	89.2	0	0.19	1.45	6.50	8.14
	78	377	376.73	376.73	919	23,216	95.2	99.0	0	0.23	1.83	7.76	9.82
	79	441	440.17	440.17	935	23,023	95.8	97.5	0	0.23	1.75	9.45	11.43
	80	398	397.82	397.82	935	21,088	95.9	96.6	0	0.21	1.70	8.50	10.41
	81	448	447.65	447.65	879	19,798	92.2	94.7	0	0.20	1.46	5.45	7.11
	82	449	448.45	448.45	934	19,599	96.0	99.1	0	0.20	1.89	6.24	8.33
	83	441	440.30	440.30	959	22,695	97.6	99.6	0	0.22	1.76	7.10	9.08
	84	392	391.19	391.19	950	24,800	97.0	99.4	0	0.24	2.22	9.68	12.14
	85	420	419.50	419.50	959	20,047	97.5	99.5	0	0.20	1.65	7.02	8.87
	86	403	402.72	402.72	936	23,362	96.3	99.0	0	0.23	1.80	6.82	8.85
	87	465	464.64	464.64	883	18,791	92.8	98.2	0	0.19	1.35	4.71	6.25
	88	408	407.23	407.23	943	20,660	96.5	99.2	0	0.20	1.72	7.07	8.99
	89	482	481.76	481.76	896	19,912	93.1	95.6	0	0.21	1.56	6.76	8.53
	90	462	461.38	461.38	908	20,531	94.9	97.5	0	0.20	1.74	7.50	9.44
	91	316	315.04	315.04	928	25,633	95.9	97.0	0	0.25	2.07	9.85	12.17
	92	442	441.48	441.48	917	21,959	95.0	98.9	0	0.22	1.93	7.53	9.68
	93	438	437.18	437.18	942	23,227	96.7	98.9	0	0.23	1.74	7.84	9.81
	94	444	443.68	443.68	962	21,856	97.8	99.5	0	0.22	1.65	8.31	10.18
	95	366	365.56	365.56	958	24,632	97.5	99.6	0	0.24	1.73	7.32	9.29
	96	391	390.28	390.28	932	22,536	95.6	96.8	0	0.22	1.75	8.11	10.08
	97	379	378.84	378.84	946	22,882	96.8	99.2	0	0.22	1.74	13.84	15.80
	98	402	401.97	401.97	949	24,982	96.9	99.5	0	0.25	2.17	9.28	11.70
	99	473	472.27	472.27	896	22,025	93.3	95.7	0	0.22	1.73	7.28	9.23

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	100	440	439.68	439.68	927	21,309	95.7	96.2	0	0.21	1.75	8.69	10.65
1	1	66	65.50	48.15	25	89	14.4	24.7	0	0.01	0.00	0.00	0.01
	2	39	38.31	37.22	19	84	10.6	21.8	0	0.02	0.00	0.00	0.02
	3	33	33.00	31.77	49	181	11.4	18.1	0	0.01	0.00	0.01	0.02
	4	47	47.00	39.32	121	383	20.1	27.7	0	0.02	0.00	0.01	0.03
	5	71	71.00	53.95	15	46	10.9	16.8	0	0.01	0.00	0.00	0.01
	6	82	82.00	62.36	11	35	20.4	32.4	0	0.01	0.00	0.00	0.01
	7	42	41.36	39.72	33	142	10.8	20.9	0	0.01	0.00	0.01	0.02
	8	40	40.00	36.85	76	258	14.9	21.0	0	0.01	0.00	0.01	0.02
	9	29	28.25	28.07	158	614	23.4	34.1	0	0.02	0.01	0.02	0.05
	10	54	54.00	45.89	41	141	13.3	21.3	0	0.01	0.00	0.01	0.02
	11	45	45.00	42.60	24	90	15.9	29.5	0	0.01	0.00	0.00	0.01
	12	83	83.00	60.34	14	38	13.7	18.9	0	0.01	0.00	0.00	0.01
	13	29	28.10	26.52	157	506	21.0	29.0	0	0.02	0.01	0.01	0.04
	14	53	52.75	48.24	11	50	15.9	36.2	0	0.01	0.00	0.00	0.01
	15	50	49.50	45.34	19	59	16.4	25.5	0	0.01	0.00	0.00	0.01
	16	33	32.50	31.77	115	399	21.9	29.0	0	0.01	0.00	0.02	0.03
	17	43	43.00	40.03	50	169	15.6	23.7	0	0.01	0.00	0.01	0.02
	18	47	47.00	42.62	21	84	9.9	19.0	0	0.01	0.00	0.00	0.01
	19	58	58.00	49.08	25	78	16.8	25.7	0	0.01	0.00	0.00	0.01
	20	23	22.79	22.68	42	181	9.3	15.8	0	0.02	0.00	0.01	0.03
	21	56	55.17	47.91	68	243	14.2	21.6	0	0.01	0.00	0.01	0.02
	22	47	46.50	37.38	64	219	14.3	21.2	0	0.01	0.00	0.01	0.02
	23	56	55.50	47.63	9	34	13.4	25.8	0	0.01	0.00	0.00	0.01
	24	56	56.00	47.55	34	106	13.8	20.8	0	0.02	0.00	0.00	0.02
	25	42	41.50	38.31	34	132	11.8	21.0	0	0.01	0.00	0.01	0.02
	26	62	61.50	51.58	24	81	11.5	19.4	0	0.01	0.00	0.00	0.01
	27	35	34.17	33.34	37	167	8.4	14.9	0	0.01	0.00	0.01	0.02
	28	41	40.50	39.23	41	141	10.9	17.3	0	0.01	0.00	0.01	0.02
	29	57	57.00	41.70	21	67	14.7	23.3	0	0.01	0.00	0.00	0.01
	30	37	36.37	35.88	20	82	10.9	21.6	0	0.02	0.00	0.00	0.02
	31	51	50.50	47.26	14	52	15.1	28.0	0	0.01	0.00	0.00	0.01
	32	38	37.18	36.96	42	158	11.8	19.9	0	0.01	0.00	0.01	0.02
	33	45	44.50	41.66	29	118	11.0	20.0	0	0.02	0.00	0.00	0.02
	34	91	91.00	61.33	17	50	11.0	16.2	0	0.01	0.00	0.00	0.01
	35	64	63.50	51.35	16	58	12.4	22.5	0	0.01	0.00	0.00	0.01
	36	56	56.00	47.68	22	80	14.2	24.9	0	0.01	0.00	0.00	0.01
	37	59	59.00	46.66	12	41	18.2	31.5	0	0.01	0.00	0.00	0.01
	38	41	40.67	38.27	27	104	12.3	22.1	0	0.02	0.00	0.00	0.02
	39	57	56.50	46.86	31	104	12.4	20.1	0	0.02	0.00	0.00	0.02
	40	51	50.50	40.21	129	447	19.6	26.6	0	0.02	0.00	0.01	0.03
	41	40	40.00	33.58	150	591	20.7	32.4	0	0.02	0.01	0.02	0.05
	42	32	31.42	31.05	40	139	20.1	29.9	0	0.01	0.00	0.01	0.02
	43	58	57.50	50.71	13	44	22.0	36.1	0	0.01	0.00	0.00	0.01
	44	37	36.33	34.18	87	382	17.7	29.0	0	0.01	0.00	0.02	0.03
	45	39	38.67	36.03	30	112	9.4	16.2	0	0.02	0.00	0.00	0.02
	46	60	59.50	46.55	23	68	10.5	15.4	0	0.01	0.00	0.00	0.01
	47	54	53.50	46.83	21	77	10.8	19.4	0	0.01	0.00	0.00	0.01
	48	61	61.00	49.68	37	134	13.0	21.2	0	0.02	0.00	0.00	0.02
	49	38	38.00	33.88	44	182	10.3	18.9	0	0.01	0.00	0.01	0.02
	50	50	50.00	42.58	32	108	15.2	24.5	0	0.01	0.00	0.00	0.01
	51	45	44.50	41.97	34	129	11.4	19.7	0	0.01	0.00	0.01	0.02
	52	38	37.76	36.61	37	153	12.6	23.8	0	0.01	0.00	0.01	0.02
	53	21	20.26	20.22	272	1,023	33.7	36.9	0	0.02	0.03	0.03	0.08
	54	63	63.00	44.97	39	132	10.1	15.6	0	0.02	0.00	0.00	0.02
	55	44	43.50	42.56	15	59	13.5	26.0	0	0.01	0.00	0.00	0.01
	56	66	66.00	48.72	33	87	22.6	28.7	0	0.01	0.00	0.00	0.01
	57	49	49.00	44.95	10	40	9.3	18.9	0	0.01	0.00	0.00	0.01
	58	45	44.89	40.69	14	55	13.3	25.9	0	0.01	0.00	0.00	0.01
	59	47	47.00	42.17	32	130	13.6	24.6	0	0.02	0.00	0.00	0.02
	60	42	41.25	38.88	61	235	13.6	22.1	0	0.01	0.00	0.01	0.02
	61	47	47.00	37.58	45	138	12.3	18.0	0	0.02	0.00	0.00	0.02
	62	68	68.00	51.77	21	74	10.7	18.8	0	0.01	0.00	0.00	0.01
	63	40	39.57	38.67	19	58	12.9	19.5	0	0.01	0.00	0.00	0.01

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	64	64	64.00	48.34	10	32	12.3	20.1	0	0.01	0.00	0.00	0.01
	65	29	28.32	28.24	79	343	13.3	23.2	0	0.02	0.01	0.00	0.03
	66	49	48.50	41.46	43	180	21.8	39.0	0	0.01	0.00	0.01	0.02
	67	36	35.50	34.35	24	94	9.6	17.5	0	0.02	0.00	0.00	0.02
	68	32	31.38	30.62	76	369	14.4	23.6	0	0.02	0.01	0.01	0.04
	69	58	57.50	47.98	10	39	14.9	29.5	0	0.01	0.00	0.00	0.01
	70	52	51.50	46.34	10	45	13.7	29.6	0	0.01	0.00	0.00	0.01
	71	41	41.00	31.23	168	603	27.5	31.1	0	0.02	0.01	0.01	0.04
	72	70	70.00	53.13	11	37	17.7	30.1	0	0.01	0.00	0.00	0.01
	73	56	55.83	47.21	23	75	16.8	26.7	0	0.01	0.00	0.00	0.01
	74	45	44.33	41.56	12	46	20.3	39.7	0	0.01	0.00	0.00	0.01
	75	81	80.50	59.89	4	14	21.1	41.2	0	0.01	0.00	0.00	0.01
	76	56	56.00	46.15	70	242	12.5	19.5	0	0.01	0.00	0.01	0.02
	77	46	46.00	39.48	71	276	13.9	21.3	0	0.02	0.00	0.01	0.03
	78	66	65.50	52.48	12	35	20.0	29.9	0	0.01	0.00	0.00	0.01
	79	51	50.50	46.59	22	77	14.7	24.8	0	0.01	0.00	0.00	0.01
	80	38	38.00	36.79	28	88	12.4	18.8	0	0.01	0.00	0.00	0.01
	81	60	60.00	48.17	24	89	13.0	23.0	0	0.01	0.00	0.00	0.01
	82	42	42.00	39.35	43	160	11.8	20.4	0	0.01	0.00	0.01	0.02
	83	25	24.27	24.19	61	280	12.3	19.1	0	0.02	0.00	0.01	0.03
	84	56	56.00	46.75	27	103	12.4	22.7	0	0.02	0.00	0.00	0.02
	85	61	61.00	44.13	30	102	14.2	22.7	0	0.02	0.00	0.00	0.02
	86	46	46.00	43.06	27	88	16.2	25.7	0	0.02	0.00	0.00	0.02
	87	69	69.00	52.20	21	61	15.2	21.8	0	0.01	0.00	0.00	0.01
	88	57	57.00	44.65	87	309	13.9	21.9	0	0.01	0.00	0.01	0.02
	89	57	56.50	45.55	33	102	12.8	19.1	0	0.02	0.00	0.00	0.02
	90	45	44.50	40.96	11	38	14.9	25.2	0	0.01	0.00	0.00	0.01
	91	54	54.00	44.52	265	990	33.3	49.1	0	0.02	0.01	0.03	0.06
	92	34	33.20	32.81	30	126	10.0	18.9	0	0.01	0.00	0.01	0.02
	93	29	29.00	27.89	48	170	12.3	20.2	0	0.01	0.00	0.01	0.02
	94	81	81.00	60.04	6	19	23.1	39.6	0	0.01	0.00	0.00	0.01
	95	76	76.00	55.37	30	103	14.1	23.3	0	0.02	0.00	0.00	0.02
	96	54	54.00	45.43	38	110	13.4	18.9	0	0.02	0.00	0.00	0.02
	97	69	68.50	51.89	15	54	20.5	36.5	0	0.01	0.00	0.00	0.01
	98	52	51.50	42.18	42	142	10.9	16.9	0	0.01	0.00	0.01	0.02
	99	37	37.00	31.60	33	123	9.2	14.7	0	0.02	0.00	0.00	0.02
	100	57	57.00	48.64	16	51	14.4	22.8	0	0.01	0.00	0.00	0.01
1	1	647	647.00	475.72	55	154	16.4	21.9	0	0.02	0.00	0.00	0.02
	2	384	383.23	372.54	21	90	11.6	23.2	0	0.02	0.00	0.00	0.02
	3	331	330.17	317.55	51	197	11.0	17.5	0	0.01	0.00	0.01	0.02
	4	482	482.00	402.21	126	397	21.0	28.1	0	0.02	0.00	0.01	0.03
	5	706	706.00	536.49	40	110	13.7	18.5	0	0.02	0.00	0.00	0.02
	6	823	822.50	626.73	23	62	15.1	20.2	0	0.01	0.00	0.00	0.01
	7	419	418.14	401.27	38	149	10.6	18.8	0	0.01	0.00	0.01	0.02
	8	398	398.00	366.19	78	262	15.3	21.1	0	0.01	0.00	0.01	0.02
	9	284	283.50	281.89	172	642	24.9	34.9	0	0.02	0.01	0.02	0.05
	10	539	538.33	457.11	61	206	15.8	23.5	0	0.01	0.00	0.01	0.02
	11	445	445.00	423.52	28	104	11.5	20.3	0	0.02	0.00	0.00	0.02
	12	841	841.00	611.59	55	123	19.6	21.8	0	0.01	0.00	0.00	0.01
	13	283	282.70	267.79	175	566	22.7	30.6	0	0.02	0.01	0.02	0.05
	14	522	521.88	477.46	11	50	15.5	35.0	0	0.01	0.00	0.00	0.01
	15	498	497.50	455.97	20	61	17.1	26.2	0	0.01	0.00	0.00	0.01
	16	331	330.10	322.28	121	406	22.2	28.6	0	0.02	0.00	0.01	0.03
	17	435	435.00	403.88	70	242	15.4	22.5	0	0.01	0.00	0.01	0.02
	18	476	476.00	430.67	24	91	11.2	20.2	0	0.01	0.00	0.00	0.01
	19	579	579.00	492.03	25	78	16.8	25.7	0	0.01	0.00	0.00	0.01
	20	232	231.42	230.23	40	176	8.8	14.9	0	0.01	0.00	0.01	0.02
	21	548	547.50	476.26	63	223	12.5	18.3	0	0.01	0.00	0.01	0.02
	22	476	475.50	380.32	64	219	14.3	21.2	0	0.01	0.00	0.01	0.02
	23	551	551.00	475.05	9	34	13.2	25.4	0	0.01	0.00	0.00	0.01
	24	562	561.50	474.91	65	167	19.9	24.5	0	0.01	0.00	0.01	0.02
	25	407	406.50	378.23	38	139	12.3	20.5	0	0.01	0.00	0.01	0.02
	26	616	616.00	517.29	82	233	18.4	24.3	0	0.01	0.00	0.01	0.02
	27	351	350.35	341.21	42	175	8.9	14.7	0	0.01	0.00	0.01	0.02

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	28	403	402.50	390.62	44	147	11.5	17.7	0	0.01	0.00	0.01	0.02
	29	573	573.00	417.62	22	69	15.2	23.7	0	0.01	0.00	0.00	0.01
	30	367	366.82	361.86	20	82	10.9	21.6	0	0.01	0.00	0.01	0.02
	31	503	503.00	470.37	19	61	15.6	24.7	0	0.01	0.00	0.00	0.01
	32	373	372.99	370.61	43	165	12.0	20.3	0	0.01	0.00	0.01	0.02
	33	444	444.00	417.32	29	118	11.0	20.0	0	0.02	0.00	0.00	0.02
	34	903	903.00	611.52	94	216	24.8	27.5	0	0.02	0.00	0.00	0.02
	35	633	632.50	512.38	23	77	12.8	21.2	0	0.01	0.00	0.00	0.01
	36	564	564.00	479.19	23	82	14.7	25.4	0	0.01	0.00	0.00	0.01
	37	592	592.00	467.24	12	41	18.2	31.5	0	0.01	0.00	0.00	0.01
	38	400	399.67	379.14	27	104	12.3	22.1	0	0.02	0.00	0.00	0.02
	39	565	565.00	467.19	32	109	12.4	20.3	0	0.01	0.00	0.00	0.01
	40	506	505.50	398.68	134	459	20.4	27.0	0	0.01	0.01	0.01	0.03
	41	405	405.00	338.46	205	756	26.6	34.4	0	0.02	0.01	0.02	0.05
	42	317	316.44	312.25	40	145	17.9	27.5	0	0.01	0.00	0.01	0.02
	43	577	577.00	508.10	13	44	22.0	36.1	0	0.01	0.00	0.00	0.01
	44	356	355.50	335.69	106	400	17.7	23.5	0	0.01	0.00	0.02	0.03
	45	377	377.00	353.15	28	102	8.0	13.3	0	0.02	0.00	0.00	0.02
	46	596	595.50	463.43	56	145	19.1	23.9	0	0.02	0.00	0.00	0.02
	47	531	530.50	466.99	23	82	11.6	20.0	0	0.01	0.00	0.00	0.01
	48	613	612.50	498.88	40	140	7.9	12.1	0	0.02	0.00	0.00	0.02
	49	369	369.00	334.92	43	171	10.1	17.3	0	0.01	0.00	0.01	0.02
	50	509	508.50	431.10	32	108	15.2	24.5	0	0.02	0.00	0.00	0.02
	51	449	449.00	422.98	39	149	12.1	20.4	0	0.02	0.00	0.00	0.02
	52	378	377.45	365.63	52	180	14.0	22.1	0	0.01	0.00	0.01	0.02
	53	204	203.07	202.56	277	1,039	34.3	37.1	0	0.02	0.02	0.03	0.07
	54	628	628.00	448.06	34	117	8.6	13.5	0	0.02	0.00	0.00	0.02
	55	432	432.00	423.18	25	84	15.2	24.9	0	0.01	0.00	0.00	0.01
	56	667	667.00	490.78	33	88	22.6	28.9	0	0.01	0.00	0.00	0.01
	57	496	495.50	452.90	10	40	9.3	18.9	0	0.01	0.00	0.00	0.01
	58	448	447.17	404.87	16	65	14.0	28.1	0	0.01	0.00	0.00	0.01
	59	468	467.50	419.13	35	130	12.6	20.9	0	0.02	0.00	0.00	0.02
	60	410	409.75	386.94	63	226	13.2	19.5	0	0.01	0.00	0.01	0.02
	61	465	464.50	372.82	45	140	12.3	18.2	0	0.01	0.00	0.01	0.02
	62	667	667.00	508.83	43	140	16.4	24.5	0	0.02	0.00	0.00	0.02
	63	388	387.43	379.08	19	58	12.9	19.5	0	0.01	0.00	0.00	0.01
	64	646	646.00	484.58	13	38	13.8	20.5	0	0.01	0.00	0.00	0.01
	65	286	285.84	284.89	125	490	18.4	26.5	0	0.02	0.01	0.03	0.06
	66	485	484.50	414.20	72	277	26.1	41.2	0	0.02	0.00	0.01	0.03
	67	352	351.20	339.51	24	94	9.6	17.5	0	0.02	0.00	0.00	0.02
	68	310	309.88	303.08	81	354	14.8	21.0	0	0.02	0.01	0.01	0.04
	69	585	584.50	484.31	10	39	14.7	29.1	0	0.01	0.00	0.00	0.01
	70	514	513.50	462.69	9	42	12.2	27.3	0	0.01	0.00	0.00	0.01
	71	414	414.00	316.89	174	622	28.5	31.4	0	0.02	0.01	0.01	0.04
	72	707	706.50	535.10	15	45	19.5	28.7	0	0.01	0.00	0.00	0.01
	73	563	562.17	474.70	48	129	14.7	19.2	0	0.02	0.00	0.00	0.02
	74	435	434.33	410.13	12	46	20.3	39.7	0	0.01	0.00	0.00	0.01
	75	809	809.00	601.10	4	14	21.1	41.2	0	0.01	0.00	0.00	0.01
	76	564	564.00	458.80	96	286	14.8	19.8	0	0.01	0.00	0.01	0.02
	77	453	452.50	390.49	69	260	13.1	19.3	0	0.01	0.00	0.01	0.02
	78	664	664.00	530.77	12	35	20.0	29.9	0	0.01	0.00	0.00	0.01
	79	520	520.00	473.55	22	77	14.5	24.5	0	0.01	0.00	0.00	0.01
	80	382	381.67	369.56	33	98	14.3	20.5	0	0.02	0.00	0.00	0.02
	81	598	597.50	479.42	24	89	13.0	23.0	0	0.02	0.00	0.00	0.02
	82	420	419.83	393.57	45	154	12.4	18.9	0	0.01	0.00	0.01	0.02
	83	241	240.58	239.73	61	280	12.3	19.1	0	0.01	0.00	0.03	0.04
	84	568	567.50	471.19	29	109	12.8	22.9	0	0.02	0.00	0.00	0.02
	85	600	600.00	434.56	30	103	14.2	22.9	0	0.02	0.00	0.00	0.02
	86	454	453.50	426.97	27	88	16.2	25.7	0	0.01	0.00	0.00	0.01
	87	691	691.00	523.76	99	218	23.7	25.7	0	0.02	0.00	0.00	0.02
	88	566	566.00	443.87	165	545	21.7	29.2	0	0.01	0.01	0.01	0.03
	89	562	561.50	449.61	41	117	14.0	19.4	0	0.01	0.00	0.01	0.02
	90	450	449.25	413.98	11	41	14.3	25.5	0	0.01	0.00	0.00	0.01
	91	532	532.00	439.85	331	1,151	40.9	49.6	0	0.02	0.02	0.03	0.07

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	92	336	335.95	331.74	30	126	10.0	18.9	0	0.01	0.00	0.01	0.02
	93	289	289.00	278.21	48	170	12.3	20.2	0	0.01	0.00	0.01	0.02
	94	800	799.50	596.78	6	20	22.2	40.0	0	0.01	0.00	0.00	0.01
	95	757	757.00	553.95	59	176	16.4	22.5	0	0.01	0.00	0.01	0.02
	96	537	537.00	451.12	64	160	22.2	26.7	0	0.02	0.00	0.00	0.02
	97	678	677.50	511.61	18	61	15.8	26.2	0	0.01	0.00	0.00	0.01
	98	514	513.83	420.62	42	141	10.9	16.7	0	0.01	0.00	0.01	0.02
	99	368	368.00	317.81	33	121	9.2	14.3	0	0.02	0.00	0.00	0.02
	100	570	570.00	485.37	16	52	13.7	22.0	0	0.01	0.00	0.00	0.01
	1	91	90.50	86.30	91	419	15.1	27.3	0	0.01	0.01	0.01	0.03
	2	80	79.50	73.22	400	2,044	49.3	53.6	0	0.04	0.04	0.06	0.14
	3	161	161.00	115.64	53	237	15.6	30.2	0	0.01	0.00	0.01	0.02
	4	74	73.62	71.95	171	843	25.4	28.2	0	0.02	0.01	0.03	0.06
	5	96	95.33	89.64	140	673	21.6	35.8	0	0.02	0.01	0.02	0.05
	6	109	108.50	96.47	98	513	17.5	28.9	0	0.02	0.01	0.01	0.04
	7	84	83.20	78.92	314	1,561	39.1	43.1	0	0.04	0.03	0.05	0.12
	8	123	123.00	102.05	19	87	10.3	22.3	0	0.01	0.00	0.00	0.01
	9	98	97.50	81.34	424	2,011	53.3	51.8	0	0.04	0.04	0.05	0.13
	10	90	90.00	86.19	89	379	15.8	25.9	0	0.02	0.00	0.01	0.03
	11	92	92.00	82.98	346	1,650	45.8	47.0	0	0.03	0.03	0.04	0.10
	12	89	88.17	83.24	70	366	14.0	23.8	0	0.02	0.00	0.01	0.03
	13	102	101.50	88.71	436	2,045	57.7	74.2	0	0.03	0.04	0.06	0.13
	14	75	74.82	73.47	89	466	15.2	22.5	0	0.02	0.01	0.02	0.05
	15	97	97.00	86.34	95	394	16.9	26.3	0	0.02	0.00	0.01	0.03
	16	84	83.17	79.94	68	305	12.5	19.1	0	0.02	0.00	0.01	0.03
	17	100	99.25	94.08	49	292	11.0	22.3	0	0.02	0.00	0.01	0.03
	18	92	91.50	87.23	87	423	15.3	24.8	0	0.02	0.01	0.01	0.04
	19	115	115.00	96.23	25	140	8.3	19.9	0	0.02	0.00	0.00	0.02
	20	124	124.00	96.88	120	471	18.4	27.2	0	0.01	0.01	0.01	0.03
	21	84	84.00	74.29	284	1,456	36.1	38.3	0	0.04	0.03	0.04	0.11
	22	95	94.50	88.19	32	154	9.8	18.8	0	0.01	0.00	0.01	0.02
	23	73	72.60	70.46	444	2,390	53.7	66.6	0	0.04	0.08	0.08	0.20
	24	98	97.60	89.77	172	847	24.8	33.9	0	0.02	0.01	0.03	0.06
	25	67	66.80	66.15	231	1,116	30.4	36.7	0	0.03	0.02	0.05	0.10
	26	71	70.32	69.34	180	930	26.5	33.5	0	0.03	0.02	0.03	0.08
	27	68	67.89	66.89	502	2,571	60.1	57.8	0	0.04	0.09	0.09	0.22
	28	89	89.00	75.50	214	1,016	29.6	36.7	0	0.02	0.01	0.04	0.07
	29	85	84.90	81.86	41	260	9.2	19.8	0	0.02	0.00	0.01	0.03
	30	60	59.20	59.10	231	1,279	33.1	37.2	0	0.03	0.03	0.04	0.10
	31	78	78.00	72.06	372	1,870	43.1	53.4	0	0.03	0.04	0.05	0.12
	32	137	137.00	104.33	355	1,278	45.2	53.8	0	0.03	0.02	0.02	0.07
	33	69	68.27	67.54	379	1,925	50.8	54.0	0	0.04	0.06	0.08	0.18
	34	77	76.40	74.94	87	570	16.1	27.1	0	0.05	0.01	0.02	0.08
	35	89	88.88	85.25	58	302	10.8	18.0	0	0.02	0.00	0.01	0.03
	36	101	101.00	86.14	304	1,324	43.1	43.6	0	0.03	0.02	0.04	0.09
	37	103	103.00	91.67	135	588	19.8	28.7	0	0.02	0.01	0.01	0.04
	38	160	160.00	119.03	15	63	9.5	19.4	0	0.01	0.00	0.00	0.01
	39	74	74.00	70.98	166	874	25.1	30.8	0	0.02	0.01	0.02	0.05
	40	98	98.00	86.71	44	234	10.3	21.7	0	0.01	0.00	0.01	0.02
	41	72	71.50	69.45	232	1,253	32.6	37.7	0	0.03	0.02	0.04	0.09
	42	99	98.67	88.53	70	375	12.9	21.1	0	0.02	0.00	0.01	0.03
	43	128	128.00	94.76	88	347	15.8	24.2	0	0.02	0.00	0.01	0.03
	44	99	98.50	92.96	90	365	19.1	31.9	0	0.02	0.00	0.01	0.03
	45	146	145.50	102.32	231	913	28.9	41.3	0	0.02	0.01	0.03	0.06
	46	74	73.41	71.47	703	3,481	75.8	83.9	0	0.04	0.11	0.18	0.33
	47	91	91.00	83.91	218	1,032	29.5	36.4	0	0.03	0.01	0.03	0.07
	48	86	85.50	78.34	234	1,092	31.0	38.5	0	0.03	0.02	0.04	0.09
	49	111	111.00	90.58	147	598	20.4	29.7	0	0.02	0.01	0.01	0.04
	50	80	79.70	78.26	209	1,111	30.6	36.2	0	0.03	0.02	0.05	0.10
	51	116	116.00	97.92	120	521	22.3	33.3	0	0.02	0.01	0.01	0.04
	52	67	66.84	66.62	140	683	21.4	29.6	0	0.02	0.01	0.02	0.05
	53	68	67.20	66.68	103	521	16.9	23.5	0	0.02	0.01	0.01	0.04
	54	92	91.50	82.71	142	581	20.7	26.4	0	0.02	0.01	0.01	0.04
	55	70	69.24	68.22	45	253	10.2	18.8	0	0.02	0.00	0.01	0.03

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	56	59	58.18	58.02	575	2,916	66.9	69.2	0	0.04	0.13	0.09	0.26
	57	84	84.00	77.23	431	1,837	53.9	57.1	0	0.03	0.03	0.04	0.10
	58	96	95.46	89.03	36	184	9.7	20.3	0	0.01	0.00	0.01	0.02
	59	84	84.00	77.81	297	1,097	36.4	45.5	0	0.02	0.02	0.02	0.06
	60	79	78.58	75.65	401	1,994	50.1	66.1	0	0.03	0.05	0.08	0.16
	61	66	65.89	65.30	233	1,202	33.6	43.3	0	0.03	0.02	0.06	0.11
	62	91	91.00	84.08	114	552	18.3	27.1	0	0.02	0.01	0.01	0.04
	63	103	102.33	94.72	92	447	19.1	35.2	0	0.01	0.01	0.01	0.03
	64	90	89.17	84.43	148	748	28.4	45.9	0	0.02	0.01	0.02	0.05
	65	142	141.50	108.53	57	257	12.6	22.1	0	0.01	0.00	0.01	0.02
	66	122	121.50	101.32	66	304	17.2	32.3	0	0.02	0.00	0.01	0.03
	67	89	88.58	85.31	47	270	11.2	21.1	0	0.02	0.00	0.01	0.03
	68	106	105.67	92.47	130	556	21.2	31.1	0	0.02	0.01	0.01	0.04
	69	103	102.50	86.09	245	939	30.7	40.8	0	0.03	0.01	0.02	0.06
	70	118	117.50	89.41	72	311	13.4	17.8	0	0.02	0.00	0.01	0.03
	71	91	91.00	85.35	175	811	25.3	31.3	0	0.02	0.01	0.02	0.05
	72	108	108.00	92.07	249	1,029	31.9	33.9	0	0.03	0.01	0.02	0.06
	73	101	100.83	91.25	98	453	18.7	27.7	0	0.02	0.01	0.01	0.04
	74	91	91.00	81.93	146	637	21.6	29.5	0	0.02	0.01	0.03	0.06
	75	119	119.00	102.53	38	186	11.1	23.1	0	0.01	0.00	0.01	0.02
	76	75	74.50	68.12	627	2,645	70.6	73.0	0	0.04	0.06	0.08	0.18
	77	96	96.00	89.09	86	404	17.4	32.6	0	0.01	0.00	0.02	0.03
	78	119	119.00	100.54	59	291	12.1	23.2	0	0.01	0.00	0.01	0.02
	79	98	97.50	86.02	620	2,689	72.2	79.1	0	0.03	0.06	0.11	0.20
	80	104	104.00	92.21	42	207	9.9	19.8	0	0.01	0.00	0.01	0.02
	81	71	70.69	70.09	657	2,991	78.2	85.4	0	0.04	0.10	0.12	0.26
	82	70	70.00	65.40	147	740	22.8	31.7	0	0.03	0.01	0.01	0.05
	83	84	83.50	79.26	189	900	26.9	33.0	0	0.02	0.01	0.03	0.06
	84	80	80.00	78.36	322	1,480	39.5	49.8	0	0.03	0.03	0.04	0.10
	85	57	56.50	56.38	684	3,060	75.6	80.8	0	0.04	0.11	0.12	0.27
	86	85	84.50	78.75	79	437	15.0	26.9	0	0.01	0.01	0.01	0.03
	87	103	103.00	82.18	292	1,266	37.9	48.3	0	0.03	0.02	0.03	0.08
	88	84	83.50	81.03	57	258	11.5	21.0	0	0.01	0.00	0.01	0.02
	89	92	91.25	86.97	37	225	9.2	19.9	0	0.01	0.00	0.01	0.02
	90	122	122.00	92.76	303	1,178	38.4	42.7	0	0.03	0.02	0.03	0.08
	91	71	70.44	69.52	98	483	17.0	26.7	0	0.02	0.01	0.01	0.04
	92	55	54.21	54.03	546	3,027	60.9	63.4	0	0.04	0.12	0.14	0.30
	93	106	106.00	84.26	447	2,200	54.2	57.2	0	0.04	0.04	0.05	0.13
	94	100	99.50	90.31	120	570	18.0	29.1	0	0.02	0.01	0.01	0.04
	95	112	112.00	94.56	127	553	21.0	32.5	0	0.02	0.01	0.01	0.04
	96	110	109.50	94.37	90	355	16.3	22.3	0	0.02	0.00	0.01	0.03
	97	61	60.76	60.45	97	464	16.5	23.7	0	0.02	0.01	0.01	0.04
	98	124	124.00	100.88	151	557	20.9	28.7	0	0.02	0.01	0.01	0.04
	99	70	69.84	69.21	222	1,150	31.2	34.8	0	0.02	0.02	0.05	0.09
	100	123	123.00	106.00	14	63	14.4	28.5	0	0.01	0.00	0.00	0.01
1	1	917	917.00	871.18	103	441	17.1	28.6	0	0.01	0.01	0.01	0.03
	2	800	799.50	735.21	407	2,097	50.1	54.0	0	0.04	0.05	0.06	0.15
	3	1,604	1,604.00	1,151.54	211	705	27.8	35.4	0	0.02	0.01	0.01	0.04
	4	732	731.94	716.23	171	843	25.4	28.2	0	0.02	0.01	0.04	0.07
	5	951	950.17	892.43	188	836	27.2	39.1	0	0.02	0.01	0.02	0.05
	6	1,077	1,077.00	957.23	101	459	16.3	23.2	0	0.02	0.01	0.00	0.03
	7	833	832.10	787.68	329	1,640	41.0	42.5	0	0.03	0.04	0.06	0.13
	8	1,237	1,236.50	1,024.52	22	93	11.8	23.6	0	0.01	0.00	0.00	0.01
	9	976	975.50	812.14	438	2,085	55.0	52.5	0	0.04	0.04	0.07	0.15
	10	905	905.00	865.56	102	416	17.6	27.3	0	0.02	0.01	0.00	0.03
	11	922	921.50	834.42	348	1,659	46.0	47.2	0	0.03	0.03	0.05	0.11
	12	877	876.50	830.06	70	368	14.0	23.9	0	0.02	0.00	0.01	0.03
	13	1,019	1,018.50	889.54	651	2,659	70.6	81.0	0	0.04	0.06	0.08	0.18
	14	744	743.61	730.43	89	466	15.2	22.5	0	0.02	0.01	0.01	0.04
	15	983	983.00	871.32	95	393	16.9	26.2	0	0.02	0.00	0.01	0.03
	16	823	822.67	791.92	61	287	11.2	17.7	0	0.02	0.00	0.01	0.03
	17	987	987.00	938.15	51	283	11.1	20.1	0	0.01	0.00	0.01	0.02
	18	912	911.50	870.27	87	422	15.3	24.7	0	0.01	0.00	0.02	0.03
	19	1,144	1,144.00	960.46	25	141	8.3	20.0	0	0.02	0.00	0.00	0.02

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	20	1,232	1,232.00	961.91	129	497	19.8	28.2	0	0.02	0.01	0.00	0.03
	21	839	839.00	740.69	338	1,630	41.6	40.3	0	0.04	0.03	0.04	0.11
	22	954	953.25	886.58	29	143	8.9	17.4	0	0.01	0.00	0.01	0.02
	23	727	726.76	704.62	520	2,618	60.4	69.6	0	0.04	0.09	0.08	0.21
	24	985	984.70	905.14	205	960	29.0	35.9	0	0.02	0.01	0.03	0.06
	25	668	667.55	661.00	231	1,120	30.4	36.8	0	0.02	0.03	0.03	0.08
	26	703	702.67	692.98	183	950	27.0	33.8	0	0.02	0.02	0.03	0.07
	27	672	671.40	662.26	506	2,602	60.6	58.1	0	0.04	0.10	0.12	0.26
	28	895	895.00	757.00	272	1,201	36.7	39.8	0	0.03	0.02	0.02	0.07
	29	853	852.20	821.55	43	270	9.4	19.7	0	0.02	0.00	0.01	0.03
	30	591	590.05	589.05	231	1,280	33.1	37.2	0	0.03	0.03	0.04	0.10
	31	772	772.00	714.83	453	2,073	50.7	55.6	0	0.03	0.04	0.05	0.12
	32	1,362	1,361.50	1,041.69	422	1,477	47.3	54.8	0	0.03	0.02	0.03	0.08
	33	679	678.52	671.30	385	1,899	48.2	51.0	0	0.03	0.06	0.06	0.15
	34	758	757.70	744.07	93	569	16.3	25.7	0	0.02	0.01	0.01	0.04
	35	888	887.62	851.93	53	286	9.9	16.9	0	0.02	0.00	0.01	0.03
	36	1,016	1,015.50	864.64	304	1,306	42.1	42.1	0	0.03	0.02	0.03	0.08
	37	1,031	1,030.83	918.93	140	599	20.4	29.0	0	0.02	0.01	0.01	0.04
	38	1,605	1,605.00	1,189.34	15	63	9.5	19.2	0	0.01	0.00	0.00	0.01
	39	740	740.00	707.14	169	897	25.5	31.2	0	0.03	0.01	0.02	0.06
	40	983	983.00	871.81	45	241	10.4	21.8	0	0.01	0.00	0.01	0.02
	41	716	715.90	695.14	233	1,264	32.6	37.7	0	0.03	0.02	0.04	0.09
	42	990	989.50	887.41	74	383	13.6	21.0	0	0.02	0.00	0.01	0.03
	43	1,257	1,257.00	931.60	93	376	16.4	24.9	0	0.02	0.00	0.01	0.03
	44	997	996.33	938.05	126	454	21.7	32.0	0	0.02	0.01	0.00	0.03
	45	1,457	1,456.50	1,023.91	275	1,076	34.1	43.2	0	0.03	0.02	0.02	0.07
	46	735	734.23	714.44	754	3,713	81.2	87.5	0	0.05	0.11	0.18	0.34
	47	916	916.00	844.41	248	1,149	33.3	38.1	0	0.03	0.02	0.04	0.09
	48	859	859.00	784.98	263	1,215	34.5	40.6	0	0.03	0.02	0.02	0.07
	49	1,112	1,112.00	909.94	162	632	22.0	30.6	0	0.02	0.01	0.01	0.04
	50	794	793.08	778.85	217	1,138	31.7	36.6	0	0.02	0.02	0.05	0.09
	51	1,145	1,145.00	964.20	134	541	22.1	30.2	0	0.02	0.01	0.01	0.04
	52	655	654.66	652.82	147	700	22.3	29.5	0	0.02	0.01	0.02	0.05
	53	677	676.93	671.34	103	521	16.9	23.5	0	0.02	0.01	0.02	0.05
	54	906	906.00	820.10	150	614	21.8	27.4	0	0.02	0.01	0.01	0.04
	55	691	690.62	680.77	45	253	10.2	18.8	0	0.02	0.00	0.01	0.03
	56	585	584.97	583.23	579	2,967	66.9	69.5	0	0.05	0.13	0.14	0.32
	57	839	839.00	767.86	431	1,837	53.9	57.1	0	0.03	0.03	0.05	0.11
	58	953	952.79	888.88	36	184	9.7	20.3	0	0.01	0.00	0.01	0.02
	59	837	836.25	774.47	314	1,140	37.9	46.4	0	0.02	0.02	0.03	0.07
	60	791	790.58	761.01	434	2,075	50.5	65.4	0	0.03	0.05	0.06	0.14
	61	661	660.43	654.54	238	1,208	32.5	41.8	0	0.03	0.03	0.04	0.10
	62	888	888.00	829.21	114	551	18.3	27.1	0	0.02	0.01	0.01	0.04
	63	1,027	1,026.83	946.65	105	438	16.8	26.7	0	0.01	0.01	0.01	0.03
	64	894	893.50	843.92	186	780	27.4	37.1	0	0.02	0.01	0.02	0.05
	65	1,407	1,406.50	1,079.48	84	369	16.3	24.6	0	0.02	0.00	0.01	0.03
	66	1,209	1,208.50	1,011.25	97	378	16.8	25.8	0	0.02	0.00	0.01	0.03
	67	881	880.33	848.51	51	285	11.7	21.2	0	0.02	0.00	0.01	0.03
	68	1,066	1,065.67	931.58	136	559	20.6	29.5	0	0.02	0.01	0.01	0.04
	69	1,000	999.50	840.11	250	966	31.3	41.3	0	0.02	0.01	0.03	0.06
	70	1,178	1,177.50	891.76	71	310	13.2	17.7	0	0.02	0.00	0.01	0.03
	71	905	905.00	848.17	176	818	25.4	31.4	0	0.02	0.01	0.02	0.05
	72	1,075	1,075.00	921.45	292	1,182	37.4	37.3	0	0.03	0.02	0.02	0.07
	73	989	988.67	897.69	110	470	17.3	24.2	0	0.02	0.01	0.01	0.04
	74	912	911.50	820.02	147	645	21.8	29.8	0	0.02	0.01	0.01	0.04
	75	1,165	1,165.00	1,010.07	40	190	11.7	23.5	0	0.01	0.00	0.01	0.02
	76	749	748.50	682.61	627	2,647	70.6	73.0	0	0.04	0.06	0.13	0.23
	77	957	956.50	888.41	116	454	18.3	28.9	0	0.01	0.01	0.01	0.03
	78	1,177	1,176.50	997.08	64	291	12.5	21.9	0	0.01	0.00	0.01	0.02
	79	992	991.50	872.52	630	2,710	71.3	78.3	0	0.04	0.07	0.08	0.19
	80	1,035	1,034.50	916.35	42	207	9.9	19.8	0	0.01	0.00	0.01	0.02
	81	710	709.67	702.86	707	3,160	76.3	84.1	0	0.04	0.09	0.13	0.26
	82	699	699.00	649.63	147	741	22.8	31.8	0	0.03	0.01	0.01	0.05
	83	843	843.00	798.86	252	1,110	35.0	37.0	0	0.03	0.01	0.03	0.07

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	84	797	797.00	781.25	356	1,600	43.7	52.7	0	0.03	0.03	0.05	0.11
	85	568	567.08	566.02	684	3,061	75.6	80.8	0	0.04	0.11	0.15	0.30
	86	835	834.50	778.10	79	437	15.0	26.9	0	0.02	0.01	0.00	0.03
	87	1,029	1,029.00	820.75	329	1,400	41.1	50.5	0	0.03	0.02	0.03	0.08
	88	851	851.00	822.79	57	258	11.5	21.0	0	0.01	0.00	0.01	0.02
	89	912	911.62	868.55	37	223	9.2	19.6	0	0.01	0.00	0.01	0.02
	90	1,221	1,221.00	928.10	324	1,265	40.9	44.4	0	0.03	0.02	0.03	0.08
	91	694	693.75	686.06	106	507	18.2	26.9	0	0.02	0.01	0.01	0.04
	92	540	539.71	538.07	552	3,038	61.1	63.3	0	0.05	0.15	0.17	0.37
	93	1,072	1,072.00	849.60	481	2,324	58.4	59.3	0	0.04	0.05	0.05	0.14
	94	992	991.50	898.48	156	668	23.1	32.4	0	0.02	0.01	0.01	0.04
	95	1,120	1,120.00	945.89	145	600	21.4	29.9	0	0.02	0.01	0.01	0.04
	96	1,083	1,083.00	934.29	83	334	15.0	20.8	0	0.02	0.00	0.01	0.03
	97	605	604.03	600.97	97	464	16.5	23.7	0	0.02	0.01	0.01	0.04
	98	1,241	1,241.00	1,011.65	163	592	22.4	29.1	0	0.02	0.01	0.01	0.04
	99	706	705.19	698.62	222	1,150	31.2	34.8	0	0.03	0.03	0.03	0.09
	100	1,233	1,233.00	1,062.55	15	69	14.4	29.0	0	0.01	0.00	0.00	0.01
1	1	159	158.07	154.96	310	2,380	41.0	54.6	0	0.04	0.05	0.09	0.18
	2	218	218.00	183.90	739	4,441	81.9	84.5	0	0.06	0.13	0.19	0.38
	3	151	150.75	148.08	834	6,957	88.1	94.3	0	0.08	0.44	0.37	0.89
	4	158	157.87	156.89	794	6,658	85.7	92.9	0	0.08	0.44	0.45	0.97
	5	191	191.00	165.70	633	5,899	74.6	72.5	0	0.08	0.24	0.16	0.48
	6	164	163.75	160.78	588	4,982	70.9	70.7	0	0.06	0.25	0.49	0.80
	7	151	150.50	147.83	878	6,855	91.6	96.4	0	0.08	0.34	0.38	0.80
	8	191	191.00	167.69	247	1,742	34.1	44.7	0	0.03	0.02	0.05	0.10
	9	175	175.00	168.13	570	4,039	66.6	75.9	0	0.05	0.14	0.18	0.37
	10	163	162.19	160.86	210	1,570	30.3	34.4	0	0.03	0.03	0.07	0.13
	11	169	168.50	149.50	831	8,183	89.4	94.2	0	0.09	0.34	0.39	0.82
	12	163	162.75	159.88	785	6,814	84.6	94.0	0	0.08	0.39	0.44	0.91
	13	169	169.00	156.09	498	3,931	60.4	65.7	0	0.06	0.12	0.16	0.34
	14	124	123.61	123.52	678	7,425	78.4	76.8	0	0.09	0.69	0.52	1.30
	15	149	148.25	146.67	394	2,949	52.4	66.1	0	0.04	0.09	0.15	0.28
	16	209	209.00	185.88	727	5,516	81.7	83.1	0	0.07	0.24	0.23	0.54
	17	168	168.00	156.91	777	7,085	86.2	90.2	0	0.08	0.28	0.30	0.66
	18	165	164.60	162.45	170	1,337	25.7	35.9	0	0.03	0.02	0.06	0.11
	19	218	217.44	193.94	428	2,560	51.0	55.5	0	0.04	0.07	0.11	0.22
	20	181	181.00	167.36	464	3,192	57.7	61.8	0	0.05	0.07	0.09	0.21
	21	141	140.21	139.94	568	5,025	69.7	70.6	0	0.06	0.34	0.34	0.74
	22	212	211.50	191.44	331	1,870	43.4	48.5	0	0.03	0.03	0.08	0.14
	23	153	153.00	146.61	832	7,369	86.7	84.1	0	0.08	0.35	0.24	0.67
	24	175	175.00	163.97	627	5,063	72.9	79.3	0	0.06	0.19	0.17	0.42
	25	205	204.50	178.92	458	2,562	55.7	57.2	0	0.05	0.05	0.10	0.20
	26	223	223.00	190.32	330	1,658	42.9	47.6	0	0.03	0.03	0.04	0.10
	27	160	159.73	158.12	279	1,914	38.6	41.7	0	0.04	0.05	0.09	0.18
	28	141	140.06	139.26	227	1,949	33.2	39.2	0	0.05	0.04	0.07	0.16
	29	203	203.00	176.79	624	3,750	68.3	74.0	0	0.06	0.12	0.09	0.27
	30	155	154.55	152.42	641	4,225	72.5	79.8	0	0.05	0.21	0.14	0.40
	31	143	142.78	141.93	588	4,404	70.2	75.4	0	0.06	0.22	0.23	0.51
	32	182	182.00	169.89	789	5,459	85.0	88.9	0	0.08	0.26	0.34	0.68
	33	203	202.50	176.68	785	6,395	86.7	85.3	0	0.07	0.25	0.31	0.63
	34	185	184.50	173.93	538	4,014	69.8	75.5	0	0.05	0.12	0.19	0.36
	35	212	211.50	173.98	731	5,635	83.6	87.5	0	0.07	0.17	0.24	0.48
	36	172	172.00	166.00	480	3,317	59.9	65.2	0	0.05	0.09	0.18	0.32
	37	188	188.00	168.99	683	4,927	78.1	79.6	0	0.07	0.18	0.15	0.40
	38	188	187.50	177.17	110	797	18.7	31.4	0	0.02	0.01	0.03	0.06
	39	205	204.50	180.11	529	3,442	61.8	69.5	0	0.05	0.09	0.12	0.26
	40	172	171.33	166.94	687	5,913	76.8	89.2	0	0.07	0.25	0.19	0.51
	41	172	171.50	150.88	747	5,544	81.4	91.6	0	0.06	0.23	0.19	0.48
	42	149	148.23	147.46	463	3,384	58.9	61.3	0	0.05	0.14	0.19	0.38
	43	159	158.94	157.21	756	6,160	82.5	90.2	0	0.07	0.33	0.38	0.78
	44	197	196.50	170.07	627	4,464	72.6	82.1	0	0.06	0.15	0.20	0.41
	45	192	191.50	178.66	209	1,536	30.4	38.1	0	0.04	0.02	0.05	0.11
	46	130	129.40	128.72	659	5,992	75.4	84.6	0	0.07	0.38	0.57	1.02
	47	196	196.00	167.95	648	5,464	75.8	77.2	0	0.07	0.21	0.19	0.47

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	48	237	237.00	200.54	612	4,101	71.7	75.1	0	0.06	0.14	0.12	0.32
	49	158	157.41	156.21	530	3,837	65.7	63.8	0	0.05	0.16	0.20	0.41
	50	197	197.00	178.56	355	2,548	45.7	45.8	0	0.05	0.05	0.14	0.24
	51	160	159.50	152.08	841	6,946	89.7	93.6	0	0.08	0.30	0.26	0.64
	52	232	232.00	203.76	392	2,603	54.1	69.0	0	0.05	0.05	0.08	0.18
	53	152	151.03	150.51	454	4,001	56.3	62.9	0	0.05	0.16	0.16	0.37
	54	183	183.00	175.10	389	2,473	51.5	49.2	0	0.04	0.05	0.10	0.19
	55	246	246.00	200.99	811	6,258	87.8	93.2	0	0.08	0.27	0.28	0.63
	56	207	207.00	176.87	650	4,729	75.5	76.0	0	0.06	0.20	0.23	0.49
	57	214	214.00	181.40	443	2,835	55.9	58.7	0	0.04	0.07	0.09	0.20
	58	182	182.00	165.81	288	1,833	38.3	48.2	0	0.04	0.03	0.03	0.10
	59	203	203.00	175.86	638	4,723	73.0	77.1	0	0.06	0.20	0.15	0.41
	60	199	199.00	178.42	506	3,326	61.9	68.0	0	0.05	0.08	0.16	0.29
	61	150	149.50	134.77	722	6,237	80.5	83.3	0	0.08	0.35	0.34	0.77
	62	182	182.00	172.96	411	2,824	53.4	61.2	0	0.04	0.06	0.12	0.22
	63	180	179.50	170.37	803	6,545	88.3	88.4	0	0.07	0.32	0.26	0.65
	64	179	178.50	168.54	609	3,925	69.8	78.4	0	0.05	0.14	0.14	0.33
	65	147	146.96	146.23	351	2,895	47.1	52.9	0	0.05	0.09	0.14	0.28
	66	157	156.44	154.71	608	4,905	72.0	72.9	0	0.07	0.24	0.21	0.52
	67	155	154.94	153.07	765	7,439	84.6	91.7	0	0.08	0.48	0.38	0.94
	68	164	163.50	154.91	683	4,797	78.1	88.0	0	0.06	0.24	0.22	0.52
	69	194	194.00	176.78	409	2,423	52.4	53.5	0	0.04	0.05	0.07	0.16
	70	159	159.00	149.49	722	4,935	77.6	85.2	0	0.06	0.21	0.20	0.47
	71	191	190.50	173.04	525	3,522	64.0	70.2	0	0.05	0.09	0.22	0.36
	72	179	179.00	172.64	245	1,915	34.8	43.6	0	0.04	0.03	0.07	0.14
	73	147	146.08	145.08	744	7,181	82.5	92.9	0	0.08	0.53	0.38	0.99
	74	161	160.25	155.27	856	6,499	89.7	95.7	0	0.07	0.32	0.35	0.74
	75	133	132.45	132.14	573	5,404	69.3	73.3	0	0.07	0.38	0.31	0.76
	76	166	165.84	164.28	265	1,727	36.4	44.3	0	0.04	0.04	0.08	0.16
	77	208	207.50	187.63	456	2,791	56.8	55.4	0	0.05	0.06	0.09	0.20
	78	146	145.94	145.07	661	5,296	74.1	78.3	0	0.06	0.29	0.55	0.90
	79	192	191.25	171.62	681	5,289	75.4	86.3	0	0.06	0.22	0.24	0.52
	80	163	162.50	152.96	690	7,380	78.5	82.8	0	0.08	0.33	0.31	0.72
	81	194	194.00	173.66	391	2,336	49.7	57.2	0	0.04	0.05	0.08	0.17
	82	196	195.33	175.01	554	4,120	65.3	61.2	0	0.06	0.13	0.17	0.36
	83	190	190.00	170.68	859	5,244	90.3	95.2	0	0.06	0.23	0.16	0.45
	84	151	150.66	149.91	771	6,624	85.5	90.5	0	0.07	0.42	0.38	0.87
	85	188	188.00	162.60	873	7,182	92.6	95.8	0	0.08	0.28	0.35	0.71
	86	163	162.50	156.61	637	4,653	73.8	71.3	0	0.06	0.20	0.20	0.46
	87	204	204.00	181.14	379	2,299	47.7	54.2	0	0.04	0.04	0.07	0.15
	88	167	166.50	158.24	735	5,528	82.3	84.4	0	0.07	0.24	0.37	0.68
	89	206	205.50	189.24	479	3,761	65.4	77.2	0	0.05	0.11	0.16	0.32
	90	182	181.86	178.69	444	3,085	58.4	58.9	0	0.05	0.11	0.22	0.38
	91	119	118.24	118.17	643	7,070	73.6	81.2	0	0.08	0.49	0.28	0.85
	92	186	186.00	171.99	496	3,389	62.9	70.3	0	0.04	0.09	0.14	0.27
	93	191	191.00	169.46	579	4,639	66.3	70.7	0	0.07	0.19	0.15	0.41
	94	187	186.50	173.61	780	7,167	84.3	80.3	0	0.09	0.36	0.33	0.78
	95	142	141.24	140.77	836	7,201	88.9	89.2	0	0.08	0.55	0.46	1.09
	96	150	149.74	149.01	649	6,000	74.6	84.5	0	0.07	0.43	0.33	0.83
	97	147	147.00	144.41	729	6,660	80.8	87.4	0	0.07	0.40	0.35	0.82
	98	157	156.65	154.89	745	7,306	82.7	91.1	0	0.08	0.47	0.43	0.98
	99	204	203.50	182.79	460	2,936	57.1	66.8	0	0.05	0.08	0.13	0.26
	100	194	194.00	169.99	627	5,394	74.5	73.3	0	0.07	0.17	0.18	0.42
1	1	1,594	1,593.61	1,561.25	313	2,421	41.4	54.9	0	0.04	0.05	0.11	0.20
	2	2,192	2,191.50	1,847.34	739	4,443	81.9	84.5	0	0.05	0.13	0.15	0.33
	3	1,513	1,512.83	1,484.76	835	6,961	88.1	94.3	0	0.08	0.46	0.41	0.95
	4	1,573	1,572.37	1,562.53	800	6,693	86.3	93.0	0	0.08	0.45	0.28	0.81
	5	1,908	1,908.00	1,657.11	638	5,969	75.1	72.8	0	0.08	0.22	0.17	0.47
	6	1,638	1,637.19	1,607.38	589	5,084	71.0	71.2	0	0.06	0.23	0.31	0.60
	7	1,499	1,499.00	1,474.61	878	6,870	91.6	96.4	0	0.08	0.32	0.68	1.08
	8	1,927	1,927.00	1,688.34	251	1,772	34.6	44.8	0	0.04	0.03	0.04	0.11
	9	1,760	1,760.00	1,683.79	570	4,046	66.6	76.0	0	0.06	0.14	0.09	0.29
	10	1,622	1,621.91	1,608.31	219	1,611	31.5	34.6	0	0.04	0.04	0.06	0.14
	11	1,687	1,687.00	1,498.16	831	8,236	89.4	94.2	0	0.09	0.33	0.38	0.80

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	12	1,626	1,625.89	1,596.55	824	7,064	87.5	94.8	0	0.08	0.40	0.38	0.86
	13	1,667	1,667.00	1,546.58	513	4,018	61.1	65.9	0	0.05	0.12	0.11	0.28
	14	1,236	1,235.42	1,234.43	684	7,492	78.9	76.9	0	0.09	0.52	0.30	0.91
	15	1,487	1,486.38	1,470.03	408	2,974	51.5	65.3	0	0.04	0.09	0.15	0.28
	16	2,093	2,092.50	1,855.75	737	5,607	79.9	82.2	0	0.07	0.21	0.24	0.52
	17	1,660	1,660.00	1,559.29	779	7,091	85.4	89.9	0	0.08	0.30	0.25	0.63
	18	1,649	1,648.02	1,626.55	170	1,332	25.3	35.5	0	0.03	0.02	0.04	0.09
	19	2,168	2,167.81	1,932.21	444	2,644	52.5	55.5	0	0.05	0.07	0.10	0.22
	20	1,801	1,800.50	1,663.81	475	3,274	59.1	62.5	0	0.05	0.08	0.10	0.23
	21	1,408	1,407.47	1,404.68	578	5,054	70.0	70.3	0	0.06	0.30	0.44	0.80
	22	2,101	2,100.50	1,903.58	370	1,997	47.0	49.6	0	0.04	0.03	0.06	0.13
	23	1,525	1,525.00	1,466.24	835	7,380	87.0	83.9	0	0.08	0.36	0.54	0.98
	24	1,739	1,739.00	1,631.37	629	5,096	73.1	79.5	0	0.06	0.17	0.16	0.39
	25	2,039	2,038.94	1,782.99	467	2,616	56.8	57.7	0	0.04	0.06	0.09	0.19
	26	2,208	2,208.00	1,891.45	349	1,725	45.4	48.7	0	0.04	0.03	0.04	0.11
	27	1,588	1,587.72	1,572.01	279	1,915	38.6	41.7	0	0.04	0.05	0.15	0.24
	28	1,395	1,394.70	1,387.13	229	1,976	33.5	39.4	0	0.04	0.05	0.08	0.17
	29	2,048	2,048.00	1,770.82	659	3,861	72.1	75.2	0	0.05	0.12	0.09	0.26
	30	1,565	1,564.02	1,541.56	644	4,233	72.9	79.9	0	0.05	0.20	0.17	0.42
	31	1,429	1,428.85	1,420.29	588	4,413	70.2	75.5	0	0.06	0.27	0.23	0.56
	32	1,816	1,815.50	1,691.83	835	5,622	87.0	88.9	0	0.08	0.24	0.41	0.73
	33	2,000	2,000.00	1,746.39	786	6,400	86.9	85.0	0	0.08	0.25	0.15	0.48
	34	1,840	1,839.50	1,723.95	552	4,040	65.5	73.1	0	0.06	0.11	0.13	0.30
	35	2,119	2,118.50	1,740.79	737	5,679	81.6	86.8	0	0.07	0.19	0.17	0.43
	36	1,732	1,731.50	1,669.00	484	3,337	60.2	65.3	0	0.05	0.09	0.13	0.27
	37	1,898	1,898.00	1,705.68	683	5,036	78.1	79.9	0	0.07	0.20	0.26	0.53
	38	1,870	1,869.83	1,769.69	110	799	18.7	31.4	0	0.02	0.01	0.03	0.06
	39	2,062	2,062.00	1,815.99	542	3,473	63.3	69.1	0	0.05	0.09	0.10	0.24
	40	1,701	1,700.50	1,661.29	687	5,959	76.8	89.2	0	0.07	0.28	0.29	0.64
	41	1,714	1,714.00	1,504.06	761	5,622	81.7	91.6	0	0.07	0.23	0.15	0.45
	42	1,480	1,479.73	1,472.17	483	3,473	61.2	62.1	0	0.05	0.15	0.17	0.37
	43	1,598	1,597.72	1,578.52	765	6,224	83.5	90.6	0	0.07	0.31	0.48	0.86
	44	1,960	1,960.00	1,698.97	628	4,515	72.7	82.3	0	0.06	0.13	0.11	0.30
	45	1,908	1,908.00	1,780.02	210	1,545	30.5	38.2	0	0.04	0.02	0.06	0.12
	46	1,279	1,278.62	1,272.45	665	6,004	75.3	84.5	0	0.07	0.39	0.61	1.07
	47	1,966	1,966.00	1,681.99	657	5,600	76.7	77.8	0	0.08	0.20	0.17	0.45
	48	2,378	2,377.50	2,008.78	618	4,195	72.5	75.6	0	0.07	0.14	0.14	0.35
	49	1,559	1,558.71	1,548.43	535	3,931	66.3	64.4	0	0.05	0.19	0.16	0.40
	50	1,950	1,950.00	1,769.44	365	2,602	47.0	46.2	0	0.05	0.05	0.06	0.16
	51	1,584	1,584.00	1,514.23	841	7,002	89.7	93.7	0	0.08	0.28	0.26	0.62
	52	2,327	2,326.33	2,040.36	471	2,945	55.8	69.0	0	0.05	0.06	0.09	0.20
	53	1,504	1,503.95	1,499.00	459	4,008	56.6	62.8	0	0.06	0.16	0.18	0.40
	54	1,815	1,815.00	1,742.33	391	2,549	51.8	49.5	0	0.05	0.06	0.06	0.17
	55	2,442	2,442.00	1,992.78	831	6,330	88.3	93.2	0	0.09	0.26	0.17	0.52
	56	2,054	2,054.00	1,758.87	655	4,889	76.1	76.6	0	0.07	0.18	0.13	0.38
	57	2,134	2,134.00	1,809.29	472	2,962	58.9	59.8	0	0.05	0.07	0.08	0.20
	58	1,833	1,833.00	1,670.62	289	1,838	38.4	48.1	0	0.04	0.03	0.03	0.10
	59	2,053	2,053.00	1,774.92	639	4,740	73.1	77.2	0	0.06	0.18	0.13	0.37
	60	1,998	1,997.50	1,788.87	506	3,327	61.7	67.8	0	0.04	0.09	0.10	0.23
	61	1,509	1,509.00	1,356.71	734	6,460	81.6	83.9	0	0.09	0.29	0.19	0.57
	62	1,810	1,809.50	1,722.93	414	2,830	53.1	60.9	0	0.04	0.06	0.09	0.19
	63	1,796	1,796.00	1,704.07	803	6,670	88.3	88.6	0	0.08	0.24	0.23	0.55
	64	1,776	1,775.50	1,679.27	609	3,936	69.8	78.5	0	0.05	0.12	0.17	0.34
	65	1,467	1,466.83	1,459.61	351	2,897	47.1	52.9	0	0.04	0.09	0.15	0.28
	66	1,556	1,555.21	1,538.13	608	4,920	72.0	72.9	0	0.06	0.26	0.21	0.53
	67	1,564	1,563.62	1,544.27	765	7,487	84.6	91.8	0	0.08	0.42	0.49	0.99
	68	1,654	1,654.00	1,560.57	690	4,850	76.1	87.1	0	0.05	0.18	0.20	0.43
	69	1,916	1,915.50	1,749.85	410	2,428	52.3	53.5	0	0.05	0.05	0.07	0.17
	70	1,594	1,593.75	1,496.55	763	5,117	82.0	87.2	0	0.06	0.19	0.26	0.51
	71	1,910	1,909.50	1,735.64	525	3,524	64.0	70.3	0	0.05	0.09	0.17	0.31
	72	1,795	1,794.50	1,730.09	255	1,934	35.9	43.5	0	0.04	0.03	0.06	0.13
	73	1,468	1,467.26	1,456.83	764	7,305	82.7	92.8	0	0.08	0.50	0.27	0.85
	74	1,603	1,602.25	1,553.48	856	6,565	89.7	95.7	0	0.07	0.32	0.32	0.71
	75	1,311	1,310.42	1,307.66	573	5,408	69.3	73.3	0	0.07	0.35	0.28	0.70

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	76	1,658	1,657.90	1,642.46	266	1,733	36.5	44.4	0	0.04	0.04	0.06	0.14
	77	2,047	2,047.00	1,855.45	468	2,880	58.3	56.1	0	0.05	0.06	0.08	0.19
	78	1,448	1,447.44	1,439.70	679	5,383	76.0	79.0	0	0.07	0.29	0.39	0.75
	79	1,895	1,894.50	1,703.51	712	5,395	78.8	87.5	0	0.07	0.21	0.22	0.50
	80	1,630	1,630.00	1,526.77	698	7,403	77.7	82.3	0	0.09	0.27	0.19	0.55
	81	1,941	1,941.00	1,736.55	395	2,362	50.0	57.4	0	0.04	0.04	0.06	0.14
	82	1,943	1,942.33	1,738.62	557	4,213	65.7	61.3	0	0.07	0.12	0.16	0.35
	83	1,905	1,904.50	1,704.39	859	5,256	89.5	94.7	0	0.06	0.17	0.23	0.46
	84	1,509	1,508.47	1,500.95	773	6,647	85.2	90.4	0	0.07	0.41	0.35	0.83
	85	1,866	1,866.00	1,617.71	873	7,181	92.6	95.6	0	0.08	0.27	0.22	0.57
	86	1,603	1,602.25	1,547.65	661	4,839	76.6	72.4	0	0.07	0.19	0.20	0.46
	87	2,041	2,041.00	1,804.82	379	2,313	47.7	54.4	0	0.04	0.04	0.09	0.17
	88	1,650	1,649.50	1,568.77	750	5,672	82.9	84.6	0	0.07	0.22	0.26	0.55
	89	2,032	2,032.00	1,874.91	512	3,896	62.5	75.1	0	0.05	0.11	0.17	0.33
	90	1,823	1,822.71	1,790.23	447	3,103	57.0	58.2	0	0.05	0.10	0.13	0.28
	91	1,185	1,184.77	1,184.12	643	7,070	73.6	81.2	0	0.08	0.51	0.23	0.82
	92	1,854	1,854.00	1,710.46	509	3,425	62.5	69.8	0	0.05	0.10	0.08	0.23
	93	1,905	1,905.00	1,691.65	657	4,961	75.0	72.7	0	0.08	0.16	0.15	0.39
	94	1,839	1,839.00	1,719.03	780	7,311	84.3	80.6	0	0.09	0.35	0.19	0.63
	95	1,399	1,398.94	1,394.48	841	7,213	87.8	88.5	0	0.08	0.57	0.28	0.93
	96	1,506	1,505.66	1,497.71	681	6,095	76.6	84.5	0	0.08	0.42	0.31	0.81
	97	1,470	1,469.25	1,449.31	760	6,788	84.1	88.4	0	0.08	0.32	0.45	0.85
	98	1,562	1,561.67	1,543.62	749	7,483	82.8	91.3	0	0.09	0.43	0.30	0.82
	99	2,042	2,042.00	1,835.92	503	3,157	59.7	66.9	0	0.05	0.08	0.10	0.23
	100	1,944	1,944.00	1,700.39	631	5,424	74.9	73.4	0	0.07	0.19	0.17	0.43
1	1	80	80.00	55.80	9	30	20.9	36.1	0	0.01	0.00	0.00	0.01
	2	52	51.67	47.53	7	27	14.3	27.6	0	0.01	0.00	0.00	0.01
	3	52	51.67	43.35	6	25	8.3	17.9	0	0.01	0.00	0.00	0.01
	4	54	53.25	49.11	10	35	15.6	27.8	0	0.01	0.00	0.00	0.01
	5	76	76.00	60.23	8	28	21.6	40.0	0	0.01	0.00	0.00	0.01
	6	95	95.00	66.61	5	16	18.5	32.0	0	0.01	0.00	0.00	0.01
	7	56	55.67	49.40	10	41	15.6	33.1	0	0.01	0.00	0.00	0.01
	8	55	55.00	47.22	8	29	13.6	24.6	0	0.01	0.00	0.00	0.01
	9	43	42.60	40.56	12	55	10.9	24.8	0	0.01	0.00	0.00	0.01
	10	63	62.83	54.08	9	35	16.1	32.4	0	0.01	0.00	0.00	0.01
	11	63	62.25	51.59	8	27	18.6	32.9	0	0.01	0.00	0.00	0.01
	12	88	87.50	65.07	6	18	22.2	36.0	0	0.01	0.00	0.00	0.01
	13	48	47.50	39.38	12	47	15.8	29.7	0	0.01	0.00	0.00	0.01
	14	67	67.00	55.89	6	21	18.2	33.3	0	0.01	0.00	0.00	0.01
	15	73	72.50	53.67	7	22	23.3	39.3	0	0.01	0.00	0.00	0.01
	16	47	46.75	43.38	14	53	17.7	33.1	0	0.01	0.00	0.00	0.01
	17	60	60.00	49.63	9	30	16.7	28.6	0	0.01	0.00	0.00	0.01
	18	66	66.00	51.61	8	29	17.4	32.2	0	0.01	0.00	0.00	0.01
	19	65	65.00	56.52	8	26	22.9	39.4	0	0.01	0.00	0.00	0.01
	20	40	39.33	36.45	10	51	12.3	31.1	0	0.01	0.00	0.00	0.01
	21	67	66.33	55.62	9	36	14.8	29.5	0	0.01	0.00	0.00	0.01
	22	52	52.00	47.64	9	33	14.8	27.7	0	0.01	0.00	0.00	0.01
	23	64	64.00	55.43	6	20	22.2	37.7	0	0.01	0.00	0.00	0.01
	24	69	68.50	55.37	8	26	19.5	32.9	0	0.01	0.00	0.00	0.01
	25	56	55.67	48.32	9	34	16.1	31.2	0	0.01	0.00	0.00	0.01
	26	84	84.00	58.43	10	34	22.7	40.0	0	0.01	0.00	0.00	0.01
	27	49	48.50	44.56	11	42	12.8	24.7	0	0.01	0.00	0.00	0.01
	28	54	54.00	49.02	8	28	17.8	31.5	0	0.01	0.00	0.00	0.01
	29	60	60.00	50.91	8	25	23.5	38.5	0	0.01	0.00	0.00	0.01
	30	57	56.50	46.50	7	26	16.3	31.0	0	0.01	0.00	0.00	0.01
	31	78	78.00	55.14	7	24	21.9	39.3	0	0.01	0.00	0.00	0.01
	32	54	53.17	47.27	11	42	19.6	36.5	0	0.01	0.00	0.00	0.01
	33	61	60.83	50.88	7	28	12.5	25.5	0	0.01	0.00	0.00	0.01
	34	93	93.00	65.80	8	26	21.1	36.1	0	0.01	0.00	0.00	0.01
	35	75	74.50	58.25	7	25	18.4	34.7	0	0.01	0.00	0.00	0.01
	36	67	67.00	55.47	8	29	22.2	41.4	0	0.01	0.00	0.00	0.01
	37	71	71.00	54.68	7	22	25.0	41.5	0	0.01	0.00	0.00	0.01
	38	56	55.33	48.30	9	34	17.6	34.7	0	0.01	0.00	0.00	0.01
	39	65	65.00	54.83	7	24	17.1	30.4	0	0.01	0.00	0.00	0.01

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	40	62	62.00	49.78	11	46	13.1	27.2	0	0.01	0.00	0.00	0.01
	41	48	48.00	44.74	12	47	12.4	23.6	0	0.01	0.00	0.00	0.01
	42	46	46.00	42.84	10	35	20.8	37.2	0	0.01	0.00	0.00	0.01
	43	73	72.50	57.75	7	21	26.9	43.8	0	0.01	0.00	0.00	0.01
	44	50	50.00	45.18	11	47	14.7	29.6	0	0.01	0.00	0.00	0.01
	45	58	57.50	46.61	9	34	19.6	35.8	0	0.01	0.00	0.00	0.01
	46	84	83.50	54.62	8	24	23.5	36.9	0	0.01	0.00	0.00	0.01
	47	62	61.75	54.81	9	33	22.5	42.9	0	0.01	0.00	0.00	0.01
	48	70	70.00	56.99	9	29	23.1	39.2	0	0.01	0.00	0.00	0.01
	49	49	48.17	44.96	13	46	16.2	28.9	0	0.01	0.00	0.00	0.01
	50	60	60.00	51.58	9	31	23.7	41.3	0	0.01	0.00	0.00	0.01
	51	58	58.00	51.10	11	39	18.0	33.1	0	0.01	0.00	0.00	0.01
	52	53	53.00	47.05	10	37	17.5	32.5	0	0.01	0.00	0.00	0.01
	53	37	36.50	34.59	15	65	9.6	20.1	0	0.01	0.00	0.00	0.01
	54	70	69.50	53.38	9	29	18.0	29.6	0	0.01	0.00	0.00	0.01
	55	58	57.50	51.56	8	29	22.2	42.0	0	0.01	0.00	0.00	0.01
	56	75	75.00	56.25	7	22	20.6	33.8	0	0.01	0.00	0.00	0.01
	57	66	66.00	53.37	7	23	20.6	35.9	0	0.01	0.00	0.00	0.01
	58	67	67.00	50.15	6	20	16.7	29.4	0	0.01	0.00	0.00	0.01
	59	64	63.50	51.26	11	39	19.0	34.2	0	0.01	0.00	0.00	0.01
	60	56	55.25	48.78	10	40	14.3	29.0	0	0.01	0.00	0.00	0.01
	61	54	53.50	47.80	10	33	22.2	36.7	0	0.01	0.00	0.00	0.01
	62	74	74.00	58.58	7	25	17.1	32.1	0	0.01	0.00	0.00	0.01
	63	57	57.00	48.59	7	23	21.9	38.3	0	0.01	0.00	0.00	0.01
	64	83	83.00	55.98	6	19	20.7	35.2	0	0.01	0.00	0.00	0.01
	65	46	46.00	40.66	14	59	13.5	28.2	0	0.01	0.00	0.00	0.01
	66	57	56.38	50.91	13	47	24.5	43.5	0	0.01	0.00	0.00	0.01
	67	54	53.50	45.34	7	33	12.5	30.3	0	0.01	0.00	0.00	0.01
	68	46	45.33	42.49	8	35	8.1	17.1	0	0.01	0.00	0.00	0.01
	69	88	88.00	55.67	6	18	23.1	36.7	0	0.01	0.00	0.00	0.01
	70	67	66.50	54.44	5	19	14.3	28.4	0	0.01	0.00	0.00	0.01
	71	46	45.75	42.97	13	48	13.1	23.6	0	0.01	0.00	0.00	0.01
	72	85	85.00	59.61	6	19	22.2	38.0	0	0.01	0.00	0.00	0.01
	73	74	74.00	55.09	8	27	19.5	34.6	0	0.01	0.00	0.00	0.01
	74	58	58.00	50.78	5	17	17.2	31.5	0	0.01	0.00	0.00	0.01
	75	100	100.00	64.73	3	11	25.0	55.0	0	0.01	0.00	0.00	0.01
	76	69	68.50	54.29	9	30	20.9	36.1	0	0.01	0.00	0.00	0.01
	77	57	56.17	49.23	12	50	14.6	30.5	0	0.01	0.00	0.00	0.01
	78	82	81.50	59.07	6	18	28.6	46.2	0	0.01	0.00	0.00	0.01
	79	68	68.00	54.60	7	25	18.4	34.2	0	0.01	0.00	0.00	0.01
	80	55	55.00	47.18	9	28	22.5	35.9	0	0.01	0.00	0.00	0.01
	81	72	71.67	55.81	11	37	26.2	46.2	0	0.01	0.00	0.00	0.01
	82	58	57.50	49.12	9	33	17.3	32.4	0	0.01	0.00	0.00	0.01
	83	40	39.58	37.61	12	49	17.6	33.1	0	0.01	0.00	0.01	0.02
	84	68	68.00	54.74	9	31	22.5	38.3	0	0.01	0.00	0.00	0.01
	85	62	62.00	52.77	6	21	14.6	26.6	0	0.01	0.00	0.00	0.01
	86	56	56.00	51.95	7	24	17.5	31.6	0	0.01	0.00	0.00	0.01
	87	77	77.00	58.89	8	24	23.5	37.5	0	0.01	0.00	0.00	0.01
	88	62	62.00	53.15	12	46	16.7	32.6	0	0.01	0.00	0.00	0.01
	89	68	68.00	53.82	10	33	23.8	41.2	0	0.01	0.00	0.00	0.01
	90	68	67.50	50.34	4	17	13.3	29.8	0	0.01	0.00	0.00	0.01
	91	62	62.00	53.25	10	40	10.9	21.4	0	0.01	0.00	0.00	0.01
	92	47	46.50	44.14	10	38	15.6	29.5	0	0.01	0.00	0.00	0.01
	93	47	46.50	40.41	10	36	18.9	34.3	0	0.01	0.00	0.00	0.01
	94	97	97.00	64.86	4	13	26.7	50.0	0	0.01	0.00	0.00	0.01
	95	84	83.50	61.28	10	35	21.3	38.5	0	0.01	0.00	0.00	0.01
	96	64	64.00	53.75	8	26	20.5	35.1	0	0.01	0.00	0.00	0.01
	97	81	81.00	59.03	6	20	20.0	35.7	0	0.01	0.00	0.00	0.01
	98	60	60.00	51.28	9	31	17.0	28.7	0	0.01	0.00	0.00	0.01
	99	48	47.50	43.20	10	35	15.4	27.1	0	0.01	0.00	0.00	0.01
	100	65	65.00	56.22	8	24	28.6	45.3	0	0.01	0.00	0.00	0.01
1	1	790	790.00	553.59	9	30	20.9	36.1	0	0.01	0.00	0.00	0.01
	2	517	516.83	475.49	7	27	14.3	27.6	0	0.01	0.00	0.00	0.01
	3	518	517.67	433.41	6	26	7.7	17.1	0	0.01	0.00	0.00	0.01

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	4	545	544.75	497.95	10	35	15.6	27.8	0	0.01	0.00	0.00	0.01
	5	757	757.00	600.03	8	28	20.5	37.8	0	0.01	0.00	0.00	0.01
	6	950	950.00	668.50	6	18	20.7	33.3	0	0.01	0.00	0.00	0.01
	7	560	559.33	497.10	10	41	15.6	33.1	0	0.01	0.00	0.00	0.01
	8	547	546.50	470.45	8	29	13.6	24.6	0	0.01	0.00	0.00	0.01
	9	428	427.17	406.55	12	55	10.9	24.8	0	0.01	0.00	0.00	0.01
	10	626	625.50	539.50	10	38	15.9	30.9	0	0.01	0.00	0.00	0.01
	11	620	620.00	514.01	8	30	16.3	31.9	0	0.01	0.00	0.00	0.01
	12	887	887.00	656.91	6	18	22.2	36.0	0	0.01	0.00	0.00	0.01
	13	478	477.50	395.78	12	50	15.8	31.1	0	0.01	0.00	0.00	0.01
	14	655	655.00	555.19	6	21	18.2	32.8	0	0.01	0.00	0.00	0.01
	15	733	732.50	538.73	7	22	23.3	39.3	0	0.01	0.00	0.00	0.01
	16	471	470.75	437.30	14	53	17.7	33.1	0	0.01	0.00	0.00	0.01
	17	604	604.00	499.06	9	31	15.5	27.2	0	0.01	0.00	0.00	0.01
	18	669	669.00	519.46	8	29	17.4	32.2	0	0.01	0.00	0.00	0.01
	19	653	653.00	566.14	8	26	22.9	39.4	0	0.01	0.00	0.00	0.01
	20	399	398.33	367.15	10	51	12.2	30.7	0	0.01	0.00	0.00	0.01
	21	658	657.83	554.07	9	37	14.5	29.8	0	0.01	0.00	0.00	0.01
	22	533	532.50	481.34	9	33	14.8	27.7	0	0.01	0.00	0.00	0.01
	23	641	641.00	553.35	6	20	22.2	37.7	0	0.01	0.00	0.00	0.01
	24	683	682.50	553.29	8	26	19.5	32.9	0	0.01	0.00	0.00	0.01
	25	552	551.33	479.50	9	34	16.1	31.2	0	0.01	0.00	0.00	0.01
	26	840	840.00	585.46	10	34	21.3	37.0	0	0.01	0.00	0.00	0.01
	27	491	490.83	451.48	11	42	12.8	24.7	0	0.01	0.00	0.00	0.01
	28	538	537.75	488.89	8	28	17.8	31.5	0	0.01	0.00	0.00	0.01
	29	603	602.50	509.56	8	25	23.5	38.5	0	0.01	0.00	0.00	0.01
	30	569	568.50	467.33	7	26	16.3	31.0	0	0.01	0.00	0.00	0.01
	31	777	777.00	549.70	7	24	21.9	39.3	0	0.01	0.00	0.00	0.01
	32	535	534.17	473.47	11	43	19.3	36.8	0	0.01	0.00	0.00	0.01
	33	615	614.67	509.33	7	28	12.5	25.5	0	0.01	0.00	0.00	0.01
	34	926	926.00	656.65	8	26	20.5	35.1	0	0.01	0.00	0.00	0.01
	35	742	741.50	581.64	8	27	20.5	36.5	0	0.01	0.00	0.00	0.01
	36	673	673.00	556.49	8	29	22.2	41.4	0	0.01	0.00	0.00	0.01
	37	713	713.00	547.25	7	22	25.0	41.5	0	0.01	0.00	0.00	0.01
	38	552	551.67	480.33	9	34	17.6	34.7	0	0.01	0.00	0.00	0.01
	39	648	648.00	547.20	7	25	16.7	30.9	0	0.01	0.00	0.00	0.01
	40	618	617.50	495.21	11	46	12.9	26.9	0	0.01	0.00	0.00	0.01
	41	484	483.33	449.41	13	51	11.9	22.9	0	0.01	0.00	0.00	0.01
	42	461	460.33	429.74	10	36	20.0	36.7	0	0.01	0.00	0.00	0.01
	43	729	728.50	578.22	7	21	26.9	43.8	0	0.01	0.00	0.00	0.01
	44	490	489.67	447.20	15	63	17.4	34.8	0	0.01	0.00	0.00	0.01
	45	565	565.00	460.68	9	34	19.6	35.8	0	0.01	0.00	0.00	0.01
	46	832	831.50	544.65	9	27	25.0	39.1	0	0.01	0.00	0.00	0.01
	47	618	617.50	547.12	9	33	22.5	42.9	0	0.01	0.00	0.00	0.01
	48	701	701.00	571.45	9	30	22.0	38.5	0	0.01	0.00	0.00	0.01
	49	478	477.92	446.60	13	46	16.2	28.9	0	0.01	0.00	0.00	0.01
	50	604	603.67	519.82	9	31	23.7	41.3	0	0.01	0.00	0.00	0.01
	51	586	585.25	513.53	9	34	14.3	27.9	0	0.01	0.00	0.00	0.01
	52	529	529.00	470.20	10	37	17.5	32.5	0	0.01	0.00	0.00	0.01
	53	366	365.88	346.21	15	65	9.6	20.1	0	0.01	0.00	0.00	0.01
	54	690	690.00	532.55	9	29	18.0	29.6	0	0.01	0.00	0.00	0.01
	55	575	575.00	513.77	8	29	21.6	40.8	0	0.01	0.00	0.00	0.01
	56	754	753.50	565.26	8	25	22.9	37.3	0	0.01	0.00	0.00	0.01
	57	667	667.00	536.32	7	23	20.6	35.9	0	0.01	0.00	0.00	0.01
	58	664	664.00	499.94	6	20	16.2	28.6	0	0.01	0.00	0.00	0.01
	59	631	631.00	510.68	11	39	18.6	33.6	0	0.01	0.00	0.00	0.01
	60	551	550.50	486.42	10	40	14.1	28.6	0	0.01	0.00	0.00	0.01
	61	535	534.50	475.73	10	33	22.2	36.7	0	0.01	0.00	0.00	0.01
	62	726	726.00	579.05	9	29	20.9	35.4	0	0.01	0.00	0.00	0.01
	63	562	561.67	480.15	7	23	21.9	38.3	0	0.01	0.00	0.00	0.01
	64	833	833.00	560.74	6	19	20.7	35.2	0	0.01	0.00	0.00	0.01
	65	464	463.83	408.57	16	66	14.0	28.7	0	0.01	0.00	0.00	0.01
	66	564	563.25	506.88	14	50	21.9	38.2	0	0.01	0.00	0.00	0.01
	67	529	529.00	450.35	7	33	12.5	30.3	0	0.01	0.00	0.00	0.01

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	68	451	451.00	422.55	8	35	7.5	15.8	0	0.01	0.00	0.00	0.01
	69	894	894.00	560.18	6	18	23.1	36.7	0	0.01	0.00	0.00	0.01
	70	665	665.00	543.81	5	19	14.3	28.4	0	0.01	0.00	0.00	0.01
	71	461	460.75	433.20	13	48	13.1	23.6	0	0.01	0.00	0.00	0.01
	72	856	856.00	598.95	6	19	21.4	36.5	0	0.01	0.00	0.00	0.01
	73	746	746.00	552.90	8	27	19.0	33.8	0	0.01	0.00	0.00	0.01
	74	579	578.50	503.70	5	17	17.2	31.5	0	0.01	0.00	0.00	0.01
	75	1,000	1,000.00	649.02	3	11	25.0	55.0	0	0.01	0.00	0.00	0.01
	76	682	682.00	540.87	9	30	20.9	36.1	0	0.01	0.00	0.00	0.01
	77	556	555.17	488.99	12	50	14.6	30.5	0	0.01	0.00	0.00	0.01
	78	826	826.00	595.27	6	18	28.6	46.2	0	0.01	0.00	0.00	0.01
	79	696	696.00	551.78	7	25	18.4	34.2	0	0.01	0.00	0.00	0.01
	80	550	550.00	473.07	9	28	22.5	35.9	0	0.01	0.00	0.00	0.01
	81	714	713.83	556.38	11	37	26.2	46.2	0	0.01	0.00	0.00	0.01
	82	575	574.50	491.31	9	33	17.3	32.4	0	0.01	0.00	0.00	0.01
	83	395	394.25	374.50	12	49	17.6	33.1	0	0.01	0.00	0.00	0.01
	84	687	687.00	550.16	10	34	24.4	41.0	0	0.01	0.00	0.00	0.01
	85	609	609.00	522.57	6	21	14.6	26.6	0	0.01	0.00	0.00	0.01
	86	556	555.25	516.75	7	24	17.5	31.6	0	0.01	0.00	0.00	0.01
	87	771	771.00	590.29	8	24	23.5	37.5	0	0.01	0.00	0.00	0.01
	88	617	617.00	529.49	14	52	17.1	32.3	0	0.01	0.00	0.00	0.01
	89	673	673.00	533.69	10	33	23.8	41.2	0	0.01	0.00	0.00	0.01
	90	681	680.50	506.76	4	17	12.9	28.8	0	0.01	0.00	0.00	0.01
	91	613	612.50	526.52	10	39	9.7	18.7	0	0.01	0.00	0.00	0.01
	92	470	469.25	444.20	10	38	15.6	29.5	0	0.01	0.00	0.00	0.01
	93	461	460.50	403.58	10	36	18.9	34.3	0	0.01	0.00	0.00	0.01
	94	957	957.00	645.83	4	13	26.7	50.0	0	0.01	0.00	0.00	0.01
	95	836	835.50	612.97	10	35	20.8	37.6	0	0.01	0.00	0.00	0.01
	96	633	633.00	535.10	8	26	20.5	35.1	0	0.01	0.00	0.00	0.01
	97	799	798.50	580.92	8	26	23.5	40.6	0	0.01	0.00	0.00	0.01
	98	598	597.50	511.94	9	31	17.0	28.7	0	0.01	0.00	0.00	0.01
	99	477	476.50	433.42	10	35	15.4	27.1	0	0.01	0.00	0.00	0.01
	100	650	649.33	561.38	8	25	27.6	45.5	0	0.01	0.00	0.00	0.01
1	1	127	127.00	104.04	12	61	9.0	22.2	0	0.01	0.00	0.00	0.01
	2	103	103.00	94.04	17	111	6.4	17.7	0	0.02	0.00	0.00	0.02
	3	171	171.00	126.33	9	47	9.5	24.2	0	0.01	0.00	0.00	0.01
	4	102	101.17	93.11	19	100	8.4	19.6	0	0.02	0.00	0.00	0.02
	5	123	123.00	106.56	14	83	8.9	24.3	0	0.01	0.00	0.00	0.01
	6	131	130.17	111.76	14	73	11.3	27.0	0	0.01	0.00	0.00	0.01
	7	114	114.00	98.42	24	140	9.2	22.3	0	0.01	0.00	0.01	0.02
	8	170	170.00	116.01	7	31	14.3	33.0	0	0.01	0.00	0.00	0.01
	9	109	109.00	100.26	21	105	8.0	16.4	0	0.02	0.00	0.00	0.02
	10	117	116.50	103.93	15	70	14.0	30.6	0	0.01	0.00	0.00	0.01
	11	115	114.12	101.51	19	103	7.6	18.3	0	0.02	0.00	0.00	0.02
	12	123	123.00	101.69	16	77	11.5	26.1	0	0.01	0.00	0.00	0.01
	13	122	121.50	105.84	15	86	7.2	18.7	0	0.01	0.00	0.00	0.01
	14	102	101.25	94.29	17	89	9.9	24.0	0	0.02	0.00	0.00	0.02
	15	120	120.00	104.04	15	72	15.2	33.3	0	0.01	0.00	0.00	0.01
	16	112	112.00	99.19	13	65	9.4	22.9	0	0.01	0.00	0.00	0.01
	17	134	133.50	109.94	14	60	10.8	22.1	0	0.01	0.00	0.00	0.01
	18	119	119.00	104.75	16	76	11.2	25.2	0	0.01	0.00	0.00	0.01
	19	138	138.00	111.57	9	41	10.0	22.0	0	0.01	0.00	0.00	0.01
	20	151	151.00	112.05	12	60	10.5	25.4	0	0.01	0.00	0.00	0.01
	21	104	103.67	95.06	26	134	9.5	20.8	0	0.01	0.00	0.01	0.02
	22	128	128.00	105.44	10	53	10.9	28.5	0	0.01	0.00	0.00	0.01
	23	100	99.72	91.97	23	132	9.8	22.7	0	0.01	0.00	0.01	0.02
	24	123	123.00	106.66	19	119	9.7	28.1	0	0.02	0.00	0.00	0.02
	25	92	91.25	88.69	23	135	9.0	23.1	0	0.01	0.00	0.01	0.02
	26	98	98.00	91.15	19	110	9.0	23.0	0	0.02	0.00	0.00	0.02
	27	94	93.36	89.26	26	169	8.3	21.1	0	0.01	0.00	0.01	0.02
	28	109	109.00	95.83	17	88	8.3	19.6	0	0.02	0.00	0.00	0.02
	29	113	112.17	100.67	10	54	7.2	18.4	0	0.01	0.00	0.00	0.01
	30	88	88.00	83.36	19	100	7.4	16.0	0	0.02	0.00	0.00	0.02
	31	105	105.00	93.20	17	89	7.1	15.6	0	0.02	0.00	0.00	0.02

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	32	159	159.00	117.70	13	65	8.8	20.3	0	0.01	0.00	0.00	0.01
	33	93	92.75	89.95	26	147	10.6	24.3	0	0.01	0.00	0.01	0.02
	34	110	109.50	95.57	16	104	9.2	27.9	0	0.02	0.00	0.00	0.02
	35	127	127.00	103.23	13	63	9.8	21.7	0	0.01	0.00	0.00	0.01
	36	133	133.00	103.91	18	83	10.3	21.3	0	0.01	0.00	0.00	0.01
	37	133	133.00	108.10	13	66	9.7	23.6	0	0.01	0.00	0.00	0.01
	38	189	189.00	128.90	6	29	11.8	29.6	0	0.01	0.00	0.00	0.01
	39	97	96.83	92.39	18	112	8.5	22.7	0	0.02	0.00	0.00	0.02
	40	130	130.00	104.31	9	44	8.7	20.7	0	0.01	0.00	0.00	0.01
	41	96	96.00	91.21	21	128	8.6	22.3	0	0.02	0.00	0.00	0.02
	42	123	123.00	105.71	14	75	11.7	29.3	0	0.01	0.00	0.00	0.01
	43	144	144.00	110.50	10	51	9.6	24.3	0	0.01	0.00	0.00	0.01
	44	130	130.00	109.08	16	73	14.8	32.9	0	0.01	0.00	0.00	0.01
	45	157	157.00	116.14	15	85	9.0	24.0	0	0.01	0.00	0.00	0.01
	46	98	97.83	92.75	22	115	8.3	19.0	0	0.01	0.00	0.01	0.02
	47	117	117.00	102.20	19	93	9.0	20.0	0	0.02	0.00	0.00	0.02
	48	105	104.33	97.98	20	104	9.4	21.7	0	0.02	0.00	0.00	0.02
	49	132	132.00	107.24	16	79	12.0	28.5	0	0.01	0.00	0.00	0.01
	50	107	107.00	97.89	23	122	9.8	22.6	0	0.02	0.00	0.00	0.02
	51	141	140.50	112.85	16	76	13.1	29.6	0	0.01	0.00	0.00	0.01
	52	92	91.75	89.08	21	110	13.1	30.1	0	0.02	0.00	0.00	0.02
	53	93	92.12	89.13	18	98	9.4	23.2	0	0.02	0.00	0.00	0.02
	54	112	111.50	101.31	13	71	8.6	22.5	0	0.01	0.00	0.00	0.01
	55	103	102.12	90.26	11	52	8.8	20.1	0	0.01	0.00	0.00	0.01
	56	85	84.61	82.53	26	155	9.1	22.0	0	0.02	0.00	0.01	0.03
	57	110	109.50	97.11	20	100	9.7	21.4	0	0.01	0.00	0.00	0.01
	58	118	118.00	106.14	11	52	11.0	25.5	0	0.01	0.00	0.00	0.01
	59	106	105.50	97.57	11	59	8.3	19.9	0	0.01	0.00	0.00	0.01
	60	103	102.67	95.95	22	124	10.0	24.1	0	0.02	0.00	0.00	0.02
	61	91	90.77	88.07	22	123	9.3	22.5	0	0.01	0.00	0.01	0.02
	62	114	113.75	102.34	17	92	10.6	27.0	0	0.01	0.00	0.00	0.01
	63	154	154.00	110.42	15	70	12.7	27.9	0	0.01	0.00	0.00	0.01
	64	121	121.00	102.60	12	67	8.8	23.8	0	0.01	0.00	0.00	0.01
	65	158	157.50	120.88	14	66	13.1	30.0	0	0.01	0.00	0.00	0.01
	66	153	153.00	115.42	11	52	11.5	26.8	0	0.01	0.00	0.00	0.01
	67	117	116.50	103.28	11	59	8.9	22.1	0	0.01	0.00	0.00	0.01
	68	125	124.50	108.71	16	80	13.1	31.1	0	0.01	0.00	0.00	0.01
	69	127	127.00	103.88	14	77	8.1	21.5	0	0.01	0.00	0.00	0.01
	70	133	133.00	106.38	11	56	10.5	25.5	0	0.01	0.00	0.00	0.01
	71	115	114.67	103.31	19	97	10.0	23.8	0	0.02	0.00	0.00	0.02
	72	130	129.50	108.39	21	117	9.4	23.3	0	0.02	0.00	0.00	0.02
	73	130	129.50	107.78	14	69	11.8	26.6	0	0.01	0.00	0.00	0.01
	74	110	110.00	100.68	17	90	10.4	25.9	0	0.01	0.00	0.00	0.01
	75	148	148.00	116.33	10	54	11.5	31.2	0	0.01	0.00	0.00	0.01
	76	97	97.00	90.21	14	80	6.2	15.4	0	0.01	0.00	0.00	0.01
	77	148	147.50	106.15	13	61	11.4	25.4	0	0.01	0.00	0.00	0.01
	78	139	139.00	114.84	14	70	12.5	29.8	0	0.01	0.00	0.00	0.01
	79	115	114.50	103.82	25	137	11.3	26.8	0	0.02	0.00	0.00	0.02
	80	127	126.50	108.51	13	62	14.4	33.9	0	0.01	0.00	0.00	0.01
	81	94	93.69	91.75	22	131	8.9	22.3	0	0.01	0.00	0.01	0.02
	82	93	93.00	88.14	18	96	9.7	23.0	0	0.01	0.00	0.00	0.01
	83	107	107.00	98.72	19	84	9.0	17.5	0	0.02	0.00	0.00	0.02
	84	115	115.00	98.00	19	97	9.4	21.0	0	0.02	0.00	0.00	0.02
	85	85	84.88	81.29	22	130	8.5	20.8	0	0.01	0.00	0.01	0.02
	86	111	110.50	98.28	15	83	10.6	27.5	0	0.01	0.00	0.00	0.01
	87	116	115.50	100.91	17	89	9.1	21.8	0	0.01	0.00	0.00	0.01
	88	118	117.50	100.02	14	61	14.1	29.6	0	0.01	0.00	0.00	0.01
	89	128	127.50	104.54	9	52	7.8	21.7	0	0.01	0.00	0.00	0.01
	90	135	135.00	108.94	16	80	9.8	22.3	0	0.01	0.00	0.00	0.01
	91	98	97.50	91.33	16	91	10.2	27.4	0	0.02	0.00	0.00	0.02
	92	81	80.88	79.49	32	188	10.0	22.4	0	0.01	0.00	0.01	0.02
	93	122	122.00	102.48	21	109	8.2	17.6	0	0.02	0.00	0.00	0.02
	94	126	126.00	107.06	18	82	12.6	27.2	0	0.02	0.00	0.00	0.02
	95	144	144.00	110.30	12	63	9.0	22.0	0	0.01	0.00	0.00	0.01

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	96	143	142.50	110.18	13	57	10.2	21.8	0	0.01	0.00	0.00	0.01
	97	90	89.62	84.39	14	87	8.8	25.9	0	0.01	0.00	0.00	0.01
	98	147	146.50	115.11	14	65	10.2	23.0	0	0.01	0.00	0.00	0.01
	99	99	98.45	91.03	19	113	8.4	21.4	0	0.02	0.00	0.00	0.02
	100	155	155.00	118.98	7	31	15.9	36.0	0	0.01	0.00	0.00	0.01
	1	1,283	1,283.00	1,046.62	12	61	9.0	22.2	0	0.01	0.00	0.00	0.01
	2	1,034	1,033.33	942.68	17	112	6.3	17.7	0	0.02	0.00	0.00	0.02
	3	1,705	1,704.50	1,259.58	9	47	8.3	21.1	0	0.01	0.00	0.00	0.01
	4	1,010	1,009.50	928.63	19	100	8.4	19.6	0	0.02	0.00	0.00	0.02
	5	1,224	1,223.50	1,062.60	15	86	9.0	23.7	0	0.01	0.00	0.00	0.01
	6	1,292	1,291.67	1,111.87	16	78	11.9	26.6	0	0.01	0.00	0.00	0.01
	7	1,140	1,140.00	983.03	26	147	10.0	23.0	0	0.01	0.00	0.01	0.02
	8	1,709	1,709.00	1,163.17	7	31	14.3	33.0	0	0.01	0.00	0.00	0.01
	9	1,088	1,088.00	1,001.61	21	105	8.0	16.3	0	0.02	0.00	0.00	0.02
	10	1,169	1,168.50	1,042.08	15	70	14.0	30.6	0	0.01	0.00	0.00	0.01
	11	1,145	1,144.88	1,018.66	19	103	7.6	18.3	0	0.02	0.00	0.00	0.02
	12	1,216	1,215.50	1,015.07	16	77	11.5	26.1	0	0.01	0.00	0.00	0.01
	13	1,223	1,222.50	1,060.22	15	86	7.0	18.1	0	0.01	0.00	0.00	0.01
	14	1,007	1,007.00	939.65	17	89	9.9	24.0	0	0.02	0.00	0.00	0.02
	15	1,213	1,213.00	1,046.40	15	72	15.2	33.3	0	0.01	0.00	0.00	0.01
	16	1,111	1,110.50	986.21	13	66	9.4	23.1	0	0.01	0.00	0.00	0.01
	17	1,330	1,330.00	1,097.39	11	51	8.3	18.5	0	0.01	0.00	0.00	0.01
	18	1,187	1,186.50	1,045.91	16	76	11.2	25.2	0	0.01	0.00	0.00	0.01
	19	1,378	1,377.50	1,114.36	9	41	9.9	21.8	0	0.01	0.00	0.00	0.01
	20	1,501	1,501.00	1,115.24	12	60	10.5	25.3	0	0.01	0.00	0.00	0.01
	21	1,033	1,032.33	947.03	26	141	8.9	20.0	0	0.01	0.00	0.01	0.02
	22	1,294	1,293.50	1,057.92	10	53	10.9	28.5	0	0.01	0.00	0.00	0.01
	23	998	997.44	919.74	24	134	10.3	22.7	0	0.01	0.00	0.01	0.02
	24	1,242	1,242.00	1,072.24	19	120	9.6	27.6	0	0.02	0.00	0.00	0.02
	25	913	912.50	886.46	23	135	9.0	23.1	0	0.02	0.00	0.00	0.02
	26	979	979.00	911.21	19	111	8.9	23.1	0	0.02	0.00	0.00	0.02
	27	928	927.71	887.59	26	169	8.3	21.1	0	0.01	0.00	0.01	0.02
	28	1,094	1,094.00	959.84	18	91	8.5	19.4	0	0.02	0.00	0.00	0.02
	29	1,124	1,123.67	1,008.95	10	54	7.1	18.2	0	0.01	0.00	0.00	0.01
	30	876	875.50	832.12	19	100	7.4	16.0	0	0.01	0.00	0.01	0.02
	31	1,042	1,042.00	927.57	16	86	6.5	14.6	0	0.02	0.00	0.00	0.02
	32	1,586	1,586.00	1,175.78	13	65	8.8	20.2	0	0.01	0.00	0.00	0.01
	33	923	922.25	894.54	26	146	10.0	22.7	0	0.01	0.00	0.01	0.02
	34	1,086	1,085.17	949.72	16	102	8.8	26.0	0	0.02	0.00	0.00	0.02
	35	1,261	1,261.00	1,031.94	12	60	9.1	20.6	0	0.01	0.00	0.00	0.01
	36	1,337	1,337.00	1,041.56	18	82	9.4	19.2	0	0.01	0.00	0.00	0.01
	37	1,339	1,339.00	1,082.67	13	66	9.7	23.6	0	0.01	0.00	0.00	0.01
	38	1,882	1,882.00	1,288.23	6	29	11.8	29.6	0	0.01	0.00	0.00	0.01
	39	963	962.83	921.89	19	115	8.9	23.2	0	0.02	0.00	0.00	0.02
	40	1,305	1,305.00	1,046.71	9	44	8.7	20.5	0	0.01	0.00	0.00	0.01
	41	960	960.00	912.61	21	129	8.6	22.4	0	0.02	0.00	0.00	0.02
	42	1,236	1,236.00	1,058.72	14	75	11.6	29.1	0	0.01	0.00	0.00	0.01
	43	1,413	1,413.00	1,092.83	10	52	9.3	23.7	0	0.01	0.00	0.00	0.01
	44	1,317	1,317.00	1,097.29	16	71	14.3	30.9	0	0.01	0.00	0.00	0.01
	45	1,579	1,578.50	1,161.98	15	86	8.5	22.8	0	0.01	0.00	0.00	0.01
	46	978	978.00	927.33	23	118	8.2	18.4	0	0.02	0.00	0.00	0.02
	47	1,181	1,181.00	1,026.03	19	95	8.7	19.6	0	0.01	0.00	0.00	0.01
	48	1,046	1,046.00	980.98	20	105	9.0	20.9	0	0.01	0.00	0.00	0.01
	49	1,326	1,326.00	1,075.60	16	79	12.0	28.5	0	0.01	0.00	0.00	0.01
	50	1,064	1,064.00	976.07	23	122	9.8	22.6	0	0.02	0.00	0.00	0.02
	51	1,381	1,381.00	1,117.16	16	76	12.5	28.3	0	0.01	0.00	0.00	0.01
	52	907	906.38	880.64	21	110	13.0	29.9	0	0.01	0.00	0.01	0.02
	53	924	924.00	894.70	18	98	9.4	23.2	0	0.01	0.00	0.00	0.01
	54	1,106	1,106.00	1,007.76	13	71	8.6	22.5	0	0.01	0.00	0.00	0.01
	55	1,021	1,020.50	901.54	11	52	8.8	20.1	0	0.01	0.00	0.00	0.01
	56	851	850.97	827.64	26	156	9.1	22.0	0	0.01	0.00	0.01	0.02
	57	1,086	1,086.00	967.75	20	100	9.7	21.4	0	0.01	0.00	0.00	0.01
	58	1,178	1,178.00	1,060.34	11	52	11.0	25.5	0	0.01	0.00	0.00	0.01
	59	1,049	1,048.50	972.92	11	59	8.3	19.9	0	0.01	0.00	0.00	0.01

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	60	1,029	1,029.00	962.93	22	124	10.0	24.1	0	0.02	0.00	0.00	0.02
	61	910	909.30	881.90	22	123	9.3	22.3	0	0.02	0.00	0.00	0.02
	62	1,124	1,123.50	1,014.61	17	92	10.6	27.0	0	0.01	0.00	0.00	0.01
	63	1,549	1,549.00	1,103.84	15	72	12.2	27.6	0	0.01	0.00	0.00	0.01
	64	1,210	1,210.00	1,025.68	15	79	9.6	24.4	0	0.01	0.00	0.00	0.01
	65	1,570	1,569.50	1,204.39	14	68	12.1	28.5	0	0.01	0.00	0.00	0.01
	66	1,529	1,529.00	1,152.73	11	52	10.4	24.1	0	0.01	0.00	0.00	0.01
	67	1,158	1,158.00	1,029.24	11	60	8.8	22.2	0	0.01	0.00	0.00	0.01
	68	1,251	1,250.50	1,092.25	16	80	13.1	31.1	0	0.01	0.00	0.00	0.01
	69	1,237	1,236.50	1,022.95	14	77	8.1	21.5	0	0.01	0.00	0.00	0.01
	70	1,327	1,327.00	1,062.01	11	56	10.5	25.5	0	0.01	0.00	0.00	0.01
	71	1,140	1,139.67	1,029.07	19	98	10.0	24.0	0	0.02	0.00	0.00	0.02
	72	1,297	1,296.50	1,084.50	21	118	9.3	23.3	0	0.02	0.00	0.00	0.02
	73	1,271	1,270.50	1,066.50	17	77	13.2	27.4	0	0.01	0.00	0.00	0.01
	74	1,105	1,105.00	1,007.37	17	90	10.4	25.9	0	0.01	0.00	0.00	0.01
	75	1,454	1,454.00	1,151.78	10	54	11.5	31.2	0	0.01	0.00	0.00	0.01
	76	972	971.62	903.16	14	80	6.2	15.4	0	0.02	0.00	0.00	0.02
	77	1,473	1,473.00	1,059.59	13	61	11.4	25.4	0	0.01	0.00	0.00	0.01
	78	1,379	1,378.50	1,142.14	14	70	12.5	29.8	0	0.01	0.00	0.00	0.01
	79	1,162	1,161.33	1,047.52	25	137	11.3	26.8	0	0.02	0.00	0.00	0.02
	80	1,258	1,257.50	1,080.75	13	62	14.4	33.9	0	0.01	0.00	0.00	0.01
	81	939	939.00	918.93	22	131	8.9	22.3	0	0.02	0.00	0.00	0.02
	82	926	925.75	878.12	18	96	9.7	23.0	0	0.02	0.00	0.00	0.02
	83	1,080	1,079.67	991.94	19	86	8.6	16.9	0	0.02	0.00	0.00	0.02
	84	1,149	1,149.00	978.25	20	100	9.9	21.6	0	0.02	0.00	0.00	0.02
	85	850	849.38	814.61	22	130	8.5	20.8	0	0.01	0.00	0.01	0.02
	86	1,088	1,087.50	975.65	15	83	10.6	27.5	0	0.01	0.00	0.00	0.01
	87	1,150	1,150.00	1,008.28	18	92	9.6	22.0	0	0.01	0.00	0.00	0.01
	88	1,202	1,201.50	1,009.69	14	61	14.1	29.6	0	0.01	0.00	0.00	0.01
	89	1,279	1,278.50	1,044.61	9	52	7.8	21.7	0	0.01	0.00	0.00	0.01
	90	1,352	1,351.50	1,089.85	18	85	10.9	23.4	0	0.01	0.00	0.00	0.01
	91	962	961.25	906.30	16	90	10.1	26.9	0	0.01	0.00	0.00	0.01
	92	807	806.41	793.25	32	188	10.0	22.4	0	0.01	0.00	0.01	0.02
	93	1,233	1,233.00	1,030.12	21	109	8.2	17.6	0	0.02	0.00	0.00	0.02
	94	1,253	1,253.00	1,067.05	16	77	11.0	25.2	0	0.01	0.00	0.00	0.01
	95	1,441	1,441.00	1,103.15	12	63	8.5	20.9	0	0.01	0.00	0.00	0.01
	96	1,403	1,403.00	1,094.60	13	57	10.2	21.8	0	0.01	0.00	0.00	0.01
	97	895	894.20	841.26	14	87	8.8	25.9	0	0.02	0.00	0.00	0.02
	98	1,470	1,470.00	1,153.25	14	66	10.1	23.2	0	0.01	0.00	0.00	0.01
	99	991	990.75	915.23	19	113	8.4	21.4	0	0.02	0.00	0.00	0.02
	100	1,552	1,552.00	1,191.76	7	32	15.6	36.4	0	0.01	0.00	0.00	0.01
1	1	209	208.50	194.68	29	225	8.8	23.5	0	0.01	0.00	0.01	0.02
	2	251	251.00	216.68	25	165	7.0	15.5	0	0.02	0.00	0.00	0.02
	3	197	196.33	189.43	42	348	9.7	21.0	0	0.02	0.00	0.01	0.03
	4	212	211.50	196.11	41	409	9.5	25.0	0	0.02	0.00	0.01	0.03
	5	238	237.50	202.80	40	284	9.0	15.6	0	0.02	0.00	0.01	0.03
	6	219	219.00	199.09	30	254	7.2	16.8	0	0.01	0.00	0.01	0.02
	7	206	206.00	189.25	42	326	9.7	20.5	0	0.02	0.00	0.01	0.03
	8	220	219.67	204.31	29	219	10.5	28.9	0	0.01	0.00	0.01	0.02
	9	226	225.50	204.65	27	212	7.3	18.7	0	0.01	0.00	0.01	0.02
	10	216	216.00	199.14	30	239	8.7	23.0	0	0.01	0.00	0.01	0.02
	11	202	202.00	190.52	53	417	11.1	20.1	0	0.03	0.00	0.01	0.04
	12	209	208.83	198.38	41	370	9.7	22.9	0	0.02	0.00	0.01	0.03
	13	208	207.17	195.47	38	287	9.6	21.8	0	0.02	0.00	0.01	0.03
	14	178	177.50	170.76	54	463	11.2	20.2	0	0.02	0.01	0.01	0.04
	15	203	202.57	188.40	28	231	8.3	23.0	0	0.01	0.00	0.01	0.02
	16	265	265.00	218.12	37	284	9.0	20.2	0	0.01	0.00	0.01	0.02
	17	208	208.00	196.06	47	399	10.3	21.1	0	0.02	0.01	0.00	0.03
	18	216	215.50	200.36	20	160	6.7	19.4	0	0.01	0.00	0.01	0.02
	19	277	277.00	224.27	33	266	9.9	27.3	0	0.01	0.00	0.01	0.02
	20	230	229.50	204.09	32	271	8.8	24.6	0	0.01	0.00	0.01	0.02
	21	190	190.00	183.20	39	310	9.4	20.0	0	0.02	0.00	0.01	0.03
	22	259	258.50	222.39	20	135	7.7	21.3	0	0.02	0.00	0.00	0.02
	23	195	194.88	188.27	50	430	10.5	20.7	0	0.02	0.00	0.02	0.04

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	24	214	213.92	201.50	38	284	9.7	21.4	0	0.01	0.00	0.02	0.03
	25	246	246.00	212.83	31	254	9.8	28.5	0	0.01	0.00	0.01	0.02
	26	277	277.00	221.53	23	152	8.5	22.0	0	0.02	0.00	0.00	0.02
	27	211	210.67	197.04	25	205	7.6	20.5	0	0.01	0.00	0.01	0.02
	28	196	195.75	182.69	24	198	7.0	19.5	0	0.01	0.00	0.01	0.02
	29	249	248.50	211.19	28	213	7.9	20.4	0	0.01	0.00	0.01	0.02
	30	213	213.00	192.71	34	251	9.6	22.9	0	0.01	0.00	0.01	0.02
	31	190	189.20	184.76	36	283	9.4	22.1	0	0.02	0.00	0.01	0.03
	32	248	248.00	206.21	40	290	9.9	22.0	0	0.02	0.00	0.01	0.03
	33	240	239.50	211.67	40	312	9.2	19.4	0	0.02	0.00	0.01	0.03
	34	229	229.00	209.58	30	253	8.9	25.4	0	0.01	0.00	0.01	0.02
	35	243	243.00	209.12	28	227	6.8	16.6	0	0.01	0.00	0.01	0.02
	36	231	230.25	203.06	30	259	8.3	23.8	0	0.01	0.00	0.01	0.02
	37	229	228.50	205.28	34	259	8.8	20.6	0	0.02	0.00	0.01	0.03
	38	245	245.00	211.51	21	148	8.8	24.7	0	0.02	0.00	0.00	0.02
	39	282	281.50	213.72	35	253	9.7	23.0	0	0.01	0.00	0.01	0.02
	40	219	218.50	203.79	36	333	8.8	23.0	0	0.02	0.00	0.01	0.03
	41	210	209.50	191.55	41	327	10.7	24.2	0	0.02	0.00	0.01	0.03
	42	198	197.56	189.02	31	223	7.6	16.4	0	0.02	0.00	0.01	0.03
	43	217	217.00	196.32	46	384	10.9	24.9	0	0.02	0.00	0.01	0.03
	44	225	225.00	206.14	32	245	8.7	21.1	0	0.01	0.00	0.01	0.02
	45	237	237.00	212.68	24	155	7.5	17.8	0	0.02	0.00	0.00	0.02
	46	178	177.95	174.72	40	380	9.9	26.2	0	0.02	0.00	0.01	0.03
	47	232	231.50	204.49	38	287	9.2	19.7	0	0.02	0.00	0.04	0.06
	48	281	281.00	229.29	29	222	8.2	20.7	0	0.01	0.00	0.01	0.02
	49	209	208.58	195.61	34	274	8.6	20.9	0	0.01	0.00	0.02	0.03
	50	252	252.00	212.55	25	202	6.8	17.3	0	0.01	0.00	0.01	0.02
	51	202	202.00	192.44	36	275	8.6	17.3	0	0.02	0.00	0.01	0.03
	52	287	286.50	231.74	24	198	7.8	24.4	0	0.01	0.00	0.01	0.02
	53	207	207.00	191.25	35	335	8.8	26.0	0	0.02	0.00	0.01	0.03
	54	231	230.62	210.00	26	189	7.2	17.9	0	0.01	0.00	0.01	0.02
	55	285	285.00	229.62	37	258	9.2	18.4	0	0.02	0.00	0.01	0.03
	56	251	251.00	211.32	37	278	9.1	20.0	0	0.02	0.00	0.01	0.03
	57	242	242.00	214.76	30	194	8.9	20.2	0	0.01	0.00	0.01	0.02
	58	225	225.00	202.86	27	193	8.8	23.5	0	0.01	0.00	0.01	0.02
	59	236	236.00	210.53	31	231	8.4	20.7	0	0.01	0.00	0.01	0.02
	60	235	235.00	212.51	30	217	9.0	21.5	0	0.01	0.00	0.01	0.02
	61	183	182.15	179.31	44	313	9.9	18.1	0	0.02	0.00	0.01	0.03
	62	264	264.00	208.35	26	193	7.6	20.1	0	0.01	0.00	0.01	0.02
	63	233	233.00	206.33	39	304	8.7	17.5	0	0.02	0.00	0.01	0.03
	64	229	228.50	204.99	33	230	8.4	17.8	0	0.01	0.00	0.01	0.02
	65	204	203.25	188.08	34	284	9.1	24.2	0	0.02	0.00	0.01	0.03
	66	201	200.60	194.42	40	329	9.8	21.8	0	0.02	0.00	0.02	0.04
	67	206	205.03	193.21	49	400	10.8	21.5	0	0.02	0.00	0.02	0.04
	68	218	218.00	194.64	30	257	8.2	22.4	0	0.02	0.00	0.01	0.03
	69	242	241.50	211.23	30	231	9.5	26.1	0	0.01	0.00	0.01	0.02
	70	195	194.70	190.49	31	236	8.7	21.2	0	0.01	0.00	0.01	0.02
	71	232	232.00	208.39	33	269	9.5	25.1	0	0.01	0.00	0.01	0.02
	72	234	233.67	208.10	29	222	8.7	23.6	0	0.01	0.00	0.01	0.02
	73	198	197.25	187.13	45	450	10.4	25.8	0	0.02	0.00	0.02	0.04
	74	220	219.50	194.85	35	339	8.3	22.9	0	0.02	0.00	0.01	0.03
	75	182	181.50	177.35	41	358	9.4	20.8	0	0.02	0.00	0.02	0.04
	76	238	238.00	201.75	25	195	8.3	22.9	0	0.01	0.00	0.01	0.02
	77	250	249.50	219.49	30	210	8.5	20.5	0	0.01	0.00	0.01	0.02
	78	203	203.00	187.13	39	304	9.8	22.7	0	0.02	0.00	0.01	0.03
	79	225	224.75	207.30	39	326	9.8	24.8	0	0.02	0.00	0.01	0.03
	80	201	200.67	193.34	44	378	9.7	19.9	0	0.02	0.00	0.01	0.03
	81	243	242.50	208.78	28	217	8.9	25.5	0	0.01	0.00	0.01	0.02
	82	239	238.17	209.90	37	303	9.3	23.6	0	0.02	0.00	0.01	0.03
	83	253	253.00	206.56	28	206	9.1	24.0	0	0.01	0.00	0.01	0.02
	84	200	199.25	190.79	41	338	9.4	20.4	0	0.02	0.00	0.01	0.03
	85	231	230.50	200.45	33	230	8.6	20.1	0	0.01	0.00	0.01	0.02
	86	216	215.50	195.92	37	298	9.5	23.2	0	0.02	0.00	0.01	0.03
	87	250	249.50	214.54	27	205	8.7	23.9	0	0.01	0.00	0.01	0.02

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	88	207	206.50	197.11	35	325	8.5	21.8	0	0.02	0.00	0.01	0.03
	89	267	267.00	221.46	28	289	8.3	28.3	0	0.01	0.00	0.01	0.02
	90	234	233.50	212.69	28	206	7.9	18.6	0	0.01	0.00	0.01	0.02
	91	169	168.52	166.68	46	473	10.0	23.6	0	0.02	0.01	0.04	0.07
	92	240	240.00	207.59	25	216	7.3	20.8	0	0.01	0.00	0.01	0.02
	93	241	241.00	205.65	30	228	7.4	15.7	0	0.02	0.00	0.01	0.03
	94	228	227.25	208.84	51	375	10.7	19.0	0	0.02	0.00	0.01	0.03
	95	194	193.50	184.29	45	410	10.0	22.0	0	0.02	0.00	0.01	0.03
	96	202	201.67	190.16	44	406	10.3	25.6	0	0.02	0.00	0.01	0.03
	97	192	191.42	186.64	46	476	10.6	26.9	0	0.02	0.01	0.01	0.04
	98	206	206.00	194.62	45	437	10.0	23.8	0	0.02	0.00	0.01	0.03
	99	253	253.00	215.78	34	279	10.4	29.3	0	0.01	0.00	0.01	0.02
	100	226	225.50	206.11	38	295	8.9	18.3	0	0.02	0.00	0.01	0.03
1	1	2,103	2,102.50	1,955.65	30	229	9.1	23.8	0	0.01	0.00	0.01	0.02
	2	2,520	2,520.00	2,173.19	25	165	7.0	15.5	0	0.02	0.00	0.00	0.02
	3	1,970	1,969.50	1,897.34	42	349	9.7	21.1	0	0.02	0.00	0.01	0.03
	4	2,106	2,105.50	1,956.29	41	410	9.5	25.0	0	0.02	0.00	0.01	0.03
	5	2,369	2,368.50	2,028.14	41	291	9.3	15.9	0	0.02	0.00	0.01	0.03
	6	2,185	2,185.00	1,990.52	30	255	7.2	16.7	0	0.02	0.00	0.01	0.03
	7	2,046	2,046.00	1,889.70	42	326	9.7	20.5	0	0.02	0.00	0.01	0.03
	8	2,214	2,213.50	2,051.78	29	220	10.4	28.8	0	0.01	0.00	0.01	0.02
	9	2,260	2,259.33	2,048.32	27	212	7.3	18.7	0	0.01	0.00	0.01	0.02
	10	2,165	2,165.00	1,991.20	30	240	8.7	22.8	0	0.01	0.00	0.01	0.02
	11	2,024	2,024.00	1,907.55	53	418	11.1	20.2	0	0.02	0.00	0.01	0.03
	12	2,087	2,086.67	1,982.11	43	377	10.2	23.1	0	0.02	0.00	0.01	0.03
	13	2,054	2,053.83	1,943.80	38	286	9.6	21.5	0	0.02	0.00	0.01	0.03
	14	1,773	1,772.93	1,707.02	54	461	11.2	19.9	0	0.03	0.01	0.01	0.05
	15	2,030	2,029.07	1,886.46	28	231	8.3	23.0	0	0.01	0.00	0.01	0.02
	16	2,641	2,641.00	2,178.89	37	281	8.7	19.3	0	0.02	0.00	0.01	0.03
	17	2,072	2,071.75	1,953.18	47	398	10.3	21.0	0	0.02	0.00	0.02	0.04
	18	2,163	2,163.00	2,005.17	20	160	6.7	19.4	0	0.02	0.00	0.00	0.02
	19	2,757	2,757.00	2,237.30	33	267	9.7	26.9	0	0.01	0.00	0.01	0.02
	20	2,275	2,275.00	2,033.40	32	272	8.8	24.4	0	0.01	0.00	0.01	0.02
	21	1,905	1,904.25	1,836.04	39	308	9.3	19.5	0	0.02	0.00	0.01	0.03
	22	2,574	2,573.50	2,215.68	20	136	7.7	21.4	0	0.02	0.00	0.00	0.02
	23	1,948	1,947.25	1,882.80	51	430	10.7	20.5	0	0.03	0.00	0.01	0.04
	24	2,128	2,127.67	2,008.70	39	288	10.0	21.6	0	0.02	0.00	0.01	0.03
	25	2,451	2,451.00	2,123.61	31	254	9.8	28.4	0	0.01	0.00	0.01	0.02
	26	2,747	2,746.50	2,206.36	23	152	8.4	21.9	0	0.02	0.00	0.00	0.02
	27	2,098	2,097.11	1,963.49	25	206	7.6	20.6	0	0.01	0.00	0.01	0.02
	28	1,949	1,948.25	1,822.79	24	199	7.0	19.5	0	0.01	0.00	0.01	0.02
	29	2,492	2,492.00	2,114.11	29	217	8.2	20.7	0	0.01	0.00	0.01	0.02
	30	2,155	2,155.00	1,940.35	34	252	9.6	22.9	0	0.01	0.00	0.01	0.02
	31	1,894	1,893.17	1,848.34	36	282	9.4	22.0	0	0.02	0.00	0.01	0.03
	32	2,463	2,463.00	2,054.80	40	286	9.6	20.8	0	0.01	0.00	0.01	0.02
	33	2,360	2,360.00	2,095.41	40	311	9.0	18.8	0	0.02	0.00	0.01	0.03
	34	2,269	2,268.50	2,078.40	30	250	8.7	24.2	0	0.01	0.00	0.01	0.02
	35	2,434	2,433.50	2,091.94	28	226	6.8	16.4	0	0.01	0.00	0.01	0.02
	36	2,318	2,317.50	2,037.46	30	259	8.3	23.8	0	0.01	0.00	0.01	0.02
	37	2,308	2,307.50	2,064.82	34	260	8.8	20.7	0	0.01	0.00	0.01	0.02
	38	2,446	2,446.00	2,113.51	21	148	8.8	24.6	0	0.02	0.00	0.00	0.02
	39	2,838	2,838.00	2,148.51	35	252	9.7	22.7	0	0.01	0.00	0.01	0.02
	40	2,173	2,172.83	2,031.78	36	334	8.8	23.1	0	0.02	0.00	0.01	0.03
	41	2,090	2,089.50	1,911.91	41	328	10.7	24.2	0	0.02	0.00	0.01	0.03
	42	1,973	1,973.00	1,888.33	31	225	7.6	16.5	0	0.01	0.00	0.01	0.02
	43	2,181	2,181.00	1,968.07	46	386	10.9	25.0	0	0.02	0.00	0.01	0.03
	44	2,247	2,246.50	2,060.07	32	246	8.7	21.1	0	0.01	0.00	0.01	0.02
	45	2,359	2,358.50	2,121.73	23	152	7.1	17.5	0	0.01	0.00	0.01	0.02
	46	1,766	1,765.73	1,735.94	40	380	9.9	26.2	0	0.02	0.00	0.02	0.04
	47	2,318	2,317.50	2,046.87	38	287	9.2	19.5	0	0.02	0.00	0.01	0.03
	48	2,824	2,824.00	2,295.47	29	223	8.2	20.8	0	0.01	0.00	0.01	0.02
	49	2,069	2,068.25	1,945.70	34	275	8.6	20.7	0	0.01	0.00	0.01	0.02
	50	2,497	2,497.00	2,113.20	26	206	7.1	17.4	0	0.01	0.00	0.01	0.02
	51	2,007	2,007.00	1,919.44	35	273	8.4	17.2	0	0.02	0.00	0.01	0.03

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type	test	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	52	2,878	2,877.50	2,319.47	24	203	7.6	24.0	0	0.01	0.00	0.01	0.02
	53	2,062	2,062.00	1,907.94	35	335	8.8	26.0	0	0.02	0.00	0.01	0.03
	54	2,298	2,297.62	2,093.40	26	189	7.2	17.8	0	0.01	0.00	0.01	0.02
	55	2,822	2,822.00	2,283.24	36	254	9.0	18.0	0	0.01	0.00	0.01	0.02
	56	2,496	2,496.00	2,105.76	38	286	9.3	20.3	0	0.02	0.00	0.01	0.03
	57	2,412	2,412.00	2,144.09	31	197	9.1	20.1	0	0.01	0.00	0.01	0.02
	58	2,268	2,267.50	2,038.12	27	193	8.8	23.5	0	0.01	0.00	0.01	0.02
	59	2,386	2,386.00	2,117.66	31	232	8.4	20.8	0	0.01	0.00	0.01	0.02
	60	2,361	2,361.00	2,128.67	30	217	9.0	21.5	0	0.01	0.00	0.01	0.02
	61	1,831	1,830.48	1,799.98	46	322	10.2	18.1	0	0.02	0.00	0.02	0.04
	62	2,627	2,627.00	2,078.40	26	193	7.6	20.1	0	0.02	0.00	0.00	0.02
	63	2,334	2,334.00	2,063.59	39	307	8.6	17.4	0	0.02	0.00	0.01	0.03
	64	2,275	2,274.67	2,045.30	31	225	7.9	17.4	0	0.01	0.00	0.01	0.02
	65	2,026	2,026.00	1,878.78	34	284	9.1	24.2	0	0.01	0.00	0.01	0.02
	66	2,001	2,000.21	1,937.43	40	330	9.8	21.9	0	0.02	0.00	0.01	0.03
	67	2,068	2,067.94	1,942.35	49	401	10.8	21.6	0	0.02	0.00	0.02	0.04
	68	2,202	2,201.50	1,955.09	30	256	8.2	22.2	0	0.01	0.00	0.01	0.02
	69	2,390	2,389.50	2,098.72	30	231	9.5	26.1	0	0.01	0.00	0.01	0.02
	70	1,949	1,948.40	1,906.11	31	236	8.7	21.2	0	0.02	0.00	0.01	0.03
	71	2,322	2,322.00	2,087.87	33	269	9.5	25.1	0	0.01	0.00	0.01	0.02
	72	2,346	2,345.67	2,083.81	32	229	9.5	24.2	0	0.01	0.00	0.01	0.02
	73	1,983	1,982.62	1,875.92	47	455	10.8	25.7	0	0.02	0.01	0.00	0.03
	74	2,197	2,196.50	1,949.07	37	345	8.7	22.9	0	0.02	0.00	0.01	0.03
	75	1,803	1,802.10	1,763.08	41	358	9.4	20.8	0	0.02	0.00	0.01	0.03
	76	2,385	2,385.00	2,017.29	25	196	8.3	23.0	0	0.01	0.00	0.01	0.02
	77	2,464	2,463.50	2,179.07	30	208	8.5	20.2	0	0.01	0.00	0.01	0.02
	78	2,017	2,017.00	1,862.90	39	305	9.8	22.8	0	0.02	0.00	0.01	0.03
	79	2,231	2,230.50	2,063.32	38	330	9.5	24.4	0	0.02	0.00	0.01	0.03
	80	2,006	2,005.50	1,929.27	44	376	9.6	19.4	0	0.02	0.00	0.01	0.03
	81	2,426	2,425.50	2,087.78	28	218	8.9	25.6	0	0.01	0.00	0.01	0.02
	82	2,367	2,366.33	2,090.31	37	307	9.3	23.5	0	0.02	0.00	0.01	0.03
	83	2,539	2,539.00	2,063.74	28	207	9.1	24.1	0	0.02	0.00	0.00	0.02
	84	1,995	1,994.50	1,909.34	39	331	8.9	19.9	0	0.02	0.00	0.01	0.03
	85	2,289	2,288.50	1,998.17	33	230	8.6	20.1	0	0.01	0.00	0.01	0.02
	86	2,128	2,128.00	1,945.20	38	301	9.7	23.4	0	0.02	0.00	0.01	0.03
	87	2,489	2,489.00	2,140.44	27	206	8.7	24.0	0	0.01	0.00	0.01	0.02
	88	2,047	2,047.00	1,960.72	35	324	8.4	21.1	0	0.02	0.00	0.01	0.03
	89	2,645	2,645.00	2,193.59	32	301	9.3	28.2	0	0.01	0.00	0.01	0.02
	90	2,343	2,342.50	2,129.43	29	208	8.1	18.4	0	0.01	0.00	0.01	0.02
	91	1,688	1,687.10	1,668.68	46	473	10.0	23.6	0	0.02	0.01	0.02	0.05
	92	2,387	2,387.00	2,068.75	25	216	7.3	20.8	0	0.01	0.00	0.01	0.02
	93	2,407	2,407.00	2,054.28	30	227	7.2	15.0	0	0.02	0.00	0.01	0.03
	94	2,254	2,253.50	2,075.36	49	372	10.3	18.8	0	0.02	0.00	0.01	0.03
	95	1,921	1,921.00	1,828.99	45	416	10.0	22.0	0	0.02	0.00	0.01	0.03
	96	2,029	2,028.83	1,907.40	45	420	10.4	26.0	0	0.02	0.00	0.01	0.03
	97	1,924	1,923.08	1,870.31	46	475	10.6	26.7	0	0.02	0.01	0.01	0.04
	98	2,053	2,053.00	1,942.21	45	441	10.0	23.9	0	0.02	0.00	0.02	0.04
	99	2,543	2,543.00	2,163.87	38	295	11.3	29.5	0	0.01	0.00	0.01	0.02
	100	2,250	2,249.50	2,061.48	38	296	8.9	18.4	0	0.02	0.00	0.01	0.03

Table A.4: Scholl results.

class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
bin1	N1C1W1.A	25	24.75	24.34	84	981	91.3	98.6	0	0.02	0.01	0.04	0.07
	N1C1W1.B	31	30.50	27.82	74	565	81.3	84.6	0	0.02	0.01	0.02	0.05
	N1C1W1.C	20	20.00	19.84	90	1,074	91.8	93.6	0	0.02	0.01	0.04	0.07
	N1C1W1.D	28	27.50	25.31	94	788	93.1	98.5	0	0.02	0.01	0.03	0.06
	N1C1W1.E	26	26.00	24.43	92	906	92.0	94.1	0	0.02	0.01	0.03	0.06
	N1C1W1.F	27	26.50	25.43	84	790	87.5	93.4	0	0.02	0.01	0.03	0.06
	N1C1W1.G	25	25.00	24.44	92	1,078	93.9	98.8	0	0.02	0.01	0.04	0.07
	N1C1W1.H	31	31.00	27.83	86	740	89.6	98.1	0	0.02	0.01	0.02	0.05
	N1C1W1.I	25	24.04	23.45	93	960	92.1	98.9	0	0.02	0.01	0.04	0.07
	N1C1W1.J	26	25.60	24.75	92	1,031	92.9	99.0	0	0.02	0.01	0.04	0.07

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N1C1W1.K	26	25.33	23.97	84	917	86.6	86.9	0	0.02	0.01	0.04	0.07
	N1C1W1.L	33	32.50	28.96	61	469	70.9	90.4	0	0.02	0.00	0.02	0.04
	N1C1W1.M	30	29.50	27.12	96	904	94.1	98.9	0	0.02	0.01	0.03	0.06
	N1C1W1.N	25	24.89	24.21	87	838	90.6	98.2	0	0.02	0.01	0.05	0.08
	N1C1W1.O	32	32.00	29.41	74	598	82.2	94.9	0	0.02	0.01	0.02	0.05
	N1C1W1.P	26	25.50	23.76	78	646	83.0	95.1	0	0.01	0.01	0.03	0.05
	N1C1W1.Q	28	28.00	25.63	70	632	78.7	84.4	0	0.02	0.01	0.01	0.04
	N1C1W1.R	25	24.30	23.84	84	852	90.3	98.4	0	0.02	0.01	0.04	0.07
	N1C1W1.S	28	28.00	26.75	93	960	93.0	99.1	0	0.01	0.01	0.04	0.06
	N1C1W1.T	28	28.00	26.37	77	553	88.5	97.5	0	0.01	0.01	0.02	0.04
	N1C1W2.A	29	28.17	26.04	31	254	41.9	59.3	0	0.01	0.00	0.01	0.02
	N1C1W2.B	30	29.75	28.26	30	239	42.9	59.3	0	0.01	0.00	0.01	0.02
	N1C1W2.C	33	32.50	30.33	30	200	41.7	52.6	0	0.01	0.00	0.01	0.02
	N1C1W2.D	31	30.50	28.39	30	190	42.9	50.1	0	0.01	0.00	0.01	0.02
	N1C1W2.E	36	35.25	31.91	21	126	32.8	49.0	0	0.01	0.00	0.01	0.02
	N1C1W2.F	30	29.50	27.91	38	294	50.0	58.7	0	0.01	0.00	0.01	0.02
	N1C1W2.G	30	29.60	27.99	33	277	44.6	63.1	0	0.01	0.00	0.01	0.02
	N1C1W2.H	33	33.00	30.91	25	176	35.7	51.9	0	0.01	0.00	0.01	0.02
	N1C1W2.I	35	35.00	31.53	24	180	37.5	60.4	0	0.01	0.00	0.01	0.02
	N1C1W2.J	34	33.33	30.41	19	127	28.4	37.9	0	0.02	0.00	0.00	0.02
	N1C1W2.K	35	34.50	30.71	26	170	40.0	53.0	0	0.02	0.00	0.00	0.02
	N1C1W2.L	31	31.00	29.06	27	195	39.1	51.9	0	0.01	0.00	0.01	0.02
	N1C1W2.M	30	30.00	27.91	30	242	44.1	63.0	0	0.01	0.00	0.01	0.02
	N1C1W2.N	33	32.50	28.94	27	209	39.1	60.8	0	0.01	0.00	0.01	0.02
	N1C1W2.O	29	28.40	27.43	31	257	42.5	58.7	0	0.02	0.00	0.01	0.03
	N1C1W2.P	33	32.67	29.98	27	226	38.6	57.8	0	0.01	0.00	0.01	0.02
	N1C1W2.Q	36	36.00	31.64	21	137	34.4	53.1	0	0.02	0.00	0.00	0.02
	N1C1W2.R	34	34.00	30.32	19	137	29.2	48.8	0	0.01	0.00	0.01	0.02
	N1C1W2.S	37	37.00	32.79	21	146	31.8	54.3	0	0.02	0.00	0.00	0.02
	N1C1W2.T	38	37.50	32.68	22	144	35.5	56.7	0	0.02	0.00	0.00	0.02
	N1C1W4.A	35	34.25	31.13	14	77	24.6	35.8	0	0.01	0.00	0.00	0.01
	N1C1W4.B	40	40.00	33.57	7	49	15.2	38.6	0	0.01	0.00	0.00	0.01
	N1C1W4.C	36	36.00	30.66	10	61	17.9	28.8	0	0.01	0.00	0.00	0.01
	N1C1W4.D	38	37.50	33.13	12	70	21.8	33.7	0	0.01	0.00	0.00	0.01
	N1C1W4.E	38	38.00	32.50	13	69	24.5	41.3	0	0.01	0.00	0.00	0.01
	N1C1W4.F	32	32.00	29.68	15	81	25.4	33.6	0	0.01	0.00	0.00	0.01
	N1C1W4.G	37	37.00	33.69	16	91	28.6	41.4	0	0.01	0.00	0.00	0.01
	N1C1W4.H	40	40.00	34.78	15	80	26.3	36.9	0	0.01	0.00	0.00	0.01
	N1C1W4.I	35	35.00	30.98	14	69	25.0	34.0	0	0.01	0.00	0.00	0.01
	N1C1W4.J	37	37.00	31.96	11	62	20.0	32.0	0	0.01	0.00	0.00	0.01
	N1C1W4.K	41	41.00	33.96	10	59	20.0	40.4	0	0.01	0.00	0.00	0.01
	N1C1W4.L	35	35.00	32.17	17	87	28.8	35.1	0	0.01	0.00	0.00	0.01
	N1C1W4.M	41	40.50	33.00	10	57	19.6	37.3	0	0.01	0.00	0.00	0.01
	N1C1W4.N	39	38.50	33.85	13	70	24.1	38.0	0	0.01	0.00	0.00	0.01
	N1C1W4.O	34	33.50	29.50	20	115	32.3	38.0	0	0.02	0.00	0.00	0.02
	N1C1W4.P	38	37.33	32.85	10	61	18.2	33.3	0	0.01	0.00	0.00	0.01
	N1C1W4.Q	34	33.50	30.56	18	90	29.0	31.9	0	0.01	0.00	0.00	0.01
	N1C1W4.R	38	37.50	33.64	12	73	21.8	37.2	0	0.01	0.00	0.00	0.01
	N1C1W4.S	36	35.50	32.23	16	91	26.7	37.8	0	0.01	0.00	0.00	0.01
	N1C1W4.T	42	41.50	32.33	12	65	23.1	40.4	0	0.01	0.00	0.00	0.01
	N1C2W1.A	21	21.00	19.45	113	1,482	94.2	95.9	0	0.02	0.02	0.03	0.07
	N1C2W1.B	26	25.50	22.96	103	907	89.6	91.2	0	0.02	0.01	0.03	0.06
	N1C2W1.C	23	22.25	21.62	104	954	86.7	97.1	0	0.01	0.01	0.06	0.08
	N1C2W1.D	21	21.00	20.52	116	1,378	95.1	99.3	0	0.02	0.02	0.05	0.09
	N1C2W1.E	17	17.00	16.63	115	1,513	95.8	99.5	0	0.02	0.02	0.05	0.09
	N1C2W1.F	22	21.50	21.15	114	1,220	94.2	99.2	0	0.02	0.02	0.06	0.10
	N1C2W1.G	21	20.12	19.90	111	1,222	92.5	98.9	0	0.02	0.02	0.05	0.09
	N1C2W1.H	23	22.75	21.86	115	1,056	95.8	99.2	0	0.02	0.01	0.06	0.09
	N1C2W1.I	27	27.00	24.33	90	681	85.7	96.9	0	0.02	0.01	0.02	0.05
	N1C2W1.J	27	27.00	24.04	108	915	89.3	97.9	0	0.02	0.01	0.03	0.06
	N1C2W1.K	24	23.50	22.47	81	807	75.7	81.6	0	0.02	0.01	0.03	0.06
	N1C2W1.L	25	24.67	22.89	106	1,075	86.9	97.3	0	0.02	0.01	0.05	0.08
	N1C2W1.M	26	26.00	23.13	106	822	93.8	99.2	0	0.01	0.01	0.03	0.05
	N1C2W1.N	21	20.02	19.93	97	986	84.3	86.4	0	0.02	0.02	0.04	0.08

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N1C2W1.O	15	14.71	14.71	119	1,625	97.5	99.8	0	0.02	0.03	0.10	0.15
	N1C2W1.P	21	20.50	20.16	114	1,484	94.2	99.3	0	0.02	0.02	0.06	0.10
	N1C2W1.Q	24	24.00	21.31	105	960	91.3	89.7	0	0.02	0.01	0.04	0.07
	N1C2W1.R	23	23.00	21.35	115	1,245	96.6	99.4	0	0.03	0.01	0.03	0.07
	N1C2W1.S	22	21.20	20.99	96	1,025	87.3	96.1	0	0.02	0.01	0.05	0.08
	N1C2W1.T	22	22.00	21.74	89	841	84.0	95.9	0	0.02	0.01	0.03	0.06
	N1C2W2.A	24	23.85	23.43	46	392	51.1	62.9	0	0.01	0.00	0.02	0.03
	N1C2W2.B	27	27.00	25.57	43	436	47.8	64.5	0	0.02	0.00	0.01	0.03
	N1C2W2.C	29	29.00	26.29	36	313	41.9	60.1	0	0.01	0.00	0.01	0.02
	N1C2W2.D	24	23.73	23.27	50	408	54.9	64.6	0	0.01	0.00	0.02	0.03
	N1C2W2.E	33	32.50	28.07	37	308	43.0	64.2	0	0.01	0.00	0.01	0.02
	N1C2W2.F	26	25.38	24.24	41	342	47.1	61.1	0	0.02	0.00	0.01	0.03
	N1C2W2.G	29	29.00	26.37	36	299	43.4	59.1	0	0.01	0.00	0.01	0.02
	N1C2W2.H	23	22.92	22.62	44	443	49.4	65.4	0	0.01	0.01	0.01	0.03
	N1C2W2.I	25	25.00	24.38	52	475	55.9	63.9	0	0.01	0.00	0.02	0.03
	N1C2W2.J	25	24.71	23.60	50	465	54.9	66.7	0	0.02	0.01	0.01	0.04
	N1C2W2.K	29	29.00	25.17	30	234	37.0	52.8	0	0.01	0.00	0.01	0.02
	N1C2W2.L	30	30.00	25.57	39	307	43.3	50.0	0	0.01	0.00	0.01	0.02
	N1C2W2.M	30	30.00	27.27	36	312	43.4	61.1	0	0.02	0.00	0.01	0.03
	N1C2W2.N	26	26.00	24.87	35	307	42.7	59.5	0	0.01	0.00	0.01	0.02
	N1C2W2.O	29	29.00	25.80	52	402	56.5	62.1	0	0.02	0.00	0.01	0.03
	N1C2W2.P	23	22.67	22.29	56	554	60.2	72.4	0	0.02	0.01	0.01	0.04
	N1C2W2.Q	30	30.00	26.00	40	348	47.1	63.5	0	0.02	0.00	0.01	0.03
	N1C2W2.R	25	24.60	24.19	47	463	51.6	66.9	0	0.01	0.00	0.02	0.03
	N1C2W2.S	24	23.55	23.07	48	391	52.2	55.3	0	0.01	0.00	0.02	0.03
	N1C2W2.T	26	25.60	24.47	44	433	49.4	65.1	0	0.01	0.00	0.02	0.03
	N1C2W4.A	29	29.00	26.52	24	169	30.8	47.6	0	0.01	0.00	0.01	0.02
	N1C2W4.B	32	31.50	27.18	24	158	31.6	42.4	0	0.02	0.00	0.00	0.02
	N1C2W4.C	30	29.50	26.73	24	166	30.8	40.7	0	0.01	0.00	0.01	0.02
	N1C2W4.D	28	27.67	25.23	26	158	33.8	37.7	0	0.01	0.00	0.01	0.02
	N1C2W4.E	30	30.00	27.28	25	172	33.3	47.8	0	0.01	0.00	0.01	0.02
	N1C2W4.F	32	32.00	27.24	17	97	25.0	35.7	0	0.01	0.00	0.00	0.01
	N1C2W4.G	30	29.33	26.53	23	148	29.9	37.0	0	0.01	0.00	0.01	0.02
	N1C2W4.H	30	30.00	27.05	22	152	28.2	35.4	0	0.01	0.00	0.01	0.02
	N1C2W4.I	35	34.50	27.70	22	144	28.9	42.7	0	0.02	0.00	0.00	0.02
	N1C2W4.J	30	30.00	25.94	22	137	28.9	35.0	0	0.02	0.00	0.00	0.02
	N1C2W4.K	32	31.50	27.73	18	106	25.4	34.6	0	0.02	0.00	0.00	0.02
	N1C2W4.L	31	30.08	27.16	26	176	33.8	40.2	0	0.01	0.00	0.01	0.02
	N1C2W4.M	31	30.50	27.46	24	168	32.4	45.0	0	0.01	0.00	0.01	0.02
	N1C2W4.N	32	31.75	27.82	23	162	30.7	40.3	0	0.01	0.00	0.01	0.02
	N1C2W4.O	30	30.00	26.52	15	89	21.4	28.9	0	0.01	0.00	0.00	0.01
	N1C2W4.P	28	28.00	25.58	29	235	36.2	49.5	0	0.01	0.00	0.01	0.02
	N1C2W4.Q	33	33.00	27.87	19	120	26.4	35.3	0	0.01	0.00	0.00	0.01
	N1C2W4.R	35	34.50	29.03	22	144	30.1	41.4	0	0.01	0.00	0.00	0.01
	N1C2W4.S	38	38.00	29.72	17	97	25.0	37.2	0	0.01	0.00	0.00	0.01
	N1C2W4.T	29	28.50	25.23	22	170	28.2	43.3	0	0.01	0.00	0.01	0.02
	N1C3W1.A	16	15.87	15.87	149	1,822	98.7	99.9	0	0.02	0.04	0.14	0.20
	N1C3W1.B	16	15.69	15.69	146	1,799	97.3	99.8	0	0.03	0.04	0.17	0.24
	N1C3W1.C	17	16.94	16.94	128	1,652	89.5	94.2	0	0.02	0.04	0.07	0.13
	N1C3W1.D	19	18.22	18.22	142	1,628	94.0	99.2	0	0.03	0.03	0.05	0.11
	N1C3W1.E	16	15.07	15.07	147	1,990	98.7	99.9	0	0.03	0.05	0.19	0.27
	N1C3W1.F	20	19.57	19.43	137	1,445	93.8	99.0	0	0.02	0.02	0.08	0.12
	N1C3W1.G	15	14.71	14.71	146	2,020	98.6	99.9	0	0.03	0.05	0.09	0.17
	N1C3W1.H	19	18.62	18.60	116	1,257	82.3	89.1	0	0.02	0.02	0.05	0.09
	N1C3W1.I	17	16.48	16.48	136	1,871	92.5	99.1	0	0.03	0.04	0.14	0.21
	N1C3W1.J	16	15.61	15.61	152	1,902	100.0	100.0	0	0.03	0.04	0.14	0.21
	N1C3W1.K	17	16.26	16.26	143	1,967	96.6	99.7	0	0.03	0.04	0.10	0.17
	N1C3W1.L	17	16.31	16.31	147	1,556	98.7	99.9	0	0.02	0.04	0.21	0.27
	N1C3W1.M	17	16.54	16.54	135	1,865	93.1	98.6	0	0.02	0.04	0.25	0.31
	N1C3W1.N	20	19.74	19.65	99	1,173	76.7	89.3	0	0.02	0.02	0.04	0.08
	N1C3W1.O	16	15.68	15.68	152	1,534	100.0	100.0	0	0.02	0.03	0.14	0.19
	N1C3W1.P	19	18.04	18.03	144	1,859	96.6	99.8	0	0.03	0.03	0.06	0.12
	N1C3W1.Q	20	19.72	19.65	123	1,388	84.2	90.3	0	0.02	0.02	0.06	0.10
	N1C3W1.R	21	20.06	19.95	122	1,284	85.9	92.4	0	0.02	0.02	0.06	0.10

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N1C3W1.S	16	15.27	15.27	146	2,008	97.3	99.7	0	0.03	0.05	0.16	0.24
	N1C3W1.T	18	17.52	17.52	123	1,383	86.0	96.6	0	0.02	0.03	0.04	0.09
	N1C3W2.A	19	18.46	18.45	84	928	68.9	74.1	0	0.02	0.02	0.03	0.07
	N1C3W2.B	20	19.65	19.51	86	883	69.9	75.2	0	0.02	0.01	0.08	0.11
	N1C3W2.C	22	22.00	21.05	64	588	57.1	68.4	0	0.01	0.01	0.02	0.04
	N1C3W2.D	20	19.65	19.60	88	936	71.5	78.2	0	0.02	0.01	0.04	0.07
	N1C3W2.E	21	20.59	20.52	83	987	68.0	78.5	0	0.02	0.01	0.09	0.12
	N1C3W2.F	23	22.12	21.56	54	572	49.5	66.1	0	0.02	0.01	0.01	0.04
	N1C3W2.G	23	22.08	21.84	69	665	58.5	70.1	0	0.02	0.01	0.02	0.05
	N1C3W2.H	23	22.00	21.86	63	694	55.3	70.2	0	0.02	0.01	0.02	0.05
	N1C3W2.I	19	18.53	18.51	65	775	55.1	65.4	0	0.01	0.01	0.04	0.06
	N1C3W2.J	22	21.24	20.95	72	812	60.5	70.4	0	0.02	0.01	0.03	0.06
	N1C3W2.K	21	20.52	20.37	68	650	59.1	67.8	0	0.01	0.01	0.03	0.05
	N1C3W2.L	21	20.38	20.29	74	834	62.2	74.8	0	0.02	0.01	0.04	0.07
	N1C3W2.M	21	20.56	20.47	72	860	61.5	73.3	0	0.02	0.01	0.04	0.07
	N1C3W2.N	22	21.44	21.23	74	792	63.8	76.2	0	0.02	0.01	0.05	0.08
	N1C3W2.O	21	20.04	20.01	73	821	61.9	73.9	0	0.02	0.01	0.04	0.07
	N1C3W2.P	18	17.63	17.61	89	989	71.8	77.0	0	0.02	0.02	0.10	0.14
	N1C3W2.Q	19	18.53	18.51	80	938	68.4	75.3	0	0.02	0.01	0.05	0.08
	N1C3W2.R	19	18.82	18.81	89	1,089	72.4	78.7	0	0.02	0.02	0.09	0.13
	N1C3W2.S	21	20.43	20.32	86	933	70.5	77.2	0	0.02	0.01	0.04	0.07
	N1C3W2.T	22	21.05	20.93	71	761	61.2	71.3	0	0.02	0.01	0.03	0.06
	N1C3W4.A	21	20.57	20.38	52	586	47.3	57.3	0	0.01	0.01	0.06	0.08
	N1C3W4.B	22	21.75	21.45	47	412	45.6	52.6	0	0.01	0.00	0.02	0.03
	N1C3W4.C	24	24.00	22.91	39	324	39.4	48.7	0	0.02	0.00	0.01	0.03
	N1C3W4.D	21	20.97	20.54	49	505	45.4	56.4	0	0.02	0.01	0.01	0.04
	N1C3W4.E	23	22.50	21.82	50	408	46.7	50.6	0	0.01	0.00	0.02	0.03
	N1C3W4.F	21	20.85	20.79	48	471	45.3	57.6	0	0.02	0.01	0.02	0.05
	N1C3W4.G	23	22.88	22.25	44	422	41.9	52.9	0	0.01	0.00	0.02	0.03
	N1C3W4.H	23	22.67	22.29	47	524	44.8	58.2	0	0.02	0.01	0.01	0.04
	N1C3W4.I	23	22.17	21.78	39	377	37.1	54.6	0	0.01	0.00	0.02	0.03
	N1C3W4.J	22	21.33	21.12	49	529	44.5	57.8	0	0.02	0.01	0.01	0.04
	N1C3W4.K	24	23.20	22.75	40	391	38.8	52.5	0	0.01	0.00	0.02	0.03
	N1C3W4.L	20	19.83	19.77	54	551	48.2	53.7	0	0.01	0.01	0.02	0.04
	N1C3W4.M	21	20.33	20.05	47	474	43.1	51.6	0	0.02	0.01	0.01	0.04
	N1C3W4.N	21	20.70	20.52	51	545	46.8	52.5	0	0.02	0.01	0.02	0.05
	N1C3W4.O	22	21.56	20.97	44	357	41.9	48.8	0	0.02	0.00	0.01	0.03
	N1C3W4.P	25	25.00	23.42	35	280	35.7	44.6	0	0.01	0.00	0.01	0.02
	N1C3W4.Q	25	25.00	23.05	41	415	39.8	56.8	0	0.02	0.00	0.01	0.03
	N1C3W4.R	22	21.75	21.32	54	549	49.1	56.1	0	0.02	0.01	0.02	0.05
	N1C3W4.S	22	21.50	21.29	53	523	49.5	58.7	0	0.02	0.01	0.01	0.04
	N1C3W4.T	24	23.94	23.52	44	419	42.3	56.5	0	0.01	0.00	0.02	0.03
	N2C1W1.A	48	47.33	46.72	96	1,889	97.0	99.8	0	0.05	0.02	0.08	0.15
	N2C1W1.B	49	48.50	47.25	98	1,621	97.0	99.8	0	0.03	0.02	0.08	0.13
	N2C1W1.C	46	45.34	45.06	95	1,587	95.0	96.7	0	0.03	0.02	0.04	0.09
	N2C1W1.D	50	49.25	48.10	101	1,937	99.0	99.9	0	0.02	0.03	0.07	0.12
	N2C1W1.E	58	57.50	53.57	96	1,159	96.0	99.6	0	0.02	0.02	0.04	0.08
	N2C1W1.F	50	49.22	48.57	102	1,569	100.0	100.0	0	0.03	0.02	0.04	0.09
	N2C1W1.G	60	59.33	53.98	98	1,478	97.0	99.7	0	0.02	0.02	0.05	0.09
	N2C1W1.H	52	51.25	50.39	91	1,212	92.9	99.0	0	0.02	0.02	0.05	0.09
	N2C1W1.I	62	62.00	57.23	96	1,188	94.1	99.2	0	0.02	0.01	0.03	0.06
	N2C1W1.J	59	59.00	54.54	92	1,230	93.9	99.1	0	0.03	0.01	0.03	0.07
	N2C1W1.K	55	55.00	52.79	96	1,656	94.1	99.6	0	0.02	0.02	0.05	0.09
	N2C1W1.L	55	55.00	51.97	102	1,353	100.0	100.0	0	0.03	0.02	0.02	0.07
	N2C1W1.M	46	45.69	45.48	96	1,357	97.0	99.8	0	0.03	0.02	0.04	0.09
	N2C1W1.N	48	47.50	46.59	101	1,786	99.0	99.9	0	0.03	0.02	0.06	0.11
	N2C1W1.O	48	47.33	46.81	102	1,778	100.0	100.0	0	0.03	0.02	0.08	0.13
	N2C1W1.P	54	54.00	51.65	100	1,491	98.0	99.8	0	0.02	0.02	0.03	0.07
	N2C1W1.Q	46	45.75	45.56	100	1,517	98.0	99.8	0	0.02	0.02	0.05	0.09
	N2C1W1.R	56	56.00	54.12	90	1,334	91.8	95.0	0	0.02	0.01	0.06	0.09
	N2C1W1.S	45	44.57	44.15	99	1,687	98.0	99.8	0	0.03	0.03	0.05	0.11
	N2C1W1.T	52	51.57	50.45	101	1,495	99.0	99.9	0	0.03	0.02	0.05	0.10
	N2C1W2.A	64	63.25	59.68	46	502	57.5	64.5	0	0.02	0.01	0.01	0.04
	N2C1W2.B	61	60.83	56.81	44	455	56.4	61.8	0	0.02	0.01	0.01	0.04

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N2C1W2.C	68	67.50	58.83	40	369	54.1	63.7	0	0.02	0.00	0.01	0.03
	N2C1W2.D	74	74.00	64.80	38	386	50.7	67.6	0	0.02	0.00	0.01	0.03
	N2C1W2.E	65	64.75	61.73	47	476	59.5	68.7	0	0.02	0.00	0.02	0.04
	N2C1W2.F	65	64.33	60.45	37	299	48.7	53.9	0	0.02	0.00	0.01	0.03
	N2C1W2.G	73	72.50	63.14	40	380	52.6	65.1	0	0.02	0.00	0.01	0.03
	N2C1W2.H	70	69.50	61.62	39	345	52.7	60.6	0	0.02	0.00	0.01	0.03
	N2C1W2.I	67	67.00	61.13	45	464	57.0	66.2	0	0.01	0.00	0.02	0.03
	N2C1W2.J	67	67.00	62.02	39	366	52.7	66.1	0	0.02	0.00	0.01	0.03
	N2C1W2.K	72	71.50	63.36	34	298	47.9	63.9	0	0.01	0.00	0.01	0.02
	N2C1W2.L	62	61.33	58.05	50	587	63.3	68.7	0	0.02	0.01	0.01	0.04
	N2C1W2.M	65	64.38	61.01	37	288	50.0	55.0	0	0.01	0.00	0.01	0.02
	N2C1W2.N	64	63.25	60.30	45	464	57.7	66.3	0	0.01	0.01	0.01	0.03
	N2C1W2.O	64	64.00	59.97	44	376	57.1	60.7	0	0.02	0.00	0.01	0.03
	N2C1W2.P	68	68.00	61.61	38	364	51.4	63.3	0	0.02	0.00	0.01	0.03
	N2C1W2.Q	65	64.50	58.55	44	411	58.7	60.5	0	0.02	0.00	0.01	0.03
	N2C1W2.R	67	66.50	62.31	46	490	59.0	63.4	0	0.02	0.00	0.02	0.04
	N2C1W2.S	66	66.00	60.64	43	419	55.8	66.6	0	0.02	0.00	0.01	0.03
	N2C1W2.T	66	66.00	61.08	42	431	55.3	65.8	0	0.02	0.00	0.01	0.03
	N2C1W4.A	73	72.50	64.62	18	110	28.1	31.4	0	0.02	0.00	0.00	0.02
	N2C1W4.B	71	70.33	63.17	20	115	31.2	35.0	0	0.02	0.00	0.00	0.02
	N2C1W4.C	77	77.00	66.21	20	122	31.2	36.6	0	0.02	0.00	0.00	0.02
	N2C1W4.D	82	82.00	68.07	19	113	30.2	37.9	0	0.01	0.00	0.00	0.01
	N2C1W4.E	73	72.25	64.22	21	134	32.8	37.0	0	0.02	0.00	0.00	0.02
	N2C1W4.F	77	77.00	65.64	22	130	33.8	36.6	0	0.02	0.00	0.00	0.02
	N2C1W4.G	71	71.00	62.48	22	127	32.8	30.8	0	0.02	0.00	0.00	0.02
	N2C1W4.H	75	75.00	64.87	20	117	31.2	34.4	0	0.02	0.00	0.00	0.02
	N2C1W4.I	73	72.50	64.12	19	116	29.2	32.2	0	0.02	0.00	0.00	0.02
	N2C1W4.J	74	73.50	64.40	19	121	29.7	35.5	0	0.02	0.00	0.00	0.02
	N2C1W4.K	70	70.00	63.74	22	132	32.8	33.1	0	0.02	0.00	0.00	0.02
	N2C1W4.L	75	74.17	65.03	21	122	32.3	35.7	0	0.02	0.00	0.00	0.02
	N2C1W4.M	72	71.50	64.29	21	126	31.8	31.7	0	0.02	0.00	0.00	0.02
	N2C1W4.N	71	70.50	64.56	23	145	34.8	36.4	0	0.02	0.00	0.00	0.02
	N2C1W4.O	80	80.00	69.41	19	121	29.7	37.3	0	0.02	0.00	0.00	0.02
	N2C1W4.P	67	67.00	59.89	22	121	32.4	30.3	0	0.02	0.00	0.00	0.02
	N2C1W4.Q	75	75.00	65.94	18	105	28.1	31.1	0	0.02	0.00	0.00	0.02
	N2C1W4.R	70	70.00	63.51	18	111	27.3	29.7	0	0.02	0.00	0.00	0.02
	N2C1W4.S	80	79.50	69.16	19	115	30.2	38.5	0	0.02	0.00	0.00	0.02
	N2C1W4.T	70	70.00	63.07	16	94	25.4	28.7	0	0.02	0.00	0.00	0.02
	N2C2W1.A	42	41.50	40.12	122	2,406	100.0	100.0	0	0.04	0.03	0.08	0.15
	N2C2W1.B	50	49.50	45.99	116	1,944	95.9	99.6	0	0.03	0.03	0.09	0.15
	N2C2W1.C	40	39.72	39.72	120	2,614	99.2	100.0	0	0.03	0.06	0.16	0.25
	N2C2W1.D	42	41.59	41.55	121	2,092	99.2	100.0	0	0.03	0.04	0.05	0.12
	N2C2W1.E	40	39.75	39.64	122	2,411	100.0	100.0	0	0.03	0.04	0.14	0.21
	N2C2W1.F	49	49.00	46.44	115	1,882	97.5	99.8	0	0.03	0.03	0.03	0.09
	N2C2W1.G	45	45.00	43.33	122	1,870	100.0	100.0	0	0.03	0.02	0.06	0.11
	N2C2W1.H	46	45.25	43.23	120	2,182	98.4	99.9	0	0.04	0.03	0.07	0.14
	N2C2W1.I	45	44.20	43.23	122	2,173	100.0	100.0	0	0.03	0.03	0.07	0.13
	N2C2W1.J	42	41.18	41.09	120	2,210	99.2	100.0	0	0.03	0.04	0.05	0.12
	N2C2W1.K	41	40.73	40.70	121	2,621	99.2	100.0	0	0.03	0.04	0.28	0.35
	N2C2W1.L	49	49.00	45.92	118	1,935	96.7	99.6	0	0.03	0.03	0.03	0.09
	N2C2W1.M	44	43.50	42.77	114	1,767	95.8	99.4	0	0.03	0.03	0.05	0.11
	N2C2W1.N	43	42.21	42.00	120	1,767	99.2	99.9	0	0.03	0.03	0.06	0.12
	N2C2W1.O	50	50.00	46.02	122	1,875	100.0	100.0	0	0.03	0.02	0.04	0.09
	N2C2W1.P	46	46.00	44.14	118	1,957	97.5	99.8	0	0.03	0.03	0.09	0.15
	N2C2W1.Q	49	49.00	45.02	117	1,755	96.7	99.6	0	0.03	0.02	0.04	0.09
	N2C2W1.R	41	40.57	40.55	121	2,351	99.2	100.0	0	0.03	0.04	0.11	0.18
	N2C2W1.S	43	43.00	42.82	105	1,770	92.1	98.6	0	0.02	0.02	0.05	0.09
	N2C2W1.T	39	38.30	38.28	119	2,231	97.5	99.8	0	0.03	0.05	0.09	0.17
	N2C2W2.A	52	52.00	49.43	58	592	62.4	66.4	0	0.01	0.01	0.02	0.04
	N2C2W2.B	56	55.50	51.73	57	724	61.3	72.8	0	0.02	0.01	0.02	0.05
	N2C2W2.C	53	52.75	50.64	64	860	66.7	77.8	0	0.02	0.01	0.02	0.05
	N2C2W2.D	51	51.00	49.15	67	867	67.7	75.0	0	0.02	0.01	0.03	0.06
	N2C2W2.E	54	53.50	49.84	57	634	60.6	68.0	0	0.01	0.01	0.02	0.04
	N2C2W2.F	48	47.38	46.54	69	944	69.0	73.5	0	0.02	0.01	0.04	0.07

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N2C2W2.G	53	52.80	50.80	70	967	71.4	76.4	0	0.02	0.01	0.04	0.07
	N2C2W2.H	53	53.00	50.70	63	852	66.3	74.3	0	0.02	0.01	0.03	0.06
	N2C2W2.I	49	48.14	47.55	69	924	70.4	75.2	0	0.02	0.01	0.04	0.07
	N2C2W2.J	56	55.50	51.79	63	892	65.6	76.4	0	0.02	0.01	0.03	0.06
	N2C2W2.K	50	49.17	48.14	61	800	63.5	74.9	0	0.02	0.01	0.03	0.06
	N2C2W2.L	52	51.50	49.69	65	873	67.0	74.7	0	0.02	0.01	0.03	0.06
	N2C2W2.M	54	53.50	50.02	71	928	71.7	74.3	0	0.02	0.01	0.04	0.07
	N2C2W2.N	51	51.00	49.22	69	913	69.7	74.0	0	0.02	0.01	0.03	0.06
	N2C2W2.O	50	49.75	48.41	65	917	67.0	75.9	0	0.02	0.01	0.04	0.07
	N2C2W2.P	50	49.17	48.03	77	1,145	75.5	77.4	0	0.03	0.01	0.04	0.08
	N2C2W2.Q	54	54.00	50.83	63	736	64.9	70.6	0	0.02	0.01	0.02	0.05
	N2C2W2.R	51	50.12	48.86	73	1,023	73.7	76.6	0	0.02	0.01	0.04	0.07
	N2C2W2.S	58	58.00	52.62	66	902	67.3	76.6	0	0.02	0.01	0.03	0.06
	N2C2W2.T	56	55.50	51.26	57	673	60.0	67.8	0	0.01	0.01	0.02	0.04
	N2C2W4.A	57	56.25	51.77	37	287	43.5	38.9	0	0.02	0.00	0.01	0.03
	N2C2W4.B	60	59.50	53.74	41	356	47.7	50.2	0	0.02	0.00	0.01	0.03
	N2C2W4.C	65	65.00	55.95	36	297	42.9	44.0	0	0.02	0.00	0.01	0.03
	N2C2W4.D	61	61.00	54.79	40	349	46.0	46.7	0	0.02	0.00	0.01	0.03
	N2C2W4.E	60	60.00	54.93	34	286	40.5	43.8	0	0.01	0.00	0.01	0.02
	N2C2W4.F	57	57.00	53.41	39	319	45.3	42.3	0	0.02	0.00	0.01	0.03
	N2C2W4.G	61	61.00	54.67	38	313	44.7	41.6	0	0.01	0.00	0.01	0.02
	N2C2W4.H	61	61.00	54.91	35	288	41.2	44.1	0	0.01	0.00	0.01	0.02
	N2C2W4.I	58	57.33	53.53	38	334	44.2	45.1	0	0.02	0.00	0.01	0.03
	N2C2W4.J	60	59.83	54.67	35	270	42.2	41.1	0	0.01	0.00	0.01	0.02
	N2C2W4.K	59	58.50	53.53	40	330	47.1	46.7	0	0.02	0.00	0.01	0.03
	N2C2W4.L	57	56.50	52.36	37	298	44.0	45.4	0	0.02	0.00	0.01	0.03
	N2C2W4.M	60	60.00	54.58	34	271	41.0	40.8	0	0.01	0.00	0.01	0.02
	N2C2W4.N	63	62.50	55.17	32	254	39.0	42.1	0	0.01	0.00	0.01	0.02
	N2C2W4.O	62	62.00	54.93	33	246	39.3	36.7	0	0.01	0.00	0.01	0.02
	N2C2W4.P	60	59.33	53.88	40	360	46.0	46.8	0	0.02	0.00	0.01	0.03
	N2C2W4.Q	62	62.00	54.35	38	305	44.7	44.9	0	0.02	0.00	0.01	0.03
	N2C2W4.R	56	55.95	52.69	45	450	49.5	49.2	0	0.02	0.01	0.01	0.04
	N2C2W4.S	55	54.83	51.61	35	275	42.2	45.3	0	0.02	0.00	0.01	0.03
	N2C2W4.T	57	57.00	52.60	40	348	46.0	46.3	0	0.02	0.00	0.01	0.03
	N2C3W1.A	35	34.84	34.84	152	3,339	100.0	100.0	0	0.04	0.08	0.27	0.39
	N2C3W1.B	35	34.01	34.01	152	3,127	100.0	100.0	0	0.04	0.08	0.30	0.42
	N2C3W1.C	35	34.35	34.35	150	3,118	98.7	99.9	0	0.03	0.08	0.31	0.42
	N2C3W1.D	37	36.45	36.45	145	3,100	97.3	99.8	0	0.04	0.08	0.19	0.31
	N2C3W1.E	34	33.70	33.70	137	2,888	92.6	97.1	0	0.03	0.08	0.22	0.33
	N2C3W1.F	35	34.35	34.35	151	3,270	99.3	100.0	0	0.04	0.09	0.31	0.44
	N2C3W1.G	33	32.51	32.51	147	3,617	97.4	99.8	0	0.05	0.09	0.31	0.45
	N2C3W1.H	35	34.89	34.89	151	3,525	99.3	100.0	0	0.04	0.09	1.67	1.80
	N2C3W1.I	34	33.29	33.29	147	3,430	97.4	99.9	0	0.04	0.09	0.34	0.47
	N2C3W1.J	33	32.74	32.74	152	3,229	100.0	100.0	0	0.04	0.08	0.23	0.35
	N2C3W1.K	36	35.84	35.84	152	3,339	100.0	100.0	0	0.04	0.09	0.29	0.42
	N2C3W1.L	35	34.53	34.53	145	2,784	96.0	99.7	0	0.04	0.08	0.23	0.35
	N2C3W1.M	31	30.07	30.07	152	3,647	100.0	100.0	0	0.04	0.09	0.29	0.42
	N2C3W1.N	32	31.45	31.45	152	3,347	100.0	100.0	0	0.04	0.09	0.27	0.40
	N2C3W1.O	35	34.23	34.23	145	3,364	96.7	99.8	0	0.04	0.09	0.29	0.42
	N2C3W1.P	35	34.26	34.26	145	3,341	96.7	99.8	0	0.04	0.09	0.34	0.47
	N2C3W1.Q	34	33.67	33.67	140	2,733	92.1	99.2	0	0.03	0.07	0.21	0.31
	N2C3W1.R	33	32.88	32.88	151	3,375	99.3	100.0	0	0.04	0.09	0.31	0.44
	N2C3W1.S	36	35.25	35.25	133	2,736	93.0	99.2	0	0.03	0.07	0.20	0.30
	N2C3W1.T	35	34.11	34.11	150	3,739	99.3	100.0	0	0.04	0.10	0.28	0.42
	N2C3W2.A	41	40.27	40.23	102	1,866	79.7	84.9	0	0.03	0.03	0.07	0.13
	N2C3W2.B	43	42.03	41.78	96	1,638	75.6	81.1	0	0.02	0.03	0.06	0.11
	N2C3W2.C	41	40.46	40.36	98	1,927	76.6	84.0	0	0.03	0.03	0.07	0.13
	N2C3W2.D	41	40.78	40.73	96	1,616	77.4	84.5	0	0.03	0.03	0.06	0.12
	N2C3W2.E	39	38.79	38.79	86	1,502	71.1	81.2	0	0.02	0.03	0.08	0.13
	N2C3W2.F	39	38.15	38.11	97	1,579	76.4	81.6	0	0.03	0.03	0.06	0.12
	N2C3W2.G	41	40.10	40.07	96	1,593	76.2	82.8	0	0.03	0.03	0.07	0.13
	N2C3W2.H	38	37.11	37.11	92	1,537	74.2	83.9	0	0.03	0.03	0.09	0.15
	N2C3W2.I	44	43.47	43.15	87	1,458	70.2	84.1	0	0.02	0.02	0.08	0.12
	N2C3W2.J	43	42.75	42.57	91	1,415	72.8	77.6	0	0.02	0.02	0.05	0.09

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N2C3W2.K	42	41.46	41.39	95	1,630	74.2	83.4	0	0.03	0.02	0.06	0.11
	N2C3W2.L	41	40.99	40.96	89	1,457	72.4	82.0	0	0.03	0.02	0.13	0.18
	N2C3W2.M	43	42.71	42.59	88	1,476	71.0	82.8	0	0.02	0.02	0.05	0.09
	N2C3W2.N	41	40.56	40.48	100	1,765	77.5	83.5	0	0.03	0.03	0.07	0.13
	N2C3W2.O	45	44.05	43.75	86	1,367	70.5	81.2	0	0.03	0.02	0.09	0.14
	N2C3W2.P	41	40.18	40.16	101	1,888	78.3	83.9	0	0.03	0.03	0.08	0.14
	N2C3W2.Q	41	40.24	40.13	97	1,734	76.4	84.0	0	0.03	0.02	0.08	0.13
	N2C3W2.R	40	39.32	39.06	97	1,561	77.0	81.6	0	0.02	0.02	0.07	0.11
	N2C3W2.S	43	43.00	42.36	97	1,645	76.4	83.6	0	0.02	0.02	0.05	0.09
	N2C3W2.T	43	42.51	42.47	93	1,778	75.0	86.7	0	0.03	0.03	0.08	0.14
	N2C3W4.A	43	42.93	42.70	71	1,008	60.7	62.0	0	0.02	0.01	0.10	0.13
	N2C3W4.B	45	44.70	44.14	69	941	59.0	63.0	0	0.02	0.01	0.04	0.07
	N2C3W4.C	42	41.42	41.18	67	962	57.8	62.5	0	0.02	0.01	0.04	0.07
	N2C3W4.D	44	43.20	42.89	67	946	58.8	65.2	0	0.02	0.01	0.04	0.07
	N2C3W4.E	47	46.50	45.36	63	794	55.8	61.1	0	0.02	0.01	0.03	0.06
	N2C3W4.F	45	44.22	43.81	67	956	58.8	63.6	0	0.02	0.01	0.04	0.07
	N2C3W4.G	44	43.65	43.06	63	796	56.2	59.1	0	0.02	0.01	0.03	0.06
	N2C3W4.H	44	43.50	42.29	69	1,028	59.5	66.1	0	0.02	0.01	0.06	0.09
	N2C3W4.I	44	43.39	43.19	69	1,032	59.0	66.0	0	0.02	0.01	0.07	0.10
	N2C3W4.J	43	42.75	41.89	69	911	60.0	60.6	0	0.02	0.01	0.04	0.07
	N2C3W4.K	47	46.33	45.25	65	888	57.0	65.3	0	0.02	0.01	0.04	0.07
	N2C3W4.L	45	44.58	44.15	68	992	59.6	67.8	0	0.03	0.01	0.06	0.10
	N2C3W4.M	44	43.46	42.87	65	989	57.0	62.2	0	0.02	0.01	0.04	0.07
	N2C3W4.N	45	44.54	43.76	65	927	56.5	60.6	0	0.02	0.01	0.04	0.07
	N2C3W4.O	45	44.62	44.06	66	891	56.9	60.5	0	0.03	0.01	0.04	0.08
	N2C3W4.P	45	44.17	43.65	61	784	54.0	58.9	0	0.02	0.01	0.05	0.08
	N2C3W4.Q	46	46.00	45.31	65	903	58.0	65.5	0	0.02	0.01	0.03	0.06
	N2C3W4.R	42	41.85	41.77	75	1,056	63.6	62.7	0	0.02	0.01	0.10	0.13
	N2C3W4.S	42	41.15	41.08	71	973	60.2	61.6	0	0.03	0.01	0.04	0.08
	N2C3W4.T	46	45.95	44.35	68	926	59.1	62.2	0	0.03	0.01	0.03	0.07
	N3C1W1.A	105	105.00	100.88	100	1,936	98.0	99.8	0	0.03	0.02	0.05	0.10
	N3C1W1.B	114	113.33	108.13	96	2,046	96.0	99.7	0	0.03	0.02	0.06	0.11
	N3C1W1.C	99	98.50	96.67	100	2,409	98.0	99.9	0	0.03	0.03	0.06	0.12
	N3C1W1.D	108	107.50	104.69	99	2,167	98.0	99.9	0	0.03	0.02	0.06	0.11
	N3C1W1.E	98	97.12	96.68	102	2,624	100.0	100.0	0	0.04	0.04	0.07	0.15
	N3C1W1.F	113	113.00	105.91	101	2,424	99.0	100.0	0	0.03	0.03	0.06	0.12
	N3C1W1.G	111	110.50	105.06	102	2,243	100.0	100.0	0	0.03	0.03	0.05	0.11
	N3C1W1.H	104	103.67	101.80	102	2,333	100.0	100.0	0	0.04	0.03	0.05	0.12
	N3C1W1.I	100	100.00	97.63	102	2,282	100.0	100.0	0	0.03	0.03	0.06	0.12
	N3C1W1.J	108	107.50	103.41	98	2,143	97.0	99.9	0	0.03	0.02	0.07	0.12
	N3C1W1.K	102	101.50	99.51	101	2,424	99.0	100.0	0	0.04	0.03	0.07	0.14
	N3C1W1.L	97	96.83	96.03	102	2,276	100.0	100.0	0	0.03	0.03	0.07	0.13
	N3C1W1.M	106	106.00	101.77	102	2,329	100.0	100.0	0	0.03	0.03	0.07	0.13
	N3C1W1.N	93	92.24	92.04	100	2,246	99.0	100.0	0	0.03	0.04	0.07	0.14
	N3C1W1.O	98	98.00	97.24	98	2,306	97.0	99.8	0	0.04	0.03	0.06	0.13
	N3C1W1.P	108	108.00	105.59	96	2,043	97.0	95.7	0	0.03	0.02	0.05	0.10
	N3C1W1.Q	98	97.17	96.00	102	2,317	100.0	100.0	0	0.03	0.04	0.05	0.12
	N3C1W1.R	99	98.33	97.38	102	2,234	100.0	100.0	0	0.03	0.03	0.07	0.13
	N3C1W1.S	100	99.75	98.09	102	2,292	100.0	100.0	0	0.03	0.03	0.09	0.15
	N3C1W1.T	102	102.00	99.94	102	2,449	100.0	100.0	0	0.03	0.03	0.06	0.12
	N3C1W2.A	125	125.00	117.67	56	721	68.3	70.4	0	0.02	0.01	0.02	0.05
	N3C1W2.B	126	126.00	114.81	57	720	70.4	71.4	0	0.02	0.01	0.01	0.04
	N3C1W2.C	125	124.83	118.76	55	705	67.9	71.4	0	0.02	0.01	0.02	0.05
	N3C1W2.D	139	139.00	125.05	57	734	68.7	69.8	0	0.02	0.01	0.01	0.04
	N3C1W2.E	132	131.50	122.38	58	753	70.7	70.8	0	0.02	0.01	0.02	0.05
	N3C1W2.F	123	123.00	116.51	49	596	61.3	69.6	0	0.02	0.01	0.01	0.04
	N3C1W2.G	132	131.33	124.37	56	734	67.5	70.0	0	0.02	0.01	0.02	0.05
	N3C1W2.H	129	129.00	121.96	54	660	66.7	70.5	0	0.02	0.01	0.02	0.05
	N3C1W2.I	126	126.00	118.96	57	724	69.5	70.6	0	0.02	0.01	0.02	0.05
	N3C1W2.J	126	125.30	119.89	52	650	64.2	68.9	0	0.01	0.01	0.02	0.04
	N3C1W2.K	120	119.75	114.52	58	738	70.7	70.2	0	0.02	0.01	0.02	0.05
	N3C1W2.L	136	136.00	126.19	53	648	66.2	68.6	0	0.02	0.01	0.01	0.04
	N3C1W2.M	136	135.50	122.72	56	755	67.5	70.4	0	0.02	0.01	0.02	0.05
	N3C1W2.N	136	136.00	124.77	56	680	69.1	70.9	0	0.01	0.01	0.02	0.04

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N3C1W2.O	127	126.50	119.33	54	683	66.7	69.1	0	0.02	0.01	0.01	0.04
	N3C1W2.P	126	125.50	118.03	56	734	68.3	70.8	0	0.02	0.01	0.01	0.04
	N3C1W2.Q	135	135.00	122.93	55	604	67.9	67.0	0	0.02	0.01	0.01	0.04
	N3C1W2.R	123	122.90	117.92	57	700	69.5	70.1	0	0.02	0.01	0.05	0.08
	N3C1W2.S	130	129.50	119.65	54	679	67.5	74.0	0	0.02	0.01	0.01	0.04
	N3C1W2.T	136	135.50	122.50	59	788	71.1	71.5	0	0.02	0.01	0.02	0.05
	N3C1W4.A	149	149.00	133.76	25	152	36.2	32.3	0	0.02	0.00	0.00	0.02
	N3C1W4.B	149	148.50	130.25	29	181	40.3	33.1	0	0.02	0.00	0.00	0.02
	N3C1W4.C	146	146.00	129.87	26	159	38.2	34.7	0	0.02	0.00	0.00	0.02
	N3C1W4.D	148	148.00	132.03	29	182	40.8	34.1	0	0.02	0.00	0.00	0.02
	N3C1W4.E	142	141.50	126.62	29	174	40.8	33.1	0	0.01	0.00	0.01	0.02
	N3C1W4.F	140	139.50	127.55	29	173	40.3	31.7	0	0.01	0.00	0.01	0.02
	N3C1W4.G	148	147.67	130.43	27	175	38.6	35.4	0	0.01	0.00	0.01	0.02
	N3C1W4.H	141	140.50	127.49	30	183	41.7	32.9	0	0.01	0.00	0.01	0.02
	N3C1W4.I	140	139.89	126.90	30	185	42.3	34.6	0	0.01	0.00	0.01	0.02
	N3C1W4.J	142	142.00	129.14	29	181	41.4	35.1	0	0.01	0.00	0.01	0.02
	N3C1W4.K	147	147.00	130.58	27	173	37.5	32.0	0	0.02	0.00	0.00	0.02
	N3C1W4.L	148	147.50	131.34	31	189	42.5	32.8	0	0.02	0.00	0.00	0.02
	N3C1W4.M	149	149.00	131.64	29	181	40.3	32.9	0	0.02	0.00	0.00	0.02
	N3C1W4.N	148	148.00	126.85	26	162	36.6	31.7	0	0.02	0.00	0.00	0.02
	N3C1W4.O	143	142.50	125.63	25	152	36.2	31.5	0	0.02	0.00	0.00	0.02
	N3C1W4.P	143	143.00	129.15	29	180	40.3	32.3	0	0.01	0.00	0.01	0.02
	N3C1W4.Q	146	145.50	132.11	30	188	41.1	32.9	0	0.01	0.00	0.01	0.02
	N3C1W4.R	145	144.50	128.36	30	186	41.1	32.7	0	0.01	0.00	0.01	0.02
	N3C1W4.S	145	145.00	128.75	26	158	37.1	32.5	0	0.01	0.00	0.01	0.02
	N3C1W4.T	146	145.50	130.34	28	169	38.9	31.2	0	0.02	0.00	0.00	0.02
	N3C2W1.A	91	90.33	88.45	121	2,888	99.2	100.0	0	0.04	0.04	0.07	0.15
	N3C2W1.B	82	81.96	81.96	122	3,105	100.0	100.0	0	0.04	0.07	0.19	0.30
	N3C2W1.C	84	83.25	83.25	122	3,133	100.0	100.0	0	0.04	0.08	0.25	0.37
	N3C2W1.D	85	85.00	83.19	122	3,165	100.0	100.0	0	0.04	0.04	0.07	0.15
	N3C2W1.E	87	87.00	84.88	119	3,330	98.3	99.9	0	0.05	0.05	0.06	0.16
	N3C2W1.F	88	88.00	86.01	122	3,061	100.0	100.0	0	0.04	0.04	0.05	0.13
	N3C2W1.G	87	86.87	86.83	122	3,009	100.0	100.0	0	0.04	0.06	0.18	0.28
	N3C2W1.H	87	86.33	85.46	122	3,027	100.0	100.0	0	0.04	0.04	0.09	0.17
	N3C2W1.I	87	86.09	86.06	121	3,373	99.2	100.0	0	0.05	0.06	0.12	0.23
	N3C2W1.J	87	86.02	85.98	122	3,286	100.0	100.0	0	0.04	0.06	0.12	0.22
	N3C2W1.K	77	76.79	76.79	121	3,470	99.2	100.0	0	0.04	0.09	0.30	0.43
	N3C2W1.L	91	90.17	88.12	122	3,002	100.0	100.0	0	0.03	0.04	0.07	0.14
	N3C2W1.M	85	84.50	83.72	120	2,955	99.2	100.0	0	0.04	0.05	0.11	0.20
	N3C2W1.N	91	91.00	86.70	122	3,111	100.0	100.0	0	0.04	0.04	0.08	0.16
	N3C2W1.O	82	81.75	81.75	122	3,216	100.0	100.0	0	0.04	0.07	0.40	0.51
	N3C2W1.P	88	87.71	87.60	121	3,289	99.2	100.0	0	0.05	0.06	0.11	0.22
	N3C2W1.Q	82	81.30	81.30	118	3,069	98.3	98.1	0	0.04	0.08	0.21	0.33
	N3C2W1.R	82	81.92	81.92	122	3,458	100.0	100.0	0	0.04	0.08	0.41	0.53
	N3C2W1.S	89	88.37	88.17	108	2,338	93.9	97.2	0	0.03	0.04	0.06	0.13
	N3C2W1.T	83	82.83	82.54	122	3,207	100.0	100.0	0	0.04	0.05	0.16	0.25
	N3C2W2.A	107	106.50	102.31	77	1,381	76.2	81.0	0	0.02	0.01	0.05	0.08
	N3C2W2.B	105	104.50	98.79	80	1,410	77.7	80.0	0	0.03	0.01	0.05	0.09
	N3C2W2.C	104	104.00	100.12	75	1,276	75.0	81.2	0	0.02	0.01	0.04	0.07
	N3C2W2.D	107	107.00	102.53	79	1,304	76.7	78.6	0	0.03	0.01	0.04	0.08
	N3C2W2.E	116	115.50	107.22	77	1,313	76.2	79.0	0	0.03	0.01	0.04	0.08
	N3C2W2.F	106	105.50	102.15	78	1,306	76.5	79.6	0	0.02	0.01	0.05	0.08
	N3C2W2.G	102	101.75	99.32	76	1,283	75.2	79.2	0	0.02	0.01	0.06	0.09
	N3C2W2.H	117	117.00	106.63	76	1,250	75.2	79.4	0	0.02	0.01	0.04	0.07
	N3C2W2.I	102	101.50	98.92	81	1,474	78.6	80.5	0	0.03	0.02	0.04	0.09
	N3C2W2.J	107	106.33	102.15	74	1,211	74.7	81.5	0	0.02	0.01	0.05	0.08
	N3C2W2.K	110	110.00	102.54	76	1,379	75.2	81.3	0	0.03	0.01	0.04	0.08
	N3C2W2.L	105	105.00	101.15	78	1,377	76.5	79.7	0	0.03	0.01	0.05	0.09
	N3C2W2.M	108	108.00	103.25	74	1,289	74.0	82.4	0	0.02	0.01	0.05	0.08
	N3C2W2.N	105	105.00	101.59	74	1,211	74.0	80.4	0	0.03	0.01	0.04	0.08
	N3C2W2.O	107	106.50	101.40	75	1,226	75.0	78.6	0	0.03	0.01	0.04	0.08
	N3C2W2.P	107	107.00	101.92	76	1,228	75.2	79.9	0	0.03	0.01	0.04	0.08
	N3C2W2.Q	105	104.50	100.21	75	1,266	74.3	79.3	0	0.02	0.01	0.05	0.08
	N3C2W2.R	110	109.50	102.46	73	1,067	72.3	76.5	0	0.03	0.01	0.03	0.07

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N3C2W2.S	107	107.00	101.50	77	1,346	75.5	79.0	0	0.02	0.01	0.05	0.08
	N3C2W2.T	107	106.50	101.27	75	1,330	75.0	82.6	0	0.03	0.01	0.04	0.08
	N3C2W4.A	113	112.50	105.80	54	527	58.1	48.4	0	0.02	0.01	0.01	0.04
	N3C2W4.B	112	111.75	105.07	49	462	53.8	46.3	0	0.02	0.00	0.01	0.03
	N3C2W4.C	132	131.50	113.83	50	475	54.9	46.8	0	0.02	0.00	0.02	0.04
	N3C2W4.D	114	113.50	106.58	51	497	55.4	47.7	0	0.02	0.00	0.02	0.04
	N3C2W4.E	110	109.67	103.89	52	519	55.9	47.7	0	0.02	0.01	0.01	0.04
	N3C2W4.F	115	114.50	106.15	54	514	58.7	48.6	0	0.02	0.00	0.02	0.04
	N3C2W4.G	122	122.00	111.17	50	462	54.9	45.1	0	0.02	0.00	0.01	0.03
	N3C2W4.H	113	112.17	105.17	52	491	56.5	47.2	0	0.02	0.00	0.02	0.04
	N3C2W4.I	115	115.00	106.02	54	528	58.1	47.2	0	0.01	0.00	0.02	0.03
	N3C2W4.J	120	120.00	108.03	48	430	53.9	48.5	0	0.02	0.00	0.01	0.03
	N3C2W4.K	117	116.50	108.54	51	470	56.0	46.5	0	0.01	0.00	0.02	0.03
	N3C2W4.L	116	116.00	107.72	49	434	54.4	43.5	0	0.02	0.00	0.01	0.03
	N3C2W4.M	120	120.00	108.94	51	464	56.0	45.0	0	0.01	0.00	0.02	0.03
	N3C2W4.N	117	117.00	107.78	51	499	55.4	48.1	0	0.02	0.00	0.01	0.03
	N3C2W4.O	113	113.00	105.67	52	500	56.5	47.8	0	0.01	0.01	0.01	0.03
	N3C2W4.P	122	122.00	110.35	54	524	58.1	47.6	0	0.01	0.00	0.02	0.03
	N3C2W4.Q	118	118.00	109.49	54	529	58.1	47.4	0	0.02	0.00	0.02	0.04
	N3C2W4.R	123	123.00	111.32	54	529	58.1	47.4	0	0.02	0.00	0.02	0.04
	N3C2W4.S	118	117.50	108.32	53	467	57.6	44.6	0	0.01	0.00	0.02	0.03
	N3C2W4.T	119	118.50	109.20	51	495	55.4	46.8	0	0.02	0.00	0.02	0.04
	N3C3W1.A	66	65.33	65.33	152	5,077	100.0	100.0	0	0.06	0.14	0.57	0.77
	N3C3W1.B	71	70.69	70.69	146	4,258	98.0	99.9	0	0.05	0.12	0.44	0.61
	N3C3W1.C	69	68.51	68.51	149	4,619	98.0	99.9	0	0.05	0.14	0.31	0.50
	N3C3W1.D	63	62.91	62.91	152	4,924	100.0	100.0	0	0.06	0.14	0.60	0.80
	N3C3W1.E	68	67.95	67.95	152	4,968	100.0	100.0	0	0.05	0.14	0.43	0.62
	N3C3W1.F	69	68.37	68.37	151	5,118	99.3	100.0	0	0.06	0.14	0.42	0.62
	N3C3W1.G	65	64.16	64.16	152	5,211	100.0	100.0	0	0.06	0.15	0.49	0.70
	N3C3W1.H	69	68.15	68.15	152	4,706	100.0	100.0	0	0.06	0.13	0.40	0.59
	N3C3W1.I	68	67.95	67.95	151	4,885	99.3	100.0	0	0.05	0.14	0.40	0.62
	N3C3W1.J	65	64.47	64.47	150	4,882	99.3	100.0	0	0.05	0.14	0.38	0.57
	N3C3W1.K	63	62.57	62.57	152	4,975	100.0	100.0	0	0.06	0.14	0.44	0.64
	N3C3W1.L	68	67.39	67.39	152	5,113	100.0	100.0	0	0.05	0.15	0.53	0.73
	N3C3W1.M	71	70.82	70.82	152	4,775	100.0	100.0	0	0.05	0.13	0.42	0.60
	N3C3W1.N	69	68.19	68.19	152	4,627	100.0	100.0	0	0.05	0.13	0.44	0.62
	N3C3W1.O	66	65.17	65.17	152	4,509	100.0	100.0	0	0.05	0.14	0.45	0.64
	N3C3W1.P	72	71.82	71.82	151	5,221	99.3	100.0	0	0.06	0.18	0.66	0.90
	N3C3W1.Q	73	72.61	72.61	152	4,325	100.0	100.0	0	0.05	0.12	0.33	0.50
	N3C3W1.R	66	65.85	65.85	152	4,815	100.0	100.0	0	0.05	0.14	0.46	0.65
	N3C3W1.S	68	67.51	67.51	152	4,567	100.0	100.0	0	0.05	0.13	0.57	0.75
	N3C3W1.T	70	69.02	69.02	152	4,857	100.0	100.0	0	0.05	0.13	0.59	0.77
	N3C3W2.A	84	83.34	83.28	105	2,454	80.8	86.7	0	0.04	0.04	0.12	0.20
	N3C3W2.B	81	80.93	80.90	104	2,381	80.0	86.0	0	0.04	0.03	0.17	0.24
	N3C3W2.C	82	81.20	81.20	110	2,494	82.7	86.4	0	0.03	0.05	0.16	0.24
	N3C3W2.D	79	78.37	78.37	112	2,733	84.2	87.5	0	0.04	0.05	0.18	0.27
	N3C3W2.E	79	78.21	78.21	112	2,784	84.2	87.7	0	0.04	0.06	0.15	0.25
	N3C3W2.F	81	80.84	80.79	108	2,632	82.4	88.6	0	0.03	0.04	0.14	0.21
	N3C3W2.G	81	80.24	80.24	110	2,787	82.7	87.6	0	0.04	0.05	0.19	0.28
	N3C3W2.H	82	81.85	81.82	112	2,702	84.2	87.3	0	0.04	0.05	0.32	0.41
	N3C3W2.I	79	78.17	78.14	112	2,493	84.2	86.4	0	0.04	0.04	0.13	0.21
	N3C3W2.J	83	82.52	82.49	111	2,768	83.5	87.6	0	0.04	0.05	0.13	0.22
	N3C3W2.K	83	82.66	82.57	105	2,311	81.4	87.2	0	0.03	0.04	0.11	0.18
	N3C3W2.L	82	81.20	81.16	108	2,510	82.4	85.7	0	0.04	0.04	0.13	0.21
	N3C3W2.M	83	82.84	82.83	103	2,402	79.8	88.1	0	0.04	0.04	0.15	0.23
	N3C3W2.N	77	76.13	76.13	110	2,621	83.3	87.0	0	0.03	0.05	0.25	0.33
	N3C3W2.O	82	81.50	81.50	112	2,782	84.2	87.6	0	0.04	0.05	0.17	0.26
	N3C3W2.P	80	79.59	79.59	106	2,434	81.5	87.5	0	0.03	0.04	0.16	0.23
	N3C3W2.Q	76	75.37	75.37	110	2,475	83.3	86.9	0	0.03	0.04	0.18	0.25
	N3C3W2.R	79	78.51	78.51	106	2,530	80.9	85.6	0	0.04	0.05	0.16	0.25
	N3C3W2.S	80	79.31	79.31	112	2,620	84.2	87.0	0	0.04	0.04	0.14	0.22
	N3C3W2.T	79	78.11	78.11	107	2,459	82.3	87.0	0	0.04	0.05	0.12	0.21
	N3C3W4.A	89	88.96	88.11	82	1,364	67.2	65.5	0	0.03	0.02	0.06	0.11
	N3C3W4.B	88	87.05	86.38	83	1,423	68.0	66.0	0	0.03	0.02	0.05	0.10

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N3C3W4.C	88	88.00	87.47	82	1,384	67.2	66.3	0	0.03	0.02	0.09	0.14
	N3C3W4.D	87	86.62	86.41	81	1,361	67.5	67.5	0	0.03	0.02	0.11	0.16
	N3C3W4.E	85	84.79	84.71	83	1,444	68.6	67.0	0	0.03	0.02	0.14	0.19
	N3C3W4.F	84	83.75	83.60	82	1,322	67.8	64.6	0	0.02	0.02	0.07	0.11
	N3C3W4.G	94	93.07	91.95	80	1,378	66.7	68.2	0	0.03	0.02	0.06	0.11
	N3C3W4.H	84	83.67	83.45	83	1,387	68.0	65.2	0	0.02	0.02	0.07	0.11
	N3C3W4.I	92	91.25	90.09	81	1,426	66.9	66.0	0	0.03	0.02	0.06	0.11
	N3C3W4.J	88	87.24	86.81	77	1,238	65.3	68.1	0	0.03	0.01	0.06	0.10
	N3C3W4.K	89	88.37	88.17	84	1,475	68.3	67.3	0	0.03	0.02	0.07	0.12
	N3C3W4.L	90	89.50	87.85	82	1,404	67.2	66.3	0	0.03	0.02	0.05	0.10
	N3C3W4.M	88	87.58	86.85	79	1,306	65.8	66.0	0	0.03	0.01	0.11	0.15
	N3C3W4.N	87	86.76	85.31	85	1,509	69.1	67.3	0	0.03	0.02	0.07	0.12
	N3C3W4.O	87	86.47	86.23	82	1,367	67.8	66.2	0	0.03	0.02	0.07	0.12
	N3C3W4.P	86	85.45	84.95	85	1,433	69.1	66.0	0	0.03	0.02	0.09	0.14
	N3C3W4.Q	87	86.85	86.19	81	1,369	66.9	66.5	0	0.03	0.02	0.07	0.12
	N3C3W4.R	87	86.64	86.39	86	1,483	69.9	67.0	0	0.03	0.02	0.07	0.12
	N3C3W4.S	84	83.08	83.03	84	1,517	68.3	67.7	0	0.03	0.02	0.08	0.13
	N3C3W4.T	85	84.05	83.91	83	1,442	68.0	66.9	0	0.03	0.02	0.15	0.20
	N4C1W1.A	240	239.80	239.43	102	2,700	100.0	100.0	0	0.03	0.05	0.10	0.18
	N4C1W1.B	262	262.00	257.61	102	2,698	100.0	100.0	0	0.04	0.03	0.10	0.17
	N4C1W1.C	241	240.64	240.11	101	2,698	99.0	100.0	0	0.04	0.05	0.12	0.21
	N4C1W1.D	246	245.78	244.73	102	2,699	100.0	100.0	0	0.04	0.04	0.09	0.17
	N4C1W1.E	272	271.50	258.11	102	2,699	100.0	100.0	0	0.04	0.03	0.04	0.11
	N4C1W1.F	265	264.40	259.73	102	2,678	100.0	100.0	0	0.04	0.04	0.09	0.17
	N4C1W1.G	259	258.50	253.64	102	2,631	100.0	100.0	0	0.04	0.03	0.06	0.13
	N4C1W1.H	251	251.00	248.47	102	2,694	100.0	100.0	0	0.04	0.03	0.06	0.13
	N4C1W1.I	262	261.50	255.78	102	2,642	100.0	100.0	0	0.04	0.03	0.06	0.13
	N4C1W1.J	288	288.00	269.15	102	2,694	100.0	100.0	0	0.04	0.03	0.05	0.12
	N4C1W1.K	253	252.50	248.54	102	2,612	100.0	100.0	0	0.04	0.03	0.06	0.13
	N4C1W1.L	258	258.00	253.43	102	2,627	100.0	100.0	0	0.04	0.03	0.05	0.12
	N4C1W1.M	246	245.12	244.22	102	2,677	100.0	100.0	0	0.03	0.04	0.07	0.14
	N4C1W1.N	256	256.00	250.91	102	2,699	100.0	100.0	0	0.04	0.03	0.08	0.15
	N4C1W1.O	258	257.58	253.75	102	2,698	100.0	100.0	0	0.04	0.04	0.09	0.17
	N4C1W1.P	271	270.50	258.44	102	2,677	100.0	100.0	0	0.04	0.03	0.05	0.12
	N4C1W1.Q	277	277.00	259.04	102	2,699	100.0	100.0	0	0.04	0.03	0.04	0.11
	N4C1W1.R	254	253.62	251.89	101	2,695	99.0	100.0	0	0.04	0.03	0.09	0.16
	N4C1W1.S	261	260.50	255.65	102	2,681	100.0	100.0	0	0.03	0.03	0.07	0.13
	N4C1W1.T	256	255.50	251.63	102	2,697	100.0	100.0	0	0.04	0.03	0.08	0.15
	N4C1W2.A	317	316.25	300.49	60	803	72.3	71.6	0	0.02	0.01	0.02	0.05
	N4C1W2.B	328	328.00	304.93	60	804	72.3	71.7	0	0.02	0.01	0.02	0.05
	N4C1W2.C	319	319.00	298.30	60	804	72.3	71.6	0	0.03	0.01	0.01	0.05
	N4C1W2.D	327	327.00	303.11	60	804	72.3	71.6	0	0.02	0.01	0.01	0.04
	N4C1W2.E	310	309.35	295.74	60	802	72.3	71.5	0	0.02	0.01	0.02	0.05
	N4C1W2.F	321	320.50	302.92	60	804	72.3	71.6	0	0.02	0.01	0.01	0.04
	N4C1W2.G	307	306.83	293.37	60	804	72.3	71.6	0	0.02	0.01	0.02	0.05
	N4C1W2.H	315	314.50	297.01	60	804	72.3	71.6	0	0.03	0.01	0.01	0.05
	N4C1W2.I	304	303.56	290.60	60	803	72.3	71.6	0	0.02	0.01	0.02	0.05
	N4C1W2.J	311	311.00	296.64	60	803	72.3	71.6	0	0.03	0.01	0.01	0.05
	N4C1W2.K	311	310.50	296.22	60	803	72.3	71.6	0	0.02	0.01	0.02	0.05
	N4C1W2.L	316	316.00	299.09	60	804	72.3	71.6	0	0.02	0.01	0.01	0.04
	N4C1W2.M	330	329.50	305.37	60	804	72.3	71.6	0	0.03	0.01	0.01	0.05
	N4C1W2.N	311	310.33	297.36	60	804	72.3	71.6	0	0.02	0.01	0.02	0.05
	N4C1W2.O	319	318.08	301.17	60	802	72.3	71.5	0	0.02	0.01	0.02	0.05
	N4C1W2.P	317	316.50	299.73	60	804	72.3	71.6	0	0.02	0.01	0.02	0.05
	N4C1W2.Q	319	318.50	302.46	60	804	72.3	71.6	0	0.02	0.01	0.02	0.05
	N4C1W2.R	319	319.00	302.40	60	803	72.3	71.6	0	0.03	0.01	0.01	0.05
	N4C1W2.S	312	312.00	298.32	60	803	72.3	71.6	0	0.03	0.01	0.01	0.05
	N4C1W2.T	323	322.50	302.13	60	804	72.3	71.6	0	0.02	0.01	0.02	0.05
	N4C1W4.A	368	367.50	330.06	31	191	42.5	32.8	0	0.01	0.00	0.01	0.02
	N4C1W4.B	349	349.00	319.63	31	191	42.5	32.8	0	0.01	0.00	0.01	0.02
	N4C1W4.C	365	365.00	324.89	31	191	42.5	32.8	0	0.02	0.00	0.00	0.02
	N4C1W4.D	359	359.00	321.77	31	191	42.5	32.8	0	0.02	0.00	0.00	0.02
	N4C1W4.E	373	372.50	332.57	31	191	42.5	32.8	0	0.01	0.00	0.01	0.02
	N4C1W4.F	369	368.50	318.64	31	190	42.5	32.6	0	0.02	0.00	0.00	0.02

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N4C1W4.G	362	362.00	327.76	31	191	42.5	32.8	0	0.01	0.00	0.01	0.02
	N4C1W4.H	359	358.17	327.82	31	191	42.5	32.8	0	0.01	0.00	0.01	0.02
	N4C1W4.I	359	359.00	327.03	31	191	42.5	32.8	0	0.01	0.00	0.01	0.02
	N4C1W4.J	368	368.00	331.96	31	191	42.5	32.8	0	0.02	0.00	0.00	0.02
	N4C1W4.K	371	370.50	331.56	31	191	42.5	32.8	0	0.02	0.00	0.00	0.02
	N4C1W4.L	355	354.50	322.81	31	191	42.5	32.8	0	0.01	0.00	0.01	0.02
	N4C1W4.M	360	360.00	325.21	31	191	42.5	32.8	0	0.01	0.00	0.01	0.02
	N4C1W4.N	363	363.00	325.73	31	191	42.5	32.8	0	0.02	0.00	0.00	0.02
	N4C1W4.O	351	351.00	319.87	31	191	42.5	32.8	0	0.01	0.00	0.01	0.02
	N4C1W4.P	363	363.00	328.44	31	191	42.5	32.8	0	0.02	0.00	0.00	0.02
	N4C1W4.Q	359	358.50	323.60	31	191	42.5	32.8	0	0.01	0.00	0.01	0.02
	N4C1W4.R	370	369.50	331.70	31	191	42.5	32.8	0	0.02	0.00	0.00	0.02
	N4C1W4.S	359	359.00	322.95	31	191	42.5	32.8	0	0.02	0.00	0.00	0.02
	N4C1W4.T	355	354.50	318.10	31	191	42.5	32.8	0	0.01	0.00	0.01	0.02
	N4C2W1.A	210	209.65	209.65	122	3,816	100.0	100.0	0	0.05	0.09	0.38	0.52
	N4C2W1.B	213	213.00	210.32	122	3,817	100.0	100.0	0	0.05	0.05	0.10	0.20
	N4C2W1.C	213	212.43	212.43	122	3,819	100.0	100.0	0	0.05	0.09	0.21	0.35
	N4C2W1.D	200	199.71	199.71	122	3,780	100.0	100.0	0	0.04	0.09	0.28	0.41
	N4C2W1.E	215	214.88	214.88	121	3,815	99.2	100.0	0	0.05	0.10	1.37	1.52
	N4C2W1.F	203	202.60	202.60	122	3,786	100.0	100.0	0	0.04	0.09	0.30	0.43
	N4C2W1.G	211	210.81	210.81	122	3,817	100.0	100.0	0	0.04	0.10	0.39	0.53
	N4C2W1.H	215	215.00	213.28	122	3,809	100.0	100.0	0	0.05	0.06	0.12	0.23
	N4C2W1.I	209	208.75	208.75	122	3,818	100.0	100.0	0	0.04	0.10	0.30	0.44
	N4C2W1.J	202	201.93	201.93	122	3,818	100.0	100.0	0	0.05	0.10	0.35	0.50
	N4C2W1.K	210	209.83	209.83	122	3,818	100.0	100.0	0	0.05	0.09	0.24	0.38
	N4C2W1.L	209	208.78	208.78	122	3,818	100.0	100.0	0	0.05	0.09	0.40	0.54
	N4C2W1.M	217	216.92	216.92	122	3,819	100.0	100.0	0	0.05	0.10	0.31	0.46
	N4C2W1.N	210	209.31	209.31	122	3,820	100.0	100.0	0	0.05	0.10	0.33	0.48
	N4C2W1.O	212	211.22	211.22	122	3,817	100.0	100.0	0	0.05	0.10	0.37	0.52
	N4C2W1.P	212	211.63	211.63	122	3,819	100.0	100.0	0	0.05	0.10	0.30	0.45
	N4C2W1.Q	210	209.79	209.79	122	3,817	100.0	100.0	0	0.05	0.10	0.36	0.51
	N4C2W1.R	212	211.82	211.82	122	3,816	100.0	100.0	0	0.05	0.09	0.33	0.47
	N4C2W1.S	210	209.12	209.12	122	3,819	100.0	100.0	0	0.05	0.09	0.30	0.44
	N4C2W1.T	212	211.02	211.02	121	3,737	99.2	100.0	0	0.05	0.10	0.27	0.42
	N4C2W2.A	253	252.50	250.16	81	1,504	78.6	80.8	0	0.03	0.02	0.05	0.10
	N4C2W2.B	254	254.00	249.22	81	1,505	78.6	80.8	0	0.02	0.02	0.04	0.08
	N4C2W2.C	249	249.00	246.94	81	1,502	78.6	80.8	0	0.03	0.02	0.03	0.08
	N4C2W2.D	258	257.75	252.28	81	1,504	78.6	80.8	0	0.03	0.02	0.04	0.09
	N4C2W2.E	257	257.00	251.22	81	1,505	78.6	80.8	0	0.02	0.02	0.04	0.08
	N4C2W2.F	272	272.00	256.00	81	1,503	78.6	80.8	0	0.03	0.02	0.04	0.09
	N4C2W2.G	252	251.50	248.38	81	1,505	78.6	80.8	0	0.03	0.02	0.04	0.09
	N4C2W2.H	255	255.00	248.44	79	1,394	77.5	79.5	0	0.02	0.01	0.04	0.07
	N4C2W2.I	262	262.00	252.37	81	1,503	78.6	80.8	0	0.03	0.02	0.03	0.08
	N4C2W2.J	256	255.83	252.07	81	1,472	78.6	80.4	0	0.03	0.02	0.05	0.10
	N4C2W2.K	259	258.50	252.73	81	1,503	78.6	80.8	0	0.03	0.02	0.05	0.10
	N4C2W2.L	263	263.00	254.06	81	1,505	78.6	80.8	0	0.02	0.01	0.05	0.08
	N4C2W2.M	261	261.00	251.42	81	1,505	78.6	80.8	0	0.03	0.01	0.04	0.08
	N4C2W2.N	264	264.00	255.04	81	1,503	78.6	80.8	0	0.03	0.02	0.05	0.10
	N4C2W2.O	253	252.25	248.21	80	1,503	77.7	80.7	0	0.03	0.02	0.04	0.09
	N4C2W2.P	266	266.00	255.51	81	1,505	78.6	80.8	0	0.03	0.01	0.04	0.08
	N4C2W2.Q	257	256.75	251.15	81	1,505	78.6	80.8	0	0.03	0.02	0.04	0.09
	N4C2W2.R	256	256.00	250.07	81	1,504	78.6	80.8	0	0.03	0.01	0.04	0.08
	N4C2W2.S	273	272.50	256.07	81	1,505	78.6	80.8	0	0.03	0.01	0.04	0.08
	N4C2W2.T	256	255.38	250.12	81	1,505	78.6	80.8	0	0.03	0.02	0.06	0.11
	N4C2W4.A	293	292.33	272.12	55	535	59.1	47.6	0	0.02	0.00	0.02	0.04
	N4C2W4.B	281	281.00	264.68	55	535	59.1	47.6	0	0.02	0.01	0.00	0.03
	N4C2W4.C	295	294.50	269.30	55	535	59.1	47.6	0	0.02	0.00	0.01	0.03
	N4C2W4.D	295	294.50	272.11	55	535	59.1	47.6	0	0.02	0.00	0.02	0.04
	N4C2W4.E	287	287.00	269.40	55	535	59.1	47.6	0	0.02	0.01	0.00	0.03
	N4C2W4.F	304	303.50	277.52	55	535	59.1	47.6	0	0.02	0.00	0.02	0.04
	N4C2W4.G	291	291.00	270.93	55	535	59.1	47.6	0	0.02	0.01	0.01	0.04
	N4C2W4.H	296	296.00	272.02	55	535	59.1	47.6	0	0.01	0.00	0.02	0.03
	N4C2W4.I	287	286.17	269.98	55	535	59.1	47.6	0	0.02	0.01	0.01	0.04
	N4C2W4.J	300	300.00	277.04	55	535	59.1	47.6	0	0.02	0.00	0.02	0.04

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N4C2W4.K	289	289.00	270.10	55	535	59.1	47.7	0	0.02	0.00	0.01	0.03
	N4C2W4.L	297	296.67	275.20	55	535	59.1	47.7	0	0.02	0.01	0.01	0.04
	N4C2W4.M	287	287.00	269.68	55	535	59.1	47.6	0	0.02	0.01	0.01	0.04
	N4C2W4.N	299	299.00	276.02	55	535	59.1	47.6	0	0.02	0.00	0.02	0.04
	N4C2W4.O	293	292.17	271.65	55	535	59.1	47.6	0	0.02	0.01	0.01	0.04
	N4C2W4.P	301	300.50	276.84	55	535	59.1	47.6	0	0.02	0.00	0.02	0.04
	N4C2W4.Q	293	293.00	270.74	55	535	59.1	47.6	0	0.02	0.00	0.01	0.03
	N4C2W4.R	300	299.50	272.92	55	535	59.1	47.6	0	0.02	0.00	0.02	0.04
	N4C2W4.S	293	292.50	271.97	55	535	59.1	47.7	0	0.02	0.00	0.02	0.04
	N4C2W4.T	287	287.00	268.96	55	535	59.1	47.6	0	0.02	0.00	0.01	0.03
	N4C3W1.A	164	163.34	163.34	152	5,852	100.0	100.0	0	0.07	0.18	0.71	0.96
	N4C3W1.B	165	164.20	164.20	152	5,871	100.0	100.0	0	0.06	0.17	0.69	0.92
	N4C3W1.C	166	165.45	165.45	152	5,871	100.0	100.0	0	0.06	0.19	0.64	0.89
	N4C3W1.D	158	157.46	157.46	152	5,875	100.0	100.0	0	0.06	0.17	0.62	0.85
	N4C3W1.E	165	164.87	164.87	152	5,874	100.0	100.0	0	0.06	0.17	0.75	0.98
	N4C3W1.F	164	163.08	163.08	152	5,736	100.0	100.0	0	0.06	0.17	0.59	0.82
	N4C3W1.G	169	168.78	168.78	152	5,872	100.0	100.0	0	0.07	0.16	0.45	0.68
	N4C3W1.H	170	169.70	169.70	152	5,873	100.0	100.0	0	0.06	0.17	0.64	0.87
	N4C3W1.I	167	166.99	166.99	152	5,872	100.0	100.0	0	0.07	0.17	0.73	0.97
	N4C3W1.J	169	168.83	168.83	152	5,873	100.0	100.0	0	0.06	0.17	0.63	0.86
	N4C3W1.K	164	163.39	163.39	152	5,748	100.0	100.0	0	0.07	0.18	0.61	0.86
	N4C3W1.L	163	162.52	162.52	152	5,874	100.0	100.0	0	0.07	0.18	0.50	0.75
	N4C3W1.M	167	166.61	166.61	152	5,643	100.0	100.0	0	0.06	0.16	0.67	0.89
	N4C3W1.N	176	175.13	175.13	152	5,775	100.0	100.0	0	0.07	0.17	0.70	0.94
	N4C3W1.O	169	168.08	168.08	152	5,798	100.0	100.0	0	0.06	0.18	0.50	0.74
	N4C3W1.P	167	166.70	166.70	152	5,804	100.0	100.0	0	0.06	0.17	0.61	0.84
	N4C3W1.Q	176	175.31	175.31	152	5,872	100.0	100.0	0	0.07	0.17	0.72	0.96
	N4C3W1.R	167	166.96	166.96	152	5,774	100.0	100.0	0	0.06	0.17	0.75	0.98
	N4C3W1.S	168	167.70	167.70	152	5,872	100.0	100.0	0	0.06	0.17	0.62	0.85
	N4C3W1.T	173	172.50	172.50	150	5,719	99.3	100.0	0	0.07	0.18	0.62	0.87
	N4C3W2.A	203	202.98	202.98	112	2,943	84.2	88.2	0	0.04	0.06	0.27	0.37
	N4C3W2.B	203	202.49	202.49	110	2,864	83.3	88.3	0	0.04	0.05	0.16	0.25
	N4C3W2.C	201	200.55	200.55	112	2,955	84.2	88.3	0	0.04	0.05	0.26	0.35
	N4C3W2.D	201	200.49	200.49	112	2,954	84.2	88.3	0	0.04	0.06	0.23	0.33
	N4C3W2.E	200	199.01	199.01	112	2,956	84.2	88.3	0	0.04	0.06	0.16	0.26
	N4C3W2.F	209	208.83	208.77	112	2,954	84.2	88.3	0	0.04	0.06	0.23	0.33
	N4C3W2.G	201	200.12	200.12	112	2,955	84.2	88.3	0	0.04	0.05	0.16	0.25
	N4C3W2.H	201	200.29	200.29	112	2,955	84.2	88.3	0	0.04	0.05	0.27	0.36
	N4C3W2.I	198	197.33	197.33	112	2,955	84.2	88.3	0	0.04	0.05	0.24	0.33
	N4C3W2.J	204	203.07	203.07	112	2,956	84.2	88.3	0	0.04	0.06	0.22	0.32
	N4C3W2.K	204	203.46	203.46	112	2,955	84.2	88.3	0	0.04	0.06	0.28	0.38
	N4C3W2.L	201	200.61	200.61	111	2,953	83.5	88.2	0	0.04	0.06	0.21	0.31
	N4C3W2.M	198	197.05	197.05	112	2,955	84.2	88.3	0	0.04	0.05	0.17	0.26
	N4C3W2.N	198	197.75	197.75	112	2,956	84.2	88.3	0	0.04	0.05	0.32	0.41
	N4C3W2.O	209	208.13	208.13	112	2,872	84.2	88.0	0	0.04	0.06	0.26	0.36
	N4C3W2.P	202	201.14	201.14	112	2,955	84.2	88.3	0	0.05	0.05	0.18	0.28
	N4C3W2.Q	199	198.30	198.30	112	2,956	84.2	88.3	0	0.04	0.05	0.14	0.23
	N4C3W2.R	194	193.81	193.81	112	2,956	84.2	88.3	0	0.05	0.05	0.24	0.34
	N4C3W2.S	196	195.97	195.97	112	2,956	84.2	88.3	0	0.04	0.05	0.21	0.30
	N4C3W2.T	195	194.36	194.36	112	2,955	84.2	88.3	0	0.04	0.06	0.20	0.30
	N4C3W4.A	216	215.65	215.41	87	1,565	70.7	67.8	0	0.03	0.02	0.06	0.11
	N4C3W4.B	215	214.14	213.01	87	1,565	70.7	67.8	0	0.03	0.02	0.06	0.11
	N4C3W4.C	218	217.12	216.63	87	1,564	70.7	67.8	0	0.03	0.02	0.07	0.12
	N4C3W4.D	215	214.73	214.46	87	1,565	70.7	67.8	0	0.03	0.02	0.07	0.12
	N4C3W4.E	219	218.10	217.55	87	1,565	70.7	67.8	0	0.03	0.02	0.07	0.12
	N4C3W4.F	222	221.02	218.79	83	1,454	68.6	67.5	0	0.03	0.02	0.04	0.09
	N4C3W4.G	222	221.56	221.07	87	1,565	70.7	67.8	0	0.03	0.02	0.16	0.21
	N4C3W4.H	219	218.37	218.18	87	1,565	70.7	67.8	0	0.03	0.02	0.07	0.12
	N4C3W4.I	224	223.15	222.05	87	1,565	70.7	67.8	0	0.03	0.02	0.05	0.10
	N4C3W4.J	213	212.22	212.04	87	1,565	70.7	67.8	0	0.03	0.02	0.06	0.11
	N4C3W4.K	215	214.04	213.58	87	1,565	70.7	67.8	0	0.03	0.02	0.07	0.12
	N4C3W4.L	219	218.21	216.98	87	1,537	70.7	67.4	0	0.03	0.02	0.07	0.12
	N4C3W4.M	216	215.14	214.65	87	1,564	70.7	67.8	0	0.03	0.02	0.06	0.11
	N4C3W4.N	225	224.71	222.88	87	1,565	70.7	67.8	0	0.03	0.02	0.12	0.17

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N4C3W4.O	226	225.69	224.57	87	1,564	70.7	67.8	0	0.03	0.02	0.13	0.18
	N4C3W4.P	219	218.46	217.23	87	1,565	70.7	67.8	0	0.03	0.02	0.05	0.10
	N4C3W4.Q	222	221.65	220.80	87	1,565	70.7	67.8	0	0.03	0.02	0.06	0.11
	N4C3W4.R	214	213.66	212.65	87	1,564	70.7	67.8	0	0.03	0.02	0.13	0.18
	N4C3W4.S	216	215.99	215.60	87	1,565	70.7	67.8	0	0.03	0.02	0.15	0.20
	N4C3W4.T	216	215.49	214.71	87	1,565	70.7	67.8	0	0.03	0.02	0.04	0.09
bin2	N1W1B1R0	18	17.41	16.93	45	555	10.7	21.8	0	0.02	0.01	0.08	0.11
	N1W1B1R1	18	17.50	16.95	43	475	9.0	14.5	0	0.02	0.01	0.02	0.05
	N1W1B1R2	19	18.30	17.18	43	557	9.9	21.4	0	0.02	0.01	0.04	0.07
	N1W1B1R3	18	17.20	16.84	45	485	11.2	18.5	0	0.02	0.01	0.02	0.05
	N1W1B1R4	17	16.67	16.41	45	444	9.9	12.8	0	0.03	0.01	0.02	0.06
	N1W1B1R5	17	16.67	16.47	45	448	9.8	14.7	0	0.03	0.01	0.01	0.05
	N1W1B1R6	17	16.92	16.76	49	570	12.2	18.9	0	0.03	0.01	0.05	0.09
	N1W1B1R7	17	16.67	16.59	47	516	9.7	14.1	0	0.03	0.01	0.02	0.06
	N1W1B1R8	18	17.08	16.82	47	598	9.9	18.5	0	0.03	0.01	0.03	0.07
	N1W1B1R9	17	16.67	16.44	43	431	9.5	13.2	0	0.03	0.01	0.01	0.05
	N1W1B2R0	17	16.59	16.59	144	1,770	22.9	29.6	0	0.05	0.03	0.17	0.25
	N1W1B2R1	17	16.34	16.33	144	1,728	22.9	30.1	0	0.05	0.03	0.09	0.17
	N1W1B2R2	17	16.50	16.50	151	1,993	24.0	31.4	0	0.06	0.04	0.06	0.16
	N1W1B2R3	16	15.82	15.81	146	1,643	23.3	28.9	0	0.04	0.03	0.06	0.13
	N1W1B2R4	17	16.48	16.46	139	1,830	22.3	30.2	0	0.05	0.03	0.07	0.15
	N1W1B2R5	17	16.62	16.61	117	1,404	19.3	28.3	0	0.04	0.03	0.06	0.13
	N1W1B2R6	17	16.25	16.25	161	1,947	25.3	29.5	0	0.05	0.04	0.08	0.17
	N1W1B2R7	18	17.28	17.23	136	1,631	22.2	29.9	0	0.04	0.02	0.07	0.13
	N1W1B2R8	16	15.79	15.79	164	1,970	25.9	31.2	0	0.05	0.03	0.21	0.29
	N1W1B2R9	18	17.43	17.32	101	1,175	17.2	25.9	0	0.04	0.01	0.06	0.11
	N1W1B3R0	17	16.14	16.13	605	7,083	74.0	76.1	0	0.09	0.37	0.31	0.77
	N1W1B3R1	17	16.92	16.91	541	6,194	67.1	74.2	0	0.08	0.29	1.29	1.66
	N1W1B3R2	15	14.70	14.70	697	10,467	80.0	86.0	0	0.12	0.55	0.53	1.20
	N1W1B3R3	16	15.95	15.94	453	4,924	59.6	60.6	0	0.07	0.18	0.19	0.44
	N1W1B3R4	19	18.18	18.15	385	3,654	51.8	56.7	0	0.06	0.10	0.14	0.30
	N1W1B3R5	16	15.71	15.71	735	9,935	83.3	87.3	0	0.11	0.55	1.90	2.56
	N1W1B3R6	16	15.05	15.04	716	9,510	81.7	86.7	0	0.11	0.61	1.61	2.33
	N1W1B3R7	18	17.98	17.93	500	5,040	65.0	70.2	0	0.07	0.17	0.67	0.91
	N1W1B3R8	16	15.64	15.64	537	6,397	66.1	73.4	0	0.08	0.29	0.28	0.65
	N1W1B3R9	17	16.40	16.40	639	7,401	75.6	80.1	0	0.09	0.44	0.39	0.92
	N1W2B1R0	11	10.43	10.29	169	2,320	24.7	32.7	0	0.05	0.04	0.23	0.32
	N1W2B1R1	11	10.29	10.21	160	2,281	23.9	30.1	0	0.06	0.04	0.44	0.54
	N1W2B1R2	11	10.03	10.03	182	2,470	26.3	26.5	0	0.07	0.05	0.21	0.33
	N1W2B1R3	11	10.39	10.28	183	2,639	26.8	33.4	0	0.06	0.05	0.19	0.30
	N1W2B1R4	11	10.53	10.37	194	2,384	28.0	30.0	0	0.06	0.04	0.51	0.61
	N1W2B1R5	10	9.89	9.89	214	3,077	30.5	31.3	0	0.07	0.07	1.50	1.64
	N1W2B1R6	11	10.07	10.05	193	2,805	28.0	30.1	0	0.07	0.06	0.22	0.35
	N1W2B1R7	11	10.06	10.05	207	3,339	29.4	32.2	0	0.07	0.07	0.37	0.51
	N1W2B1R8	10	9.95	9.95	189	2,109	27.4	28.7	0	0.05	0.04	0.74	0.83
	N1W2B1R9	11	10.10	10.08	173	2,438	25.0	28.1	0	0.06	0.05	0.23	0.34
	N1W2B2R0	10	9.78	9.78	597	11,901	73.6	72.2	0	0.14	0.50	2.38	3.02
	N1W2B2R1	11	10.40	10.40	524	8,435	66.2	66.2	0	0.11	0.30	1.43	1.84
	N1W2B2R2	10	9.23	9.23	612	13,409	74.4	73.1	0	0.16	0.55	1.47	2.18
	N1W2B2R3	11	10.08	10.08	564	10,803	70.4	69.7	0	0.13	0.41	1.03	1.57
	N1W2B2R4	10	9.75	9.75	573	10,570	71.3	70.7	0	0.13	0.47	1.03	1.63
	N1W2B2R5	10	9.91	9.91	583	11,472	72.6	70.3	0	0.14	0.48	1.60	2.22
	N1W2B2R6	10	9.51	9.51	594	12,670	73.2	72.7	0	0.15	0.53	2.35	3.03
	N1W2B2R7	10	9.76	9.76	577	10,047	72.2	70.3	0	0.13	0.42	2.67	3.22
	N1W2B2R8	11	10.49	10.49	551	9,271	68.7	65.3	0	0.12	0.37	1.52	2.01
	N1W2B2R9	11	10.21	10.21	561	9,634	70.7	68.8	0	0.13	0.38	1.83	2.34
	N1W2B3R0	10	9.76	9.76	810	14,928	87.1	95.7	0	0.16	1.12	11.38	12.66
	N1W2B3R1	11	10.14	10.14	744	14,463	84.4	92.5	0	0.16	0.96	6.13	7.25
	N1W2B3R2	10	9.93	9.93	803	14,510	86.6	84.9	0	0.17	0.98	5.16	6.31
	N1W2B3R3	11	10.15	10.15	692	13,605	81.4	87.9	0	0.15	0.86	1.67	2.68
	N1W2B3R4	10	9.04	9.04	778	16,405	85.8	94.7	0	0.17	1.10	2.28	3.55
	N1W2B3R5	10	9.52	9.52	806	16,056	88.3	93.1	0	0.18	1.06	13.91	15.15
	N1W2B3R6	11	10.19	10.19	774	14,206	86.3	94.7	0	0.16	0.98	2.95	4.09
	N1W2B3R7	10	9.30	9.30	827	17,429	89.2	93.1	0	0.20	1.22	8.17	9.59

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N1W2B3R8	11	10.29	10.29	742	15,311	84.5	94.2	0	0.16	1.00	2.02	3.18
	N1W2B3R9	10	9.55	9.55	810	16,598	87.3	91.7	0	0.19	1.15	10.45	11.79
	N1W3B1R0	7	6.94	6.94	339	3,855	44.5	36.9	0	0.08	0.13	2.14	2.35
	N1W3B1R1	8	7.33	7.31	355	4,678	46.2	44.2	0	0.07	0.13	1.01	1.21
	N1W3B1R2	7	7.00	7.00	424	5,941	53.0	44.1	0	0.10	0.21	3.18	3.49
	N1W3B1R3	8	7.04	7.04	359	5,232	45.5	44.0	0	0.09	0.18	1.25	1.52
	N1W3B1R4	8	7.01	7.01	384	5,279	48.3	40.7	0	0.10	0.18	0.84	1.12
	N1W3B1R5	8	7.09	7.09	367	4,830	46.8	42.8	0	0.08	0.17	0.43	0.68
	N1W3B1R6	8	7.23	7.23	386	5,468	48.4	47.0	0	0.09	0.18	1.03	1.30
	N1W3B1R7	8	7.09	7.09	383	5,374	48.2	40.3	0	0.09	0.19	0.63	0.91
	N1W3B1R8	7	6.99	6.99	409	6,094	51.1	43.1	9	0.11	0.23	6.19	6.53
	N1W3B1R9	8	7.01	7.01	395	5,898	49.6	44.2	0	0.10	0.22	1.17	1.49
	N1W3B2R0	8	7.16	7.16	693	15,949	80.4	83.1	0	0.18	0.86	1.81	2.85
	N1W3B2R1	8	7.34	7.34	703	14,551	81.6	81.6	0	0.16	0.86	3.23	4.25
	N1W3B2R2	8	7.13	7.13	708	15,836	81.8	84.4	0	0.17	0.95	2.15	3.27
	N1W3B2R3	7	6.92	6.92	724	16,659	82.9	80.5	0	0.19	0.98	15.60	16.77
	N1W3B2R4	7	6.73	6.73	703	14,705	82.0	80.8	0	0.16	0.84	1.94	2.94
	N1W3B2R5	7	6.78	6.78	741	17,866	84.4	83.8	0	0.20	1.07	15.25	16.52
	N1W3B2R6	8	7.25	7.25	717	15,976	82.3	82.6	0	0.17	0.85	2.10	3.12
	N1W3B2R7	8	7.31	7.31	723	15,902	82.5	80.3	0	0.18	0.82	4.71	5.71
	N1W3B2R8	8	7.20	7.20	691	14,533	80.1	78.1	0	0.17	0.85	2.93	3.95
	N1W3B2R9	8	7.04	7.04	732	17,013	83.9	84.8	0	0.19	0.99	10.86	12.04
	N1W3B3R0	7	6.88	6.88	870	23,708	91.7	98.3	0	0.26	1.57	24.04	25.87
	N1W3B3R1	8	7.28	7.28	849	21,535	90.8	97.6	0	0.22	1.64	3.29	5.15
	N1W3B3R2	7	6.58	6.58	879	23,101	93.1	97.2	0	0.23	1.51	3.90	5.64
	N1W3B3R3	8	7.44	7.44	842	20,105	89.5	96.9	0	0.21	1.44	16.66	18.31
	N1W3B3R4	8	7.29	7.29	836	22,094	90.6	96.9	0	0.24	1.47	21.22	22.93
	N1W3B3R5	7	6.41	6.41	866	20,956	91.8	97.4	0	0.22	1.55	3.56	5.33
	N1W3B3R6	7	6.99	6.99	913	22,331	94.9	98.7	1	0.23	1.66	78.33	80.22
	N1W3B3R7	8	7.53	7.53	834	19,506	89.3	97.3	0	0.21	1.36	5.79	7.36
	N1W3B3R8	7	6.29	6.29	905	22,240	94.1	96.2	0	0.23	1.71	21.51	23.45
	N1W3B3R9	7	6.86	6.86	919	22,029	95.3	98.8	0	0.24	1.67	17.30	19.21
	N1W4B1R0	6	5.48	5.48	554	7,413	65.6	51.3	0	0.11	0.33	1.28	1.72
	N1W4B1R1	6	5.60	5.60	496	5,643	59.0	49.1	0	0.09	0.23	3.14	3.46
	N1W4B1R2	6	5.52	5.52	572	7,099	67.6	50.2	0	0.11	0.31	2.67	3.09
	N1W4B1R3	6	5.52	5.52	528	6,087	63.0	50.1	0	0.09	0.27	2.30	2.66
	N1W4B1R4	6	5.55	5.55	523	6,219	62.6	49.0	0	0.10	0.27	1.06	1.43
	N1W4B1R5	6	5.45	5.45	562	8,248	66.4	51.0	0	0.13	0.37	7.36	7.86
	N1W4B1R6	6	5.48	5.48	526	6,848	62.5	54.6	0	0.10	0.27	1.74	2.11
	N1W4B1R7	6	5.56	5.56	554	6,413	66.1	51.1	0	0.09	0.27	0.64	1.00
	N1W4B1R8	6	5.38	5.38	534	6,097	63.8	49.0	0	0.10	0.28	0.56	0.94
	N1W4B1R9	6	5.41	5.41	553	7,767	65.5	51.3	0	0.11	0.34	1.90	2.35
	N1W4B2R0	6	5.71	5.71	795	18,344	88.0	89.1	0	0.20	1.19	19.19	20.58
	N1W4B2R1	6	5.05	5.05	805	18,366	89.0	88.4	0	0.20	1.22	2.76	4.18
	N1W4B2R2	6	5.34	5.34	807	22,436	88.8	90.4	0	0.24	1.39	4.17	5.80
	N1W4B2R3	6	5.71	5.71	795	19,761	87.9	87.2	0	0.22	1.30	19.93	21.45
	N1W4B2R4	6	5.61	5.61	793	18,423	87.9	87.4	0	0.20	1.23	2.89	4.32
	N1W4B2R5	6	5.91	5.91	785	20,297	87.2	89.8	0	0.22	1.31	9.14	10.67
	N1W4B2R6	6	5.56	5.56	788	20,061	87.7	88.6	0	0.22	1.34	15.56	17.12
	N1W4B2R7	6	5.19	5.19	785	16,430	87.2	88.7	0	0.18	1.08	2.19	3.45
	N1W4B2R8	6	5.98	5.98	776	19,080	86.4	88.3	16	0.21	1.20	51.26	52.67
	N1W4B2R9	6	5.28	5.28	800	20,959	88.4	89.8	0	0.21	1.34	7.04	8.59
	N1W4B3R0	6	5.19	5.19	946	25,837	96.7	99.4	0	0.26	1.86	8.52	10.64
	N1W4B3R1	6	5.54	5.54	922	25,236	95.3	99.3	0	0.27	1.78	38.05	40.10
	N1W4B3R2	7	6.31	6.31	863	19,793	91.9	96.3	0	0.21	1.34	3.14	4.69
	N1W4B3R3	6	5.40	5.40	898	27,262	93.9	99.0	0	0.29	1.75	11.37	13.41
	N1W4B3R4	6	5.52	5.52	932	26,024	95.8	99.2	0	0.26	1.79	9.51	11.56
	N1W4B3R5	7	6.06	6.06	886	22,280	93.2	96.6	0	0.22	1.62	3.91	5.75
	N1W4B3R6	6	5.53	5.53	913	25,018	94.6	98.9	0	0.25	1.95	4.74	6.94
	N1W4B3R7	6	5.29	5.29	936	26,880	96.3	99.1	0	0.27	1.81	5.59	7.67
	N1W4B3R8	6	5.80	5.80	902	23,642	94.3	99.2	0	0.25	1.67	31.60	33.52
	N1W4B3R9	6	5.82	5.82	857	23,307	91.2	96.8	0	0.25	1.61	24.77	26.63
	N2W1B1R0	34	33.33	32.89	81	1,049	15.7	15.4	0	0.06	0.01	0.07	0.14
	N2W1B1R1	34	33.51	33.40	75	1,311	14.6	20.8	0	0.05	0.02	0.09	0.16

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N2W1B1R2	34	33.33	32.87	74	861	14.6	14.3	0	0.04	0.01	0.05	0.10
	N2W1B1R3	34	33.34	33.34	77	1,160	15.3	19.1	0	0.04	0.02	0.07	0.13
	N2W1B1R4	34	33.33	32.91	82	1,131	15.9	16.8	0	0.06	0.02	0.05	0.13
	N2W1B1R5	34	33.33	32.62	77	1,094	14.8	17.4	0	0.05	0.01	0.06	0.12
	N2W1B1R6	34	33.75	33.49	79	1,264	15.0	20.6	0	0.05	0.02	0.08	0.15
	N2W1B1R7	34	33.33	33.17	70	1,056	14.1	18.6	0	0.05	0.01	0.06	0.12
	N2W1B1R8	34	33.42	33.36	77	1,251	15.5	21.5	0	0.05	0.02	0.09	0.16
	N2W1B1R9	34	33.33	33.24	80	1,266	15.4	18.7	0	0.05	0.02	0.06	0.13
	N2W1B2R0	36	35.11	34.84	202	4,333	30.9	40.6	0	0.09	0.06	0.17	0.32
	N2W1B2R1	33	32.70	32.70	312	6,910	44.3	44.7	0	0.13	0.20	1.46	1.79
	N2W1B2R2	35	34.10	34.03	267	5,582	38.9	42.6	0	0.11	0.11	0.21	0.43
	N2W1B2R3	35	34.37	34.28	265	5,701	38.7	44.4	0	0.11	0.10	0.23	0.44
	N2W1B2R4	33	32.05	32.05	330	7,928	46.4	48.6	0	0.14	0.19	1.36	1.69
	N2W1B2R5	34	33.97	33.93	250	5,211	36.9	41.6	0	0.10	0.10	1.19	1.39
	N2W1B2R6	35	34.33	34.25	261	5,662	38.3	45.5	0	0.11	0.12	0.27	0.50
	N2W1B2R7	33	32.37	32.37	317	7,068	44.9	46.4	0	0.13	0.17	0.67	0.97
	N2W1B2R8	34	33.97	33.94	257	5,636	37.9	45.2	3	0.11	0.11	1.73	1.95
	N2W1B2R9	33	32.79	32.79	295	6,591	42.0	43.3	0	0.13	0.15	1.01	1.29
	N2W1B3R0	35	34.38	34.38	727	17,368	82.7	93.0	0	0.20	1.38	7.57	9.15
	N2W1B3R1	32	31.95	31.95	794	21,682	87.1	92.0	0	0.25	1.66	28.16	30.07
	N2W1B3R2	35	34.89	34.88	758	18,307	84.8	93.2	0	0.21	1.38	12.55	14.14
	N2W1B3R3	33	32.04	32.04	816	21,585	89.1	95.0	0	0.23	1.87	3.29	5.39
	N2W1B3R4	34	33.14	33.14	786	18,631	86.8	93.8	0	0.20	1.76	5.66	7.62
	N2W1B3R5	34	33.04	33.04	735	18,950	84.0	88.3	0	0.21	1.61	2.49	4.31
	N2W1B3R6	31	30.61	30.61	844	24,899	90.9	95.5	0	0.27	2.00	8.64	10.91
	N2W1B3R7	30	29.97	29.97	804	23,818	88.4	94.9	40	0.28	1.82	149.31	151.41
	N2W1B3R8	34	33.46	33.46	777	20,486	86.2	94.1	0	0.23	1.75	7.08	9.06
	N2W1B3R9	29	28.86	28.86	844	26,045	90.7	95.9	0	0.29	2.04	33.36	35.69
	N2W2B1R0	20	19.82	19.82	258	4,927	35.6	32.4	0	0.11	0.13	1.05	1.29
	N2W2B1R1	20	19.80	19.80	255	4,890	35.3	32.7	0	0.10	0.13	0.70	0.93
	N2W2B1R2	21	20.33	20.25	250	4,954	34.3	34.6	0	0.10	0.12	0.60	0.82
	N2W2B1R3	21	20.14	20.10	246	4,174	34.2	29.8	0	0.10	0.10	0.55	0.75
	N2W2B1R4	21	20.22	20.18	253	4,777	34.8	32.5	0	0.10	0.12	0.57	0.79
	N2W2B1R5	21	20.67	20.48	251	4,702	34.7	35.6	0	0.10	0.11	0.57	0.78
	N2W2B1R6	21	20.30	20.21	247	4,822	34.3	33.2	0	0.10	0.13	0.57	0.80
	N2W2B1R7	21	20.14	20.11	253	5,131	34.9	33.0	0	0.11	0.13	0.52	0.76
	N2W2B1R8	21	20.44	20.34	222	4,572	31.2	35.1	0	0.10	0.10	0.49	0.69
	N2W2B1R9	20	19.94	19.94	244	4,594	34.1	34.4	0	0.10	0.13	0.65	0.88
	N2W2B2R0	21	20.70	20.70	630	21,032	75.9	78.9	0	0.25	0.98	26.03	27.26
	N2W2B2R1	21	20.27	20.27	641	21,415	77.0	78.8	0	0.26	1.21	6.48	7.95
	N2W2B2R2	21	20.33	20.33	657	26,855	78.4	81.9	0	0.33	1.54	7.67	9.54
	N2W2B2R3	22	21.19	21.19	629	20,353	76.2	80.2	0	0.24	0.98	6.57	7.79
	N2W2B2R4	22	21.62	21.62	618	20,017	75.6	79.0	0	0.24	0.98	5.58	6.80
	N2W2B2R5	20	19.91	19.91	654	22,649	77.9	77.2	0	0.28	1.24	23.50	25.02
	N2W2B2R6	21	20.46	20.46	635	23,979	76.7	82.0	0	0.29	1.32	28.59	30.20
	N2W2B2R7	20	19.41	19.41	652	24,286	78.2	79.2	0	0.28	1.26	10.41	11.95
	N2W2B2R8	21	20.47	20.47	632	20,411	76.9	79.5	0	0.25	1.08	5.07	6.40
	N2W2B2R9	20	19.92	19.92	663	26,020	78.9	78.9	0	0.32	1.43	31.22	32.97
	N2W2B3R0	21	20.18	20.18	888	34,366	92.9	98.4	0	0.37	3.06	5.72	9.15
	N2W2B3R1	20	19.42	19.42	900	37,440	94.1	98.5	0	0.42	3.07	45.07	48.56
	N2W2B3R2	21	20.75	20.75	854	34,437	91.0	96.5	0	0.39	2.71	44.38	47.48
	N2W2B3R3	19	18.62	18.62	909	39,308	94.8	98.5	0	0.42	3.10	20.48	24.00
	N2W2B3R4	21	20.41	20.41	895	36,377	94.1	98.3	0	0.39	3.12	30.03	33.54
	N2W2B3R5	21	20.25	20.25	888	32,654	93.4	98.4	0	0.35	2.58	9.61	12.54
	N2W2B3R6	20	19.67	19.67	881	36,713	93.0	97.7	0	0.42	2.86	35.84	39.12
	N2W2B3R7	21	21.00	21.00	896	35,152	93.9	98.8	1	0.39	2.70	88.72	91.81
	N2W2B3R8	20	19.88	19.88	892	35,989	93.7	98.1	0	0.40	2.70	56.74	59.84
	N2W2B3R9	20	19.70	19.70	892	38,179	93.7	98.4	0	0.43	2.88	47.93	51.24
	N2W3B1R0	15	14.33	14.33	442	8,143	54.8	46.2	0	0.13	0.32	1.53	1.98
	N2W3B1R1	15	14.27	14.27	452	7,846	55.7	46.2	0	0.13	0.33	0.77	1.23
	N2W3B1R2	15	14.45	14.45	455	8,591	55.8	46.8	0	0.13	0.36	1.05	1.54
	N2W3B1R3	14	13.91	13.91	441	7,996	54.6	46.3	0	0.12	0.33	1.65	2.10
	N2W3B1R4	15	14.16	14.16	454	8,100	55.9	44.8	0	0.14	0.36	1.13	1.63
	N2W3B1R5	15	14.04	14.04	433	7,401	53.7	43.7	0	0.12	0.28	1.09	1.49

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N2W3B1R6	15	14.25	14.25	444	8,040	55.2	44.3	0	0.13	0.32	0.94	1.39
	N2W3B1R7	14	14.00	14.00	444	7,576	54.6	44.4	0	0.13	0.30	3.77	4.20
	N2W3B1R8	15	14.01	14.01	460	8,393	56.2	44.2	0	0.14	0.33	0.92	1.39
	N2W3B1R9	15	14.30	14.30	452	8,641	55.9	47.8	0	0.14	0.35	1.26	1.75
	N2W3B2R0	15	14.15	14.15	776	29,928	86.8	86.3	0	0.32	2.02	6.25	8.59
	N2W3B2R1	14	13.93	13.93	784	33,686	87.6	88.7	0	0.36	2.31	18.38	21.05
	N2W3B2R2	15	14.44	14.44	768	29,422	86.3	87.2	0	0.33	1.99	9.44	11.76
	N2W3B2R3	15	14.14	14.14	772	32,317	86.4	87.6	0	0.34	2.18	5.32	7.84
	N2W3B2R4	15	14.05	14.05	775	32,042	87.0	90.3	0	0.34	2.20	13.30	15.84
	N2W3B2R5	14	13.60	13.60	776	31,566	86.4	87.0	0	0.34	2.14	9.14	11.62
	N2W3B2R6	14	13.74	13.74	776	29,806	86.7	86.1	0	0.34	2.14	39.89	42.37
	N2W3B2R7	15	14.42	14.42	768	29,329	86.4	85.7	0	0.32	2.12	6.41	8.85
	N2W3B2R8	14	13.72	13.72	772	29,796	86.6	87.8	0	0.32	2.07	12.13	14.52
	N2W3B2R9	15	14.24	14.24	774	30,502	87.0	87.3	0	0.33	2.14	8.46	10.93
	N2W3B3R0	15	14.12	14.12	918	42,393	95.2	98.1	0	0.43	3.69	9.71	13.83
	N2W3B3R1	13	12.49	12.49	934	48,649	96.2	99.3	0	0.52	4.15	75.73	80.40
	N2W3B3R2	14	13.66	13.66	926	48,370	95.8	99.4	0	0.53	3.95	80.22	84.70
	N2W3B3R3	14	13.62	13.62	930	46,448	95.8	99.5	0	0.51	3.94	79.58	84.03
	N2W3B3R4	15	14.45	14.45	934	46,214	96.2	99.4	0	0.50	3.70	72.59	76.79
	N2W3B3R5	15	14.24	14.24	938	43,543	96.3	99.3	0	0.44	4.06	9.85	14.35
	N2W3B3R6	15	14.16	14.16	894	39,373	93.0	90.4	0	0.43	3.05	7.82	11.30
	N2W3B3R7	13	12.44	12.44	938	47,341	96.3	99.4	0	0.48	4.06	19.10	23.64
	N2W3B3R8	15	14.04	14.04	934	42,887	96.1	98.6	0	0.46	3.61	41.94	46.01
	N2W3B3R9	14	13.30	13.30	924	44,687	95.1	97.6	0	0.49	4.15	70.96	75.60
	N2W4B1R0	12	11.27	11.27	613	10,200	71.7	56.3	0	0.13	0.55	2.09	2.77
	N2W4B1R1	12	11.09	11.09	616	10,224	71.9	55.9	0	0.13	0.57	1.08	1.78
	N2W4B1R2	12	11.30	11.30	596	10,105	69.9	55.2	0	0.14	0.54	1.31	1.99
	N2W4B1R3	12	11.22	11.22	616	10,759	72.0	56.8	0	0.15	0.59	1.68	2.42
	N2W4B1R4	12	11.18	11.18	600	9,580	70.2	54.4	0	0.13	0.56	1.35	2.04
	N2W4B1R5	12	11.19	11.19	595	10,056	69.4	57.4	0	0.13	0.55	1.45	2.13
	N2W4B1R6	11	10.98	10.98	595	10,132	69.8	54.9	0	0.14	0.57	2.25	2.96
	N2W4B1R7	12	11.10	11.10	605	10,226	70.5	54.7	0	0.14	0.56	1.06	1.76
	N2W4B1R8	11	10.94	10.94	623	10,714	72.5	55.0	0	0.16	0.56	7.80	8.52
	N2W4B1R9	11	10.94	10.94	571	8,911	67.2	52.8	0	0.13	0.48	0.89	1.50
	N2W4B2R0	11	10.65	10.65	842	35,800	91.1	89.4	0	0.36	2.49	12.28	15.13
	N2W4B2R1	11	10.67	10.67	841	37,306	90.8	90.6	0	0.39	2.64	15.23	18.26
	N2W4B2R2	11	10.93	10.93	843	37,449	91.0	90.8	0	0.41	2.62	60.88	63.91
	N2W4B2R3	11	10.42	10.42	857	39,738	92.0	90.3	0	0.40	3.06	7.64	11.10
	N2W4B2R4	12	11.30	11.30	816	30,188	89.4	91.5	0	0.32	1.99	12.41	14.72
	N2W4B2R5	12	11.26	11.26	824	31,661	89.9	90.8	0	0.33	2.32	12.35	15.00
	N2W4B2R6	12	11.15	11.15	835	35,089	90.5	91.0	0	0.36	2.58	14.09	17.03
	N2W4B2R7	11	10.85	10.85	851	36,943	91.7	90.2	0	0.42	2.80	62.83	66.05
	N2W4B2R8	12	11.75	11.75	826	31,638	90.0	92.0	0	0.32	2.34	19.04	21.70
	N2W4B2R9	11	10.83	10.83	835	30,833	90.9	88.1	0	0.32	2.28	10.10	12.70
	N2W4B3R0	12	11.03	11.03	939	47,189	96.4	99.6	0	0.49	3.87	21.52	25.88
	N2W4B3R1	12	11.13	11.13	949	47,576	97.0	99.7	0	0.48	3.85	11.33	15.66
	N2W4B3R2	12	11.84	11.84	943	43,066	96.4	98.7	0	0.46	3.81	92.20	96.47
	N2W4B3R3	11	10.99	10.99	951	48,266	97.3	99.7	8	0.54	4.12	262.51	267.17
	N2W4B3R4	12	11.16	11.16	956	49,475	97.4	99.6	0	0.49	4.09	27.86	32.44
	N2W4B3R5	10	9.98	9.98	965	52,243	97.9	99.7	0	0.57	4.39	121.17	126.13
	N2W4B3R6	11	10.62	10.62	956	50,924	97.3	99.7	0	0.54	4.40	66.96	71.90
	N2W4B3R7	11	10.75	10.75	941	50,729	96.4	98.1	0	0.55	4.14	92.01	96.70
	N2W4B3R8	11	10.88	10.88	943	46,572	96.8	99.5	0	0.50	3.88	80.17	84.55
	N2W4B3R9	12	11.76	11.76	930	45,646	95.9	99.2	0	0.46	3.60	10.73	14.79
	N3W1B1R0	67	66.67	66.55	115	2,427	20.4	22.9	0	0.08	0.04	0.14	0.26
	N3W1B1R1	67	66.67	66.58	112	2,424	20.3	21.0	0	0.09	0.04	0.17	0.30
	N3W1B1R2	67	66.67	66.01	120	2,364	21.0	19.4	0	0.09	0.03	0.15	0.27
	N3W1B1R3	67	66.67	66.53	120	2,527	21.2	21.0	0	0.09	0.04	0.17	0.30
	N3W1B1R4	67	66.67	66.49	121	2,643	21.2	22.6	0	0.09	0.04	0.35	0.48
	N3W1B1R5	67	66.67	66.31	113	2,262	20.1	20.9	0	0.08	0.03	0.24	0.35
	N3W1B1R6	68	67.03	66.80	123	2,478	21.5	20.4	0	0.09	0.05	0.22	0.36
	N3W1B1R7	67	66.67	64.98	106	1,890	19.1	18.0	0	0.08	0.02	0.10	0.20
	N3W1B1R8	67	66.67	66.33	117	2,386	20.9	20.8	0	0.09	0.03	0.13	0.25
	N3W1B1R9	67	66.67	66.19	119	2,459	20.9	20.6	0	0.09	0.03	0.16	0.28

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N3W1B2R0	67	66.46	66.46	420	16,704	56.5	56.7	0	0.27	0.69	3.28	4.24
	N3W1B2R1	68	67.23	67.21	400	15,967	54.3	58.3	0	0.25	0.47	1.47	2.19
	N3W1B2R2	65	64.43	64.43	429	17,569	57.5	57.3	0	0.28	0.67	3.97	4.92
	N3W1B2R3	65	64.53	64.53	432	17,307	57.8	56.1	0	0.28	0.72	2.76	3.76
	N3W1B2R4	68	67.68	67.60	406	17,235	54.9	58.1	0	0.28	0.47	2.81	3.56
	N3W1B2R5	65	64.54	64.54	414	16,820	55.7	56.6	0	0.27	0.62	4.43	5.32
	N3W1B2R6	66	65.78	65.78	427	17,409	57.5	58.0	0	0.28	0.68	4.13	5.09
	N3W1B2R7	66	65.81	65.81	410	17,017	55.4	58.6	0	0.27	0.74	3.73	4.74
	N3W1B2R8	66	65.58	65.58	404	16,228	54.7	57.4	0	0.28	0.58	5.72	6.58
	N3W1B2R9	66	65.81	65.81	414	16,641	56.0	57.9	0	0.27	0.65	4.03	4.95
	N3W1B3R0	68	67.98	67.98	828	42,587	89.9	96.8	0	0.49	4.03	67.74	72.26
	N3W1B3R1	66	65.92	65.92	800	43,054	88.3	97.1	0	0.49	4.11	71.21	75.81
	N3W1B3R2	67	66.66	66.66	836	42,415	90.5	97.0	0	0.46	4.07	21.27	25.80
	N3W1B3R3	66	65.73	65.73	850	42,793	91.3	96.5	0	0.49	3.86	76.68	81.03
	N3W1B3R4	68	67.61	67.61	857	44,411	91.9	97.5	0	0.50	3.84	22.20	26.54
	N3W1B3R5	65	64.00	64.00	863	43,896	91.8	97.3	0	0.49	4.39	11.14	16.02
	N3W1B3R6	66	65.39	65.39	827	40,230	89.8	97.2	0	0.45	3.67	20.19	24.31
	N3W1B3R7	61	60.22	60.22	875	51,975	92.9	97.7	0	0.56	4.74	29.71	35.01
	N3W1B3R8	65	64.91	64.91	856	45,031	91.5	97.5	0	0.50	4.25	75.97	80.72
	N3W1B3R9	70	69.40	69.40	806	38,144	88.5	94.5	0	0.43	2.95	21.52	24.90
	N3W2B1R0	41	40.27	40.22	287	7,162	38.3	33.2	0	0.16	0.22	1.11	1.49
	N3W2B1R1	41	40.44	40.34	274	6,592	37.0	35.3	0	0.13	0.18	0.72	1.03
	N3W2B1R2	41	40.10	40.08	282	7,055	37.8	35.0	0	0.15	0.22	0.87	1.24
	N3W2B1R3	41	40.17	40.14	274	6,638	36.9	32.6	0	0.15	0.18	1.18	1.51
	N3W2B1R4	40	39.58	39.58	287	6,872	38.5	32.6	0	0.15	0.21	1.36	1.72
	N3W2B1R5	41	40.01	40.01	279	6,416	37.6	32.2	0	0.15	0.19	0.73	1.07
	N3W2B1R6	41	40.42	40.33	282	7,096	37.9	34.2	0	0.15	0.21	1.01	1.37
	N3W2B1R7	41	40.18	40.14	261	5,871	35.5	33.3	0	0.13	0.17	0.99	1.29
	N3W2B1R8	40	40.00	40.00	285	6,929	38.2	32.6	0	0.15	0.23	1.12	1.50
	N3W2B1R9	41	40.23	40.19	285	6,711	38.3	32.7	0	0.15	0.21	1.04	1.40
	N3W2B2R0	40	39.90	39.90	715	43,927	82.5	83.3	0	0.52	3.03	16.28	19.83
	N3W2B2R1	40	39.65	39.65	726	46,377	83.3	81.7	0	0.57	3.34	19.56	23.47
	N3W2B2R2	39	38.80	38.80	717	42,850	82.6	80.1	0	0.51	2.92	25.97	29.40
	N3W2B2R3	39	38.48	38.48	727	44,968	83.3	81.0	0	0.53	3.28	16.50	20.31
	N3W2B2R4	41	40.15	40.15	713	41,221	82.7	82.0	0	0.50	2.82	18.21	21.53
	N3W2B2R5	40	39.38	39.38	708	39,735	82.1	83.8	0	0.45	2.75	13.67	16.87
	N3W2B2R6	40	39.74	39.74	711	41,491	82.4	81.3	0	0.50	2.88	18.51	21.89
	N3W2B2R7	40	39.72	39.72	710	43,405	82.3	84.7	0	0.51	2.91	13.75	17.17
	N3W2B2R8	42	41.58	41.58	703	40,858	81.8	82.6	0	0.49	2.45	14.62	17.56
	N3W2B2R9	40	39.62	39.62	705	40,517	81.9	83.2	0	0.47	2.89	29.64	33.00
	N3W2B3R0	42	41.92	41.92	918	63,275	95.1	99.2	0	0.72	5.68	177.24	183.64
	N3W2B3R1	40	39.14	39.14	923	67,479	95.6	98.8	0	0.73	6.50	36.84	44.07
	N3W2B3R2	39	38.70	38.70	923	73,800	95.6	99.2	0	0.86	6.92	159.93	167.71
	N3W2B3R3	39	38.76	38.76	914	68,813	95.0	99.2	0	0.74	6.50	42.37	49.61
	N3W2B3R4	42	41.87	41.87	920	65,503	95.2	99.3	0	0.75	6.13	171.58	178.46
	N3W2B3R5	40	39.96	39.96	931	72,597	96.2	98.4	0	0.82	6.94	206.71	214.47
	N3W2B3R6	40	39.94	39.94	924	68,834	95.6	99.4	0	0.80	6.24	182.19	189.23
	N3W2B3R7	42	41.15	41.15	908	66,844	94.5	97.9	0	0.74	6.23	36.08	43.05
	N3W2B3R8	39	38.29	38.29	930	70,258	95.9	98.3	0	0.77	6.67	41.69	49.13
	N3W2B3R9	39	38.46	38.46	919	72,345	95.2	99.3	0	0.77	6.99	43.44	51.20
	N3W3B1R0	28	27.99	27.99	482	10,332	58.5	46.8	0	0.17	0.48	8.07	8.72
	N3W3B1R1	28	27.95	27.95	483	10,515	58.6	46.9	0	0.17	0.47	8.06	8.70
	N3W3B1R2	29	28.28	28.28	489	10,623	59.3	47.0	0	0.16	0.49	2.74	3.39
	N3W3B1R3	29	28.16	28.16	485	10,440	58.9	47.7	0	0.17	0.46	2.09	2.72
	N3W3B1R4	29	28.15	28.15	480	10,493	58.3	47.2	0	0.17	0.48	2.41	3.06
	N3W3B1R5	29	28.12	28.12	480	10,458	58.4	48.1	0	0.17	0.51	2.07	2.75
	N3W3B1R6	29	28.07	28.07	482	9,793	58.8	46.6	0	0.15	0.44	2.47	3.06
	N3W3B1R7	29	28.47	28.47	466	9,759	57.0	45.4	0	0.16	0.44	2.66	3.26
	N3W3B1R8	29	28.13	28.13	478	10,541	58.0	47.0	0	0.17	0.49	2.62	3.28
	N3W3B1R9	29	28.35	28.35	480	10,146	58.4	49.7	0	0.15	0.44	2.24	2.83
	N3W3B2R0	29	28.36	28.36	816	52,285	89.6	90.0	0	0.57	4.22	23.01	27.80
	N3W3B2R1	29	28.04	28.04	817	50,844	89.4	89.0	0	0.55	4.02	25.93	30.50
	N3W3B2R2	28	27.87	27.87	816	49,686	89.4	88.8	0	0.55	4.14	50.60	55.29
	N3W3B2R3	29	28.27	28.27	813	48,137	89.2	89.5	0	0.53	3.77	18.06	22.36

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N3W3B2R4	30	29.14	29.14	811	51,147	89.1	90.7	0	0.56	3.95	21.21	25.72
	N3W3B2R5	29	28.26	28.26	818	50,810	89.7	90.5	0	0.56	4.13	22.59	27.28
	N3W3B2R6	29	28.40	28.40	809	47,182	89.1	89.6	0	0.52	3.80	24.46	28.78
	N3W3B2R7	28	27.61	27.61	822	51,205	89.6	89.0	0	0.57	3.66	25.98	30.21
	N3W3B2R8	28	27.60	27.60	803	47,887	88.6	89.7	0	0.51	3.94	23.11	27.56
	N3W3B2R9	29	28.48	28.48	812	50,588	89.2	90.4	0	0.56	3.72	27.24	31.52
	N3W3B3R0	27	26.37	26.37	948	80,288	97.0	99.7	0	0.83	7.92	48.60	57.35
	N3W3B3R1	27	26.26	26.26	949	79,962	97.2	98.9	0	0.87	7.97	57.36	66.20
	N3W3B3R2	29	28.94	28.94	956	81,746	97.5	99.6	0	0.95	8.58	286.32	295.85
	N3W3B3R3	29	28.47	28.47	946	78,443	96.6	98.6	0	0.82	7.69	46.75	55.26
	N3W3B3R4	29	28.49	28.49	940	79,034	96.4	99.7	0	0.84	7.37	48.88	57.09
	N3W3B3R5	28	27.25	27.25	949	78,484	97.1	99.3	0	0.81	7.74	57.88	66.43
	N3W3B3R6	29	28.19	28.19	952	78,115	97.1	99.7	0	0.84	7.06	41.74	49.64
	N3W3B3R7	28	28.00	28.00	958	82,101	97.7	99.7	0	0.85	8.05	62.51	71.41
	N3W3B3R8	29	28.86	28.86	936	76,552	96.3	99.6	0	0.80	7.58	54.22	62.60
	N3W3B3R9	29	28.87	28.87	958	79,984	97.6	99.7	0	0.86	8.12	46.01	54.99
	N3W4B1R0	23	22.21	22.21	636	11,712	73.9	56.4	0	0.16	0.56	3.16	3.88
	N3W4B1R1	23	22.25	22.25	638	11,831	74.1	56.7	0	0.16	0.71	3.43	4.30
	N3W4B1R2	23	22.46	22.46	641	11,865	74.4	56.8	0	0.16	0.62	4.56	5.34
	N3W4B1R3	23	22.27	22.27	582	10,350	68.4	53.5	0	0.15	0.54	12.12	12.81
	N3W4B1R4	23	22.21	22.21	648	11,818	75.3	56.7	0	0.16	0.63	3.30	4.09
	N3W4B1R5	23	22.18	22.18	646	11,873	75.0	56.9	0	0.16	0.64	3.24	4.04
	N3W4B1R6	23	22.49	22.49	632	11,716	73.5	58.2	0	0.15	0.65	3.62	4.42
	N3W4B1R7	23	22.22	22.22	635	11,804	73.8	56.7	0	0.16	0.65	3.03	3.84
	N3W4B1R8	23	22.52	22.52	635	11,789	73.8	56.6	0	0.17	0.62	5.93	6.72
	N3W4B1R9	23	22.18	22.18	630	11,606	73.3	56.4	0	0.16	0.61	3.48	4.25
	N3W4B2R0	23	22.53	22.53	862	49,000	92.3	92.5	0	0.51	3.87	37.02	41.40
	N3W4B2R1	22	21.83	21.83	862	49,263	92.3	92.9	0	0.54	4.17	233.33	238.04
	N3W4B2R2	22	21.34	21.34	871	51,676	92.9	92.5	0	0.55	3.91	19.10	23.56
	N3W4B2R3	22	21.69	21.69	869	51,665	92.7	92.0	0	0.54	4.18	31.88	36.60
	N3W4B2R4	22	21.88	21.88	866	50,927	92.5	92.5	0	0.54	3.97	22.27	26.78
	N3W4B2R5	23	22.76	22.76	870	52,033	92.8	92.9	0	0.57	4.57	232.84	237.98
	N3W4B2R6	22	21.85	21.85	869	50,534	92.7	93.1	0	0.51	4.27	28.14	32.92
	N3W4B2R7	22	21.64	21.64	875	50,943	93.1	91.8	0	0.55	4.41	50.71	55.67
	N3W4B2R8	23	22.13	22.13	875	52,020	93.1	91.8	0	0.55	4.27	28.84	33.66
	N3W4B2R9	22	21.20	21.20	875	51,650	93.1	91.9	0	0.53	4.90	27.62	33.05
	N3W4B3R0	24	23.93	23.93	962	78,241	97.9	99.8	0	0.83	7.98	65.43	74.24
	N3W4B3R1	22	21.58	21.58	969	79,266	98.2	99.8	0	0.84	8.31	48.22	57.37
	N3W4B3R2	23	22.51	22.51	956	73,248	97.5	99.7	0	0.74	7.40	49.84	57.98
	N3W4B3R3	24	23.91	23.91	967	74,523	98.1	99.8	0	0.80	7.33	74.10	82.23
	N3W4B3R4	23	22.43	22.43	963	77,230	97.9	99.8	0	0.80	7.65	50.17	58.62
	N3W4B3R5	23	22.08	22.08	965	81,216	97.9	99.8	0	0.84	7.52	57.51	65.87
	N3W4B3R6	23	22.48	22.48	951	80,267	96.9	99.8	0	0.82	7.80	62.54	71.16
	N3W4B3R7	21	20.75	20.75	967	82,382	98.1	99.8	0	0.84	8.36	52.09	61.29
	N3W4B3R8	23	22.45	22.45	962	79,502	97.8	99.8	0	0.82	8.19	51.66	60.67
	N3W4B3R9	23	22.05	22.05	959	75,582	97.6	98.0	0	0.80	7.72	45.15	53.67
	N4W1B1R0	167	166.67	164.06	135	3,454	22.7	21.6	0	0.12	0.05	0.15	0.32
	N4W1B1R1	167	166.67	163.83	139	3,405	23.4	21.2	0	0.13	0.04	0.15	0.32
	N4W1B1R2	167	166.67	164.87	136	3,361	22.9	21.2	0	0.12	0.05	0.18	0.35
	N4W1B1R3	167	166.67	165.57	137	3,439	23.1	21.5	0	0.12	0.05	0.19	0.36
	N4W1B1R4	167	166.67	165.27	138	3,463	23.2	22.3	0	0.12	0.05	0.21	0.38
	N4W1B1R5	167	166.67	164.09	138	3,413	23.2	21.3	0	0.13	0.04	0.15	0.32
	N4W1B1R6	167	166.67	166.56	138	3,485	23.2	22.2	0	0.12	0.07	0.31	0.50
	N4W1B1R7	167	166.67	164.61	137	3,504	23.3	23.0	0	0.12	0.04	0.17	0.33
	N4W1B1R8	167	166.67	164.28	136	3,425	22.9	21.8	0	0.12	0.04	0.16	0.32
	N4W1B1R9	168	167.41	166.94	136	3,444	22.9	21.9	0	0.12	0.07	0.33	0.52
	N4W1B2R0	164	163.31	163.31	560	42,388	69.7	65.3	0	0.66	2.49	11.47	14.62
	N4W1B2R1	170	169.31	169.11	555	41,836	69.5	66.8	0	0.66	1.84	9.11	11.61
	N4W1B2R2	164	163.33	163.33	551	40,485	69.0	66.5	0	0.64	2.46	23.45	26.55
	N4W1B2R3	166	165.59	165.59	573	45,728	70.8	66.0	0	0.73	3.10	40.37	44.20
	N4W1B2R4	165	164.25	164.25	560	42,980	69.7	67.0	0	0.66	2.82	14.87	18.35
	N4W1B2R5	161	160.73	160.73	563	42,479	69.9	64.2	0	0.68	2.54	31.14	34.36
	N4W1B2R6	168	167.87	167.81	542	40,935	68.0	65.3	0	0.66	1.75	6.64	9.05
	N4W1B2R7	168	167.25	167.23	549	42,005	69.0	67.1	0	0.65	2.35	6.79	9.79

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N4W1B2R8	167	166.38	166.38	541	39,545	68.3	66.0	0	0.62	2.52	10.42	13.56
	N4W1B2R9	169	168.33	168.25	550	40,309	69.1	64.9	0	0.64	2.05	10.12	12.81
	N4W1B3R0	166	165.17	165.17	910	106,185	95.0	98.5	0	1.29	11.20	74.27	86.76
	N4W1B3R1	171	170.27	170.27	889	105,020	93.9	98.7	0	1.24	11.25	81.36	93.85
	N4W1B3R2	172	171.73	171.73	896	101,028	94.3	98.4	0	1.19	11.30	73.04	85.53
	N4W1B3R3	170	169.18	169.18	886	92,730	93.8	97.9	0	1.12	9.13	55.98	66.23
	N4W1B3R4	158	157.10	157.10	902	109,167	94.5	98.6	0	1.33	11.51	81.73	94.57
	N4W1B3R5	162	161.08	161.08	904	101,020	94.7	98.4	0	1.20	11.12	70.87	83.19
	N4W1B3R6	169	168.78	168.78	885	100,114	93.6	98.7	0	1.22	10.40	66.02	77.64
	N4W1B3R7	163	162.04	162.04	912	101,674	95.2	98.4	0	1.19	11.42	76.09	88.70
	N4W1B3R8	170	169.97	169.97	906	96,644	94.9	98.5	16	1.18	9.74	1,258.98	1,269.90
	N4W1B3R9	162	161.45	161.45	891	102,409	93.9	98.7	0	1.28	11.23	192.04	204.55
	N4W2B1R0	101	100.89	100.69	299	7,590	39.7	33.7	0	0.17	0.22	1.18	1.57
	N4W2B1R1	101	100.72	100.56	299	7,604	39.7	33.7	0	0.16	0.21	1.62	1.99
	N4W2B1R2	101	100.51	100.40	295	7,598	39.2	33.7	0	0.17	0.20	1.24	1.61
	N4W2B1R3	100	100.00	100.00	299	7,603	39.7	33.7	0	0.17	0.23	2.00	2.40
	N4W2B1R4	101	100.21	100.17	299	7,605	39.7	33.7	0	0.17	0.22	1.52	1.91
	N4W2B1R5	103	102.45	101.84	299	7,596	39.7	33.7	0	0.17	0.19	3.46	3.82
	N4W2B1R6	102	101.15	100.89	299	7,604	39.7	33.7	0	0.17	0.20	1.30	1.67
	N4W2B1R7	101	100.19	100.15	299	7,597	39.7	33.7	0	0.16	0.21	1.72	2.09
	N4W2B1R8	101	100.58	100.46	296	7,592	39.3	33.6	0	0.16	0.20	1.35	1.71
	N4W2B1R9	101	100.46	100.36	299	7,607	39.7	33.7	0	0.17	0.21	1.28	1.66
	N4W2B2R0	101	100.03	100.03	768	66,403	86.2	83.7	0	0.79	5.56	43.13	49.48
	N4W2B2R1	100	99.87	99.87	773	67,834	86.6	84.7	0	0.86	5.48	83.94	90.28
	N4W2B2R2	102	102.00	102.00	768	65,385	86.3	84.4	0	0.84	4.93	202.81	208.58
	N4W2B2R3	102	101.27	101.27	770	66,254	86.3	84.0	0	0.81	5.21	38.11	44.13
	N4W2B2R4	100	99.23	99.23	777	67,701	86.9	84.8	0	0.81	5.17	39.82	45.80
	N4W2B2R5	102	101.88	101.88	763	65,319	85.9	84.7	0	0.82	5.07	288.46	294.35
	N4W2B2R6	103	102.33	102.33	772	66,822	86.5	84.7	0	0.83	5.36	36.75	42.94
	N4W2B2R7	101	100.80	100.80	769	65,938	86.3	83.7	0	0.83	5.19	69.48	75.50
	N4W2B2R8	100	99.13	99.13	780	68,981	87.1	84.4	0	0.82	5.57	41.29	47.68
	N4W2B2R9	102	101.55	101.55	765	64,451	86.2	84.5	0	0.82	4.89	297.47	303.18
	N4W2B3R0	101	100.60	100.60	946	127,294	97.0	99.6	0	1.48	16.78	129.96	148.22
	N4W2B3R1	101	100.87	100.87	946	130,447	97.0	99.6	0	1.51	16.72	180.48	198.71
	N4W2B3R2	100	99.23	99.23	946	133,615	97.0	99.6	0	1.55	16.64	113.27	131.46
	N4W2B3R3	100	99.94	99.94	942	123,579	96.8	99.4	0	1.49	15.71	231.52	248.72
	N4W2B3R4	100	99.39	99.39	947	127,959	97.0	99.5	0	1.46	15.71	96.14	113.31
	N4W2B3R5	101	100.55	100.55	945	132,622	97.0	99.6	0	1.52	16.70	110.41	128.63
	N4W2B3R6	99	98.24	98.24	941	132,123	96.8	99.6	0	1.56	19.06	108.01	128.63
	N4W2B3R7	101	100.21	100.21	949	133,247	97.2	99.6	0	1.56	17.88	99.23	118.67
	N4W2B3R8	99	98.25	98.25	939	131,749	96.7	99.7	0	1.53	15.75	120.89	138.17
	N4W2B3R9	102	101.28	101.28	943	129,628	96.8	99.6	0	1.50	15.84	113.38	130.72
	N4W3B1R0	71	70.60	70.60	490	10,772	59.4	47.1	0	0.17	0.44	2.51	3.12
	N4W3B1R1	71	70.84	70.84	490	10,784	59.4	47.2	0	0.17	0.46	2.52	3.15
	N4W3B1R2	71	70.89	70.89	490	10,785	59.4	47.2	0	0.18	0.44	10.68	11.30
	N4W3B1R3	71	70.10	70.10	491	10,778	59.5	47.2	0	0.18	0.47	4.67	5.32
	N4W3B1R4	71	70.42	70.42	490	10,788	59.4	47.2	0	0.18	0.47	17.97	18.62
	N4W3B1R5	71	70.63	70.63	490	10,782	59.4	47.2	0	0.17	0.42	2.84	3.43
	N4W3B1R6	71	70.11	70.11	490	10,783	59.4	47.2	0	0.17	0.45	2.54	3.16
	N4W3B1R7	71	70.08	70.08	490	10,788	59.4	47.2	0	0.18	0.44	3.20	3.82
	N4W3B1R8	71	70.12	70.12	490	10,776	59.4	47.2	0	0.18	0.46	10.28	10.92
	N4W3B1R9	71	70.98	70.98	490	10,782	59.4	47.2	0	0.17	0.45	2.51	3.13
	N4W3B2R0	71	70.64	70.64	848	67,243	91.4	90.2	0	0.77	6.09	312.51	319.37
	N4W3B2R1	71	70.04	70.04	848	66,525	91.4	90.1	0	0.76	5.88	116.09	122.73
	N4W3B2R2	71	70.30	70.30	852	68,661	91.6	90.3	0	0.78	6.12	275.38	282.28
	N4W3B2R3	71	70.41	70.41	855	68,355	91.8	90.2	0	0.78	5.91	44.07	50.76
	N4W3B2R4	73	72.22	72.22	841	64,751	91.0	90.6	0	0.72	5.85	59.56	66.13
	N4W3B2R5	72	71.08	71.08	850	67,144	91.5	90.1	0	0.76	6.11	322.68	329.55
	N4W3B2R6	71	70.32	70.32	846	65,010	91.4	90.1	0	0.73	5.72	114.15	120.60
	N4W3B2R7	70	69.78	69.78	853	67,935	91.7	90.3	0	0.78	5.96	303.25	309.99
	N4W3B2R8	72	71.24	71.24	853	68,909	91.7	90.4	0	0.79	6.23	289.16	296.18
	N4W3B2R9	70	69.68	69.68	853	67,218	91.7	90.1	0	0.77	5.95	325.12	331.84
	N4W3B3R0	72	71.95	71.95	964	123,161	97.9	99.8	0	1.37	15.99	185.00	202.36
	N4W3B3R1	71	70.41	70.41	965	128,535	98.1	99.8	0	1.42	16.44	141.07	158.93

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	N4W3B3R2	72	71.39	71.39	969	130,205	98.3	99.8	0	1.45	16.12	132.45	150.02
	N4W3B3R3	71	70.44	70.44	967	129,330	98.1	99.8	0	1.45	17.63	125.22	144.30
	N4W3B3R4	73	72.13	72.13	961	131,323	97.9	99.8	0	1.50	17.96	145.08	164.54
	N4W3B3R5	73	72.78	72.78	967	127,304	98.1	99.8	0	1.43	16.96	233.15	251.54
	N4W3B3R6	73	72.48	72.48	965	131,904	98.1	99.8	0	1.48	16.89	131.90	150.27
	N4W3B3R7	74	73.47	73.47	967	126,356	98.2	99.8	0	1.38	16.44	128.49	146.31
	N4W3B3R8	72	71.76	71.76	966	129,280	98.0	99.8	0	1.40	16.19	126.22	143.81
	N4W3B3R9	71	70.36	70.36	967	131,772	98.1	99.8	0	1.44	15.43	132.19	149.06
	N4W4B1R0	56	55.36	55.36	651	12,020	75.6	57.1	0	0.18	0.62	14.87	15.67
	N4W4B1R1	56	55.58	55.58	651	12,031	75.6	57.1	0	0.16	0.58	23.49	24.23
	N4W4B1R2	56	55.63	55.63	651	12,026	75.6	57.1	0	0.16	0.59	23.52	24.27
	N4W4B1R3	56	55.56	55.56	651	12,032	75.6	57.1	0	0.18	0.60	24.12	24.90
	N4W4B1R4	56	55.44	55.44	651	12,026	75.6	57.1	0	0.17	0.59	22.73	23.49
	N4W4B1R5	56	55.66	55.66	651	12,029	75.6	57.1	0	0.16	0.59	22.53	23.28
	N4W4B1R6	56	55.74	55.74	651	12,029	75.6	57.1	0	0.16	0.60	3.22	3.98
	N4W4B1R7	56	55.54	55.54	651	12,022	75.6	57.0	0	0.17	0.57	21.76	22.50
	N4W4B1R8	56	55.59	55.59	651	12,026	75.6	57.1	8	0.17	0.60	36.46	37.23
	N4W4B1R9	55	54.85	54.85	649	12,025	75.4	57.0	0	0.17	0.58	20.88	21.63
	N4W4B2R0	55	54.62	54.62	884	61,126	93.6	92.7	0	0.67	6.10	245.04	251.81
	N4W4B2R1	57	56.08	56.08	886	62,508	93.8	93.6	0	0.67	5.36	57.20	63.23
	N4W4B2R2	57	56.22	56.22	886	62,384	93.8	93.4	0	0.68	5.39	273.27	279.34
	N4W4B2R3	57	56.19	56.19	888	62,425	93.9	92.8	0	0.69	5.54	253.89	260.12
	N4W4B2R4	56	55.24	55.24	888	62,729	93.9	92.8	0	0.71	5.55	267.30	273.56
	N4W4B2R5	55	54.88	54.88	888	62,506	93.9	92.8	0	0.69	5.34	250.65	256.68
	N4W4B2R6	56	55.06	55.06	888	62,781	93.9	92.8	0	0.69	5.46	271.01	277.16
	N4W4B2R7	57	56.09	56.09	884	61,097	93.6	92.7	0	0.68	5.23	64.84	70.75
	N4W4B2R8	55	54.69	54.69	884	60,261	93.6	92.6	0	0.66	5.51	257.79	263.96
	N4W4B2R9	56	55.15	55.15	888	62,742	93.9	92.8	0	0.70	5.63	299.90	306.23
	N4W4B3R0	55	54.04	54.04	974	119,827	98.5	99.9	0	1.28	16.00	118.51	135.79
	N4W4B3R1	54	53.73	53.73	977	119,728	98.7	99.9	0	1.28	15.86	134.04	151.18
	N4W4B3R2	57	56.38	56.38	977	120,017	98.7	99.9	0	1.29	17.48	124.52	143.29
	N4W4B3R3	56	55.45	55.45	977	118,018	98.7	99.9	0	1.27	15.26	111.57	128.10
	N4W4B3R4	55	54.46	54.46	977	122,623	98.7	99.9	0	1.29	15.98	114.11	131.38
	N4W4B3R5	55	54.56	54.56	976	121,307	98.6	99.9	0	1.38	16.30	792.97	810.65
	N4W4B3R6	58	57.41	57.41	975	117,545	98.6	99.8	0	1.31	15.59	699.65	716.55
	N4W4B3R7	57	56.62	56.62	977	123,126	98.7	99.9	0	1.30	16.46	127.56	145.32
	N4W4B3R8	57	56.84	56.84	973	116,830	98.5	99.9	0	1.23	14.44	116.65	132.32
	N4W4B3R9	56	55.30	55.30	977	119,514	98.7	99.9	0	1.26	16.82	121.28	139.36
bin3	HARD0	56	55.01	54.40	1,964	93,615	3.9	6.0	0	17.10	3.49	7.42	28.01
	HARD1	57	56.04	55.39	1,760	78,160	3.6	5.2	0	16.81	2.67	9.49	28.97
	HARD2	56	55.98	55.56	1,693	73,098	3.5	4.9	0	16.44	2.46	9.91	28.81
	HARD3	55	54.92	54.42	1,963	89,007	4.0	5.7	0	17.15	3.33	9.43	29.91
	HARD4	57	56.44	55.51	1,631	71,838	3.4	5.0	0	15.80	2.39	8.00	26.19
	HARD5	56	55.52	54.81	1,896	87,875	3.9	5.9	0	16.48	3.14	21.48	41.10
	HARD6	57	56.10	55.44	1,726	77,377	3.6	5.3	0	16.10	2.62	11.79	30.51
	HARD7	55	54.25	53.99	2,201	99,635	4.4	5.7	0	19.12	3.72	10.98	33.82
	HARD8	57	56.09	55.54	1,743	76,255	3.6	5.2	0	16.19	2.65	6.10	24.94
	HARD9	56	55.47	55.00	1,885	83,297	3.8	5.3	0	17.13	2.76	14.40	34.29

Table A.5: Schwerin/Waescher/Gau results.

class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
SCH.WAE1	BPP 1	18	17.54	17.33	200	3,676	28.6	29.0	0	0.09	0.07	0.31	0.47
	BPP 2	18	17.57	17.37	216	3,803	30.9	30.3	0	0.09	0.09	0.36	0.54
	BPP 3	18	17.60	17.34	209	3,660	30.4	29.9	0	0.08	0.08	0.32	0.48
	BPP 4	18	17.74	17.48	199	3,450	28.9	29.3	0	0.08	0.07	0.24	0.39
	BPP 5	18	17.60	17.46	198	3,375	28.4	29.1	0	0.08	0.08	0.53	0.69
	BPP 6	18	17.93	17.60	208	3,903	29.8	30.0	0	0.08	0.08	0.86	1.02
	BPP 7	18	17.79	17.53	198	3,224	28.6	29.4	0	0.07	0.07	0.26	0.40
	BPP 8	18	17.81	17.59	201	3,466	28.8	28.6	0	0.08	0.07	0.27	0.42
	BPP 9	18	17.88	17.55	211	3,798	30.8	33.1	0	0.08	0.07	0.51	0.66
	BPP 10	18	17.57	17.34	197	3,315	28.6	32.7	0	0.07	0.07	0.30	0.44

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	BPP 11	18	17.93	17.66	200	3,572	28.8	29.8	0	0.08	0.08	0.51	0.67
	BPP 12	18	17.73	17.50	204	3,780	29.6	31.0	0	0.09	0.08	0.31	0.48
	BPP 13	18	17.66	17.42	203	3,630	29.5	32.5	0	0.08	0.08	0.92	1.08
	BPP 14	18	17.91	17.46	196	3,489	28.6	31.5	0	0.07	0.07	0.28	0.42
	BPP 15	18	17.28	17.17	204	3,498	29.7	29.9	0	0.08	0.08	0.34	0.50
	BPP 16	18	17.59	17.37	207	3,579	30.0	29.3	0	0.09	0.09	0.28	0.46
	BPP 17	18	17.88	17.57	206	3,662	29.8	31.1	0	0.08	0.08	0.63	0.79
	BPP 18	18	17.91	17.57	195	3,200	28.6	30.1	0	0.07	0.06	0.44	0.57
	BPP 19	18	17.96	17.66	199	3,463	28.7	31.4	0	0.07	0.07	0.85	0.99
	BPP 20	18	17.88	17.59	211	3,981	30.4	32.5	0	0.08	0.09	0.25	0.42
	BPP 21	18	17.71	17.47	207	3,680	29.7	30.8	0	0.08	0.08	0.76	0.92
	BPP 22	18	17.90	17.56	207	3,800	30.1	32.7	0	0.08	0.08	0.32	0.48
	BPP 23	18	17.88	17.53	200	3,419	29.2	29.9	0	0.08	0.07	1.17	1.32
	BPP 24	18	17.55	17.40	206	3,608	29.6	30.0	0	0.08	0.08	0.60	0.76
	BPP 25	18	17.81	17.53	197	3,338	28.6	30.4	0	0.08	0.07	0.52	0.67
	BPP 26	18	17.85	17.61	203	3,320	29.5	30.2	0	0.07	0.07	0.27	0.41
	BPP 27	18	17.80	17.56	210	3,649	30.2	31.1	0	0.08	0.08	0.58	0.74
	BPP 28	18	17.76	17.43	200	3,506	29.4	29.2	0	0.08	0.07	0.27	0.42
	BPP 29	18	17.80	17.48	207	3,444	30.3	29.4	0	0.08	0.08	0.64	0.80
	BPP 30	18	17.63	17.40	194	3,678	27.8	29.3	0	0.09	0.08	0.32	0.49
	BPP 31	18	17.85	17.56	201	3,481	29.3	30.6	0	0.08	0.07	0.79	0.94
	BPP 32	18	17.65	17.40	206	3,310	29.8	29.7	0	0.08	0.07	0.30	0.45
	BPP 33	18	17.84	17.59	201	3,629	28.8	32.7	0	0.08	0.08	0.26	0.42
	BPP 34	18	17.91	17.62	212	3,978	30.3	30.2	0	0.10	0.09	0.75	0.94
	BPP 35	18	17.33	17.23	206	3,372	29.8	28.8	0	0.08	0.08	0.24	0.40
	BPP 36	18	17.57	17.39	207	3,669	30.0	29.7	0	0.09	0.08	0.55	0.72
	BPP 37	18	17.97	17.56	192	3,298	27.9	31.5	0	0.07	0.07	0.30	0.44
	BPP 38	18	17.67	17.46	209	3,442	30.2	31.1	0	0.08	0.07	0.34	0.49
	BPP 39	18	17.46	17.30	208	3,655	30.0	29.7	0	0.09	0.08	0.31	0.48
	BPP 40	18	17.58	17.35	207	3,472	30.0	28.7	0	0.08	0.08	0.26	0.42
	BPP 41	18	17.56	17.35	206	3,628	29.9	30.5	0	0.08	0.08	0.26	0.42
	BPP 42	18	17.71	17.46	198	3,395	28.7	31.0	0	0.08	0.07	0.75	0.90
	BPP 43	18	17.77	17.50	201	3,440	29.1	31.9	0	0.07	0.07	0.44	0.58
	BPP 44	18	17.74	17.55	209	3,655	29.9	30.8	0	0.08	0.08	0.29	0.45
	BPP 45	18	17.65	17.43	211	3,716	30.4	31.4	0	0.08	0.08	0.30	0.46
	BPP 46	18	17.80	17.51	205	3,897	29.6	30.2	0	0.09	0.09	1.10	1.28
	BPP 47	18	17.32	17.19	193	3,005	28.9	28.0	0	0.07	0.07	0.23	0.37
	BPP 48	18	17.85	17.60	203	3,520	29.2	29.6	0	0.08	0.07	0.25	0.40
	BPP 49	18	17.59	17.45	198	3,213	28.6	28.2	0	0.08	0.07	0.49	0.64
	BPP 50	18	17.68	17.48	209	3,369	30.2	30.1	0	0.08	0.08	0.56	0.72
	BPP 51	18	17.59	17.39	216	3,758	31.0	30.1	0	0.09	0.08	0.62	0.79
	BPP 52	18	17.60	17.45	197	3,378	28.8	28.7	0	0.08	0.07	0.24	0.39
	BPP 53	18	17.58	17.39	213	3,622	30.6	29.8	0	0.08	0.08	0.38	0.54
	BPP 54	18	17.89	17.63	195	3,193	28.5	29.5	0	0.08	0.07	0.42	0.57
	BPP 55	18	17.26	17.16	202	3,915	29.0	30.4	0	0.08	0.08	0.32	0.48
	BPP 56	18	17.73	17.42	196	3,287	28.5	30.9	0	0.07	0.07	0.84	0.98
	BPP 57	18	17.84	17.48	200	3,557	29.4	31.5	0	0.08	0.08	0.21	0.37
	BPP 58	18	17.46	17.29	204	3,353	29.5	30.8	0	0.08	0.06	0.20	0.34
	BPP 59	18	17.67	17.44	205	3,382	29.6	30.0	0	0.08	0.08	0.20	0.36
	BPP 60	18	17.63	17.41	200	3,612	28.7	31.0	0	0.08	0.07	0.25	0.40
	BPP 61	18	17.58	17.33	189	2,967	27.8	27.3	0	0.07	0.06	0.25	0.38
	BPP 62	18	17.59	17.37	195	3,290	28.3	30.1	0	0.08	0.08	0.30	0.46
	BPP 63	18	17.76	17.48	206	3,660	29.6	31.8	0	0.08	0.08	1.20	1.36
	BPP 64	18	17.79	17.55	209	3,892	30.0	29.9	0	0.09	0.09	0.50	0.68
	BPP 65	18	17.71	17.49	204	3,648	29.5	29.8	0	0.08	0.08	0.83	0.99
	BPP 66	18	17.86	17.61	209	3,768	30.1	29.7	0	0.09	0.09	0.32	0.50
	BPP 67	18	17.70	17.46	205	3,528	29.6	29.2	0	0.09	0.07	0.83	0.99
	BPP 68	18	17.69	17.47	202	3,268	29.1	30.8	0	0.07	0.06	0.49	0.62
	BPP 69	18	17.90	17.56	204	3,682	29.6	32.7	0	0.08	0.08	0.68	0.84
	BPP 70	18	17.93	17.61	208	3,899	30.1	31.8	0	0.08	0.08	0.22	0.38
	BPP 71	18	17.84	17.57	200	3,250	29.3	30.1	0	0.07	0.07	1.71	1.85
	BPP 72	18	17.81	17.61	213	3,766	30.6	30.0	0	0.09	0.09	0.29	0.47
	BPP 73	18	17.69	17.45	199	3,274	28.8	29.3	0	0.08	0.07	0.98	1.13
	BPP 74	18	17.93	17.53	171	2,913	25.0	27.8	0	0.07	0.05	0.18	0.30

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	BPP 75	18	17.93	17.61	203	3,667	29.4	30.4	0	0.09	0.08	0.35	0.52
	BPP 76	18	17.72	17.47	200	3,481	28.8	28.7	0	0.08	0.08	0.90	1.06
	BPP 77	18	17.34	17.18	205	3,741	29.4	29.5	0	0.08	0.09	0.29	0.46
	BPP 78	18	17.96	17.71	181	2,685	26.6	27.0	0	0.07	0.05	0.14	0.26
	BPP 79	18	17.77	17.47	200	3,651	29.0	30.4	0	0.08	0.08	0.53	0.69
	BPP 80	18	17.38	17.24	203	3,378	29.7	30.4	0	0.08	0.07	0.29	0.44
	BPP 81	18	17.92	17.64	199	3,384	29.2	31.1	0	0.08	0.07	0.23	0.38
	BPP 82	18	17.70	17.46	204	3,302	29.5	31.9	0	0.07	0.07	0.31	0.45
	BPP 83	18	17.98	17.63	196	3,492	28.0	29.2	0	0.08	0.07	0.23	0.38
	BPP 84	18	17.64	17.43	214	3,732	31.0	30.2	0	0.09	0.09	0.72	0.90
	BPP 85	18	17.91	17.62	211	3,525	30.5	30.8	0	0.08	0.07	0.73	0.88
	BPP 86	18	17.61	17.43	199	3,434	28.7	29.8	0	0.07	0.07	0.31	0.45
	BPP 87	18	17.62	17.40	207	3,697	29.8	31.9	0	0.08	0.08	0.36	0.52
	BPP 88	18	17.77	17.47	188	3,179	28.0	30.0	0	0.07	0.07	0.27	0.41
	BPP 89	18	17.63	17.41	197	3,500	28.7	31.9	0	0.08	0.08	0.51	0.67
	BPP 90	18	17.57	17.39	202	3,313	29.7	27.9	0	0.08	0.07	0.35	0.50
	BPP 91	18	17.79	17.52	212	3,746	30.4	30.0	0	0.08	0.08	0.58	0.74
	BPP 92	18	17.58	17.33	204	3,680	29.2	30.8	0	0.08	0.08	0.28	0.44
	BPP 93	18	17.82	17.50	202	3,688	29.1	30.4	0	0.09	0.08	0.28	0.45
	BPP 94	18	17.76	17.49	204	3,654	29.7	30.9	0	0.08	0.07	0.32	0.47
	BPP 95	18	17.61	17.41	199	3,433	29.1	30.9	0	0.08	0.07	0.24	0.39
	BPP 96	18	17.76	17.53	213	3,461	31.0	30.7	0	0.08	0.07	0.25	0.40
	BPP 97	18	17.51	17.32	208	3,464	30.2	29.6	0	0.08	0.08	0.30	0.46
	BPP 98	18	17.48	17.33	205	3,452	29.5	29.6	0	0.08	0.08	0.31	0.47
	BPP 99	18	17.47	17.30	207	3,877	30.0	31.4	0	0.09	0.09	0.25	0.43
	BPP 100	18	17.43	17.26	206	3,227	29.9	28.3	0	0.08	0.07	0.32	0.47
SCH_WAE2	BPP 1	22	21.47	21.11	208	3,615	30.2	30.8	0	0.08	0.08	0.26	0.42
	BPP 2	22	21.58	21.14	206	3,828	29.6	31.8	0	0.09	0.08	0.38	0.55
	BPP 3	22	21.03	20.79	204	3,833	29.2	30.6	0	0.09	0.08	0.31	0.48
	BPP 4	22	21.29	20.95	212	3,604	30.3	30.0	0	0.08	0.08	0.28	0.44
	BPP 5	22	21.27	21.01	216	3,984	30.8	31.7	0	0.09	0.09	0.41	0.59
	BPP 6	22	21.44	21.04	218	3,967	31.1	30.7	0	0.09	0.09	0.41	0.59
	BPP 7	22	21.66	21.30	205	3,446	29.8	28.7	0	0.09	0.07	0.61	0.77
	BPP 8	21	20.87	20.65	204	3,481	29.7	30.0	0	0.08	0.08	0.31	0.47
	BPP 9	22	21.48	21.13	219	4,091	31.2	31.1	0	0.08	0.09	0.30	0.47
	BPP 10	21	20.89	20.71	203	3,712	29.3	30.1	0	0.09	0.09	0.89	1.07
	BPP 11	22	21.49	21.14	216	3,944	30.9	30.8	0	0.09	0.08	0.67	0.84
	BPP 12	22	21.73	21.26	211	3,986	30.3	31.3	0	0.08	0.08	0.90	1.06
	BPP 13	22	21.33	21.07	217	3,857	31.1	30.9	0	0.09	0.09	0.37	0.55
	BPP 14	22	21.06	20.83	205	3,936	29.3	30.7	0	0.09	0.08	0.30	0.47
	BPP 15	22	21.37	20.97	206	3,382	30.1	28.8	0	0.08	0.07	0.33	0.48
	BPP 16	22	21.29	21.04	208	3,828	29.7	30.6	0	0.09	0.08	0.26	0.43
	BPP 17	22	21.31	20.99	201	3,413	29.4	29.0	0	0.08	0.08	0.22	0.38
	BPP 18	22	21.51	21.09	208	3,863	29.6	30.3	0	0.09	0.08	0.37	0.54
	BPP 19	22	21.37	21.05	212	3,937	30.4	30.6	0	0.09	0.09	0.38	0.56
	BPP 20	22	21.88	21.34	198	3,890	28.7	29.6	0	0.09	0.08	1.13	1.30
	BPP 21	22	21.11	20.93	215	3,970	30.6	30.0	0	0.09	0.08	0.42	0.59
	BPP 22	22	21.32	21.04	204	3,308	29.8	29.6	0	0.07	0.06	0.25	0.38
	BPP 23	22	21.29	21.01	219	4,126	31.2	31.6	0	0.09	0.09	0.29	0.47
	BPP 24	22	21.53	21.21	211	3,494	30.3	29.1	0	0.08	0.08	0.27	0.43
	BPP 25	22	21.42	21.06	205	4,008	29.5	30.6	0	0.09	0.09	0.40	0.58
	BPP 26	22	21.22	20.83	207	3,783	29.8	32.3	0	0.08	0.08	0.30	0.46
	BPP 27	21	20.90	20.72	206	3,874	29.8	30.2	0	0.09	0.09	0.73	0.91
	BPP 28	22	21.34	20.95	208	3,991	29.8	31.1	0	0.09	0.08	0.29	0.46
	BPP 29	21	20.92	20.69	210	3,678	30.3	29.6	0	0.09	0.08	0.66	0.83
	BPP 30	22	21.04	20.82	210	3,689	30.1	30.3	0	0.09	0.08	0.36	0.53
	BPP 31	22	21.20	21.00	205	3,411	29.4	28.6	0	0.08	0.07	0.31	0.46
	BPP 32	22	21.49	21.18	218	3,847	31.5	30.1	0	0.09	0.08	0.33	0.50
	BPP 33	22	21.43	21.08	212	3,832	30.5	31.3	0	0.09	0.09	0.24	0.42
	BPP 34	22	21.02	20.77	207	3,618	29.9	31.1	0	0.08	0.08	0.40	0.56
	BPP 35	22	21.58	21.16	207	4,084	29.6	31.6	0	0.09	0.09	0.73	0.91
	BPP 36	22	21.34	21.08	201	3,470	28.9	29.1	0	0.08	0.08	0.31	0.47
	BPP 37	22	21.34	21.13	218	3,717	31.4	30.0	0	0.09	0.09	0.28	0.46
	BPP 38	22	21.43	21.18	206	3,525	29.6	29.8	0	0.09	0.07	0.39	0.55

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	BPP 39	22	21.45	21.03	201	3,473	29.3	29.1	0	0.08	0.07	0.23	0.38
	BPP 40	22	21.44	21.11	214	4,088	30.7	30.9	0	0.09	0.09	0.33	0.51
	BPP 41	22	21.09	20.84	212	3,816	30.5	30.1	0	0.08	0.08	0.32	0.48
	BPP 42	22	21.24	20.95	207	3,516	30.3	30.5	0	0.08	0.07	0.33	0.48
	BPP 43	22	21.44	21.07	204	3,810	29.2	29.9	0	0.09	0.08	0.25	0.42
	BPP 44	22	21.47	21.05	213	3,747	30.6	31.1	0	0.08	0.08	0.31	0.47
	BPP 45	22	21.46	21.02	207	3,837	29.8	30.9	0	0.08	0.08	0.33	0.49
	BPP 46	22	21.33	20.95	215	3,635	30.9	29.5	0	0.08	0.08	0.28	0.44
	BPP 47	22	21.11	20.78	201	3,734	29.3	30.1	0	0.09	0.09	0.32	0.50
	BPP 48	22	21.03	20.82	204	3,513	29.6	29.9	0	0.08	0.07	0.30	0.45
	BPP 49	22	21.32	21.05	200	3,550	29.3	29.8	0	0.08	0.07	0.29	0.44
	BPP 50	22	21.14	20.96	206	3,483	29.5	28.5	0	0.09	0.07	0.22	0.38
	BPP 51	22	21.41	21.05	205	3,508	29.8	29.1	0	0.08	0.07	0.30	0.45
	BPP 52	22	21.33	21.02	211	4,002	30.4	31.5	0	0.09	0.10	0.32	0.51
	BPP 53	22	21.39	21.04	213	4,117	30.5	31.2	0	0.09	0.08	0.41	0.58
	BPP 54	22	21.47	21.03	207	3,657	29.8	31.4	0	0.08	0.08	0.21	0.37
	BPP 55	22	21.57	21.07	217	4,086	31.1	30.7	0	0.09	0.08	0.44	0.61
	BPP 56	21	20.97	20.84	208	3,662	29.8	30.7	0	0.08	0.08	1.09	1.25
	BPP 57	22	21.22	20.88	206	3,754	29.6	30.1	0	0.08	0.08	0.35	0.51
	BPP 58	22	21.40	21.07	210	3,795	30.0	29.5	0	0.08	0.08	0.52	0.68
	BPP 59	22	21.11	20.86	208	3,982	29.9	30.2	0	0.09	0.09	0.36	0.54
	BPP 60	22	21.56	21.10	218	4,147	31.3	30.9	0	0.10	0.10	0.35	0.55
	BPP 61	22	21.54	21.05	200	3,644	29.2	31.2	0	0.08	0.08	0.31	0.47
	BPP 62	22	21.24	20.85	203	3,514	29.4	29.4	0	0.08	0.08	0.29	0.45
	BPP 63	22	21.25	20.97	209	3,708	30.6	31.2	0	0.08	0.08	0.29	0.45
	BPP 64	22	21.01	20.85	207	3,378	29.9	30.8	0	0.07	0.08	0.24	0.39
	BPP 65	22	21.43	21.04	207	3,882	29.8	31.7	0	0.08	0.08	0.30	0.46
	BPP 66	22	21.28	21.02	206	3,808	29.8	29.9	0	0.08	0.09	0.30	0.47
	BPP 67	22	21.52	21.08	205	3,905	29.5	30.5	0	0.09	0.08	0.37	0.54
	BPP 68	22	21.59	21.17	211	3,714	30.4	30.2	0	0.08	0.08	0.29	0.45
	BPP 69	22	21.19	20.88	205	3,586	29.8	29.2	0	0.08	0.07	0.30	0.45
	BPP 70	22	21.41	21.03	209	3,920	30.1	31.8	0	0.09	0.07	0.40	0.56
	BPP 71	21	20.99	20.81	211	3,470	30.4	28.6	0	0.08	0.07	0.64	0.79
	BPP 72	22	21.43	21.09	206	3,565	30.0	29.0	0	0.08	0.08	0.85	1.01
	BPP 73	22	21.43	21.12	209	3,834	29.9	30.6	0	0.09	0.10	0.39	0.58
	BPP 74	22	21.14	20.85	212	3,612	30.9	29.6	0	0.08	0.08	0.26	0.42
	BPP 75	22	21.21	20.96	208	3,720	30.0	30.0	0	0.09	0.08	0.33	0.50
	BPP 76	22	21.75	21.34	214	4,170	30.7	31.7	0	0.09	0.09	0.45	0.63
	BPP 77	22	21.26	20.94	207	3,751	30.1	30.7	0	0.09	0.08	0.33	0.50
	BPP 78	22	21.03	20.80	209	3,809	30.1	29.6	0	0.09	0.08	0.37	0.54
	BPP 79	21	20.80	20.67	209	3,907	29.9	29.8	0	0.09	0.09	0.62	0.80
	BPP 80	22	21.87	21.32	206	4,133	29.9	31.3	0	0.09	0.09	0.33	0.51
	BPP 81	22	21.70	21.25	197	3,700	28.4	31.2	0	0.08	0.08	0.27	0.43
	BPP 82	22	21.41	21.13	201	3,237	29.1	28.1	0	0.08	0.07	0.33	0.48
	BPP 83	22	21.24	20.94	216	3,993	30.9	30.3	0	0.09	0.10	0.39	0.58
	BPP 84	22	21.72	21.32	202	3,741	29.1	30.4	0	0.08	0.08	0.53	0.69
	BPP 85	22	21.30	20.94	207	3,737	29.7	30.1	0	0.08	0.08	0.33	0.49
	BPP 86	22	21.28	21.00	213	3,934	30.6	30.0	0	0.09	0.09	0.33	0.51
	BPP 87	22	21.31	20.97	215	3,756	31.1	30.3	0	0.08	0.08	0.37	0.53
	BPP 88	22	21.62	21.21	208	3,749	29.8	30.4	0	0.09	0.07	0.85	1.01
	BPP 89	22	21.27	21.01	207	3,553	30.0	29.2	0	0.09	0.08	0.40	0.57
	BPP 90	22	21.20	20.92	210	3,951	30.2	30.6	0	0.09	0.09	0.26	0.44
	BPP 91	22	21.11	20.88	205	3,537	29.8	28.5	0	0.09	0.08	0.31	0.48
	BPP 92	22	21.30	21.00	214	4,024	30.5	31.1	0	0.09	0.09	0.38	0.56
	BPP 93	22	21.04	20.82	209	3,894	30.0	30.2	0	0.09	0.09	0.39	0.57
	BPP 94	22	21.19	20.95	212	3,974	30.2	30.4	0	0.10	0.09	0.29	0.48
	BPP 95	22	21.36	20.98	214	3,950	30.7	31.8	0	0.08	0.08	0.40	0.56
	BPP 96	22	21.15	20.89	213	3,898	30.6	30.1	0	0.09	0.09	0.36	0.54
	BPP 97	22	21.55	21.16	214	3,877	31.0	30.5	0	0.09	0.08	0.62	0.79
	BPP 98	21	20.83	20.69	221	3,892	31.6	29.5	0	0.09	0.09	0.43	0.61
	BPP 99	22	21.47	21.08	207	4,020	29.8	30.6	0	0.10	0.09	0.33	0.52
	BPP 100	22	21.12	20.82	210	3,799	30.4	30.3	0	0.08	0.09	0.37	0.54
WAE_GAU1	TEST 5	28	27.99	27.99	6,828	80,420	73.8	74.9	40	1.13	114.61	3,581.90	3,697.64
	TEST 22	15	14.00	14.00	7,480	59,467	83.3	93.6	0	0.68	46.75	6.61	54.04

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class	instance	z^*	lb^{lp}	lb^{sp}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
WAE_GAU2	TEST 54	14	14.00	14.00	8,827	240,298	91.2	95.9	1	3.06	292.69	5,378.55	5,674.30
	TEST 55	15	15.00	15.00	8,326	227,254	89.7	96.1	81	2.98	286.53	27,634.22	27,923.73
	TEST 55	20	19.99	19.99	8,922	330,747	92.8	97.9	24	4.28	359.96	20,572.62	20,936.86
	TEST 58	20	20.00	20.00	7,137	72,127	81.2	85.0	0	0.89	101.09	1,013.63	1,115.61
	TEST 65	16	15.00	15.00	5,011	34,603	67.2	80.7	1	0.42	29.16	4.07	33.65
	TEST 68	12	12.00	12.00	8,495	209,594	90.7	93.3	0	2.61	210.36	224.55	437.52
	TEST 75	13	13.00	13.00	9,201	200,717	94.2	93.4	0	2.61	223.16	3,294.24	3,520.01
	TEST 84	16	15.98	15.98	7,173	163,721	81.7	86.5	0	2.25	142.43	1,105.01	1,249.69
	TEST 95	16	16.00	16.00	8,940	262,861	93.8	95.7	0	3.34	291.37	379.07	673.78
	TEST 97	12	11.99	11.99	8,825	160,266	91.0	94.1	0	1.96	170.38	142.62	314.96
	TEST 14	23	23.00	23.00	7,761	106,995	86.7	86.8	0	1.35	105.83	41.20	148.38
	TEST 30	27	27.00	27.00	7,565	87,963	84.1	91.9	0	1.05	140.64	9.84	151.53
	TEST 44	14	14.00	14.00	9,106	274,252	94.8	97.9	0	3.50	318.84	7,300.28	7,622.62
	TEST 49	11	10.99	10.99	9,381	204,066	96.0	98.9	59	2.51	276.18	36,629.75	36,908.44
	TEST 82	24	23.98	23.97	7,055	59,029	78.3	87.4	0	0.73	67.46	58.06	126.25

Table A.6: Cardinality constrained bin packing results in uniform classes.

instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120.00	7	48	47.27	47.19	17.14	97	1,815	26.9	52.5	0	0.03	0.03	0.11	0.17
	6	48	47.27	47.19	20.00	97	1,815	26.9	52.5	0	0.03	0.03	0.11	0.17
	5	48	47.27	47.19	24.00	95	1,747	27.1	50.9	0	0.04	0.03	0.09	0.16
	4	48	47.27	47.19	30.00	94	1,593	29.9	48.9	0	0.04	0.02	0.08	0.14
	3	48	47.27	47.19	40.00	65	850	25.7	30.9	0	0.03	0.01	0.04	0.08
	2	60	60.00	47.19	60.00	18	128	10.7	8.0	0	0.02	0.00	0.00	0.02
u120.01	7	49	48.05	48.03	17.14	96	1,858	24.4	53.1	0	0.04	0.03	0.09	0.16
	6	49	48.05	48.03	20.00	96	1,858	24.4	53.1	0	0.04	0.03	0.09	0.16
	5	49	48.05	48.03	24.00	98	1,807	26.3	52.5	0	0.04	0.03	0.08	0.15
	4	49	48.05	48.03	30.00	91	1,622	27.8	50.1	0	0.03	0.02	0.09	0.14
	3	49	48.05	48.03	40.00	63	926	24.2	33.6	0	0.03	0.01	0.09	0.13
	2	60	60.00	48.03	60.00	17	130	9.9	7.8	0	0.02	0.00	0.00	0.02
u120.02	7	46	45.29	45.29	17.14	103	2,104	26.1	49.2	0	0.04	0.04	0.16	0.24
	6	46	45.29	45.29	20.00	103	2,104	26.1	49.2	0	0.04	0.04	0.16	0.24
	5	46	45.29	45.29	24.00	102	2,018	27.3	48.0	0	0.04	0.04	0.16	0.24
	4	46	45.29	45.29	30.00	94	1,694	28.6	43.2	0	0.04	0.03	0.14	0.21
	3	46	45.29	45.29	40.00	69	879	26.3	27.2	0	0.03	0.01	0.22	0.26
	2	60	60.00	45.29	60.00	17	134	9.8	7.4	0	0.03	0.00	0.00	0.03
u120.03	7	49	48.63	48.57	17.14	100	2,221	26.0	52.5	0	0.04	0.03	0.10	0.17
	6	49	48.63	48.57	20.00	100	2,221	26.0	52.5	0	0.04	0.03	0.10	0.17
	5	49	48.63	48.57	24.00	99	2,139	26.8	51.1	0	0.04	0.03	0.11	0.18
	4	49	48.63	48.57	30.00	96	1,926	29.0	48.6	0	0.04	0.03	0.09	0.16
	3	49	48.63	48.57	40.00	70	1,116	26.2	33.2	0	0.03	0.01	0.07	0.11
	2	60	60.00	48.57	60.00	20	147	11.1	7.0	0	0.03	0.00	0.00	0.03
u120.04	7	50	49.09	49.03	17.14	99	1,903	23.6	52.0	0	0.04	0.03	0.07	0.14
	6	50	49.09	49.03	20.00	94	1,877	22.8	51.5	0	0.04	0.03	0.10	0.17
	5	50	49.09	49.03	24.00	93	1,818	24.2	51.0	0	0.04	0.03	0.10	0.17
	4	50	49.09	49.03	30.00	94	1,710	28.0	50.9	0	0.03	0.02	0.11	0.16
	3	50	49.09	49.03	40.00	66	968	24.8	33.9	0	0.03	0.01	0.05	0.09
	2	60	60.00	49.03	60.00	20	138	11.4	7.9	0	0.02	0.00	0.00	0.02
u120.05	7	48	47.49	47.48	17.14	100	1,932	24.6	52.0	0	0.04	0.03	0.08	0.15
	6	48	47.49	47.48	20.00	97	1,914	24.0	51.5	0	0.04	0.03	0.11	0.18
	5	48	47.49	47.48	24.00	98	1,836	25.8	50.4	0	0.03	0.03	0.09	0.15
	4	48	47.49	47.48	30.00	91	1,653	27.3	48.5	0	0.04	0.02	0.10	0.16
	3	48	47.49	47.48	40.00	68	952	25.8	33.1	0	0.03	0.01	0.06	0.10
	2	60	60.00	47.48	60.00	19	133	10.9	7.7	0	0.02	0.00	0.00	0.02
u120.06	7	48	47.58	47.58	17.14	101	2,145	25.4	53.8	0	0.04	0.04	0.14	0.22
	6	48	47.58	47.58	20.00	101	2,145	25.4	53.8	0	0.04	0.04	0.14	0.22
	5	48	47.58	47.58	24.00	102	2,087	27.1	53.0	0	0.04	0.04	0.15	0.23
	4	48	47.58	47.58	30.00	95	1,840	28.5	49.3	0	0.04	0.03	0.19	0.26
	3	48	47.58	47.58	40.00	70	1,070	26.3	33.6	0	0.03	0.01	0.06	0.10

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120.07	2	60	60.00	47.58	60.00	19	141	10.7	7.2	0	0.03	0.00	0.00	0.03
	7	49	48.66	48.63	17.14	92	1,910	24.9	57.3	0	0.03	0.03	0.08	0.14
	6	49	48.66	48.63	20.00	92	1,910	24.9	57.3	0	0.03	0.03	0.08	0.14
	5	49	48.66	48.63	24.00	93	1,857	26.1	56.2	0	0.04	0.03	0.10	0.17
	4	49	48.66	48.63	30.00	94	1,751	29.4	55.5	0	0.03	0.03	0.09	0.15
	3	49	48.66	48.63	40.00	70	1,134	27.0	40.9	0	0.03	0.02	0.06	0.11
u120.08	2	60	60.00	48.63	60.00	21	142	12.1	7.7	0	0.03	0.00	0.00	0.03
	7	50	49.91	49.85	17.14	101	2,221	25.0	53.8	0	0.04	0.03	0.13	0.20
	6	50	49.91	49.85	20.00	101	2,221	25.0	53.8	0	0.04	0.04	0.13	0.21
	5	50	49.91	49.85	24.00	99	2,142	25.8	52.7	0	0.04	0.03	0.17	0.24
	4	50	49.91	49.85	30.00	99	1,967	29.3	51.2	0	0.03	0.03	0.21	0.27
	3	50	49.91	49.85	40.00	73	1,144	27.0	34.7	0	0.03	0.02	0.07	0.12
u120.09	2	60	60.00	49.85	60.00	21	150	11.7	7.3	0	0.03	0.00	0.00	0.03
	7	46	45.80	45.80	17.14	104	2,172	24.4	51.7	0	0.04	0.04	0.13	0.21
	6	46	45.80	45.80	20.00	97	2,128	23.2	50.9	0	0.04	0.04	0.13	0.21
	5	46	45.80	45.80	24.00	100	2,080	25.7	50.9	0	0.04	0.04	0.12	0.20
	4	46	45.80	45.80	30.00	98	1,841	28.9	48.0	0	0.04	0.03	0.14	0.21
	3	46	45.80	45.80	40.00	68	1,013	25.4	31.4	0	0.03	0.01	0.13	0.17
u120.10	2	60	60.00	45.80	60.00	20	144	11.3	7.4	0	0.03	0.00	0.00	0.03
	7	52	51.28	51.20	17.14	92	1,898	24.4	54.1	0	0.03	0.03	0.08	0.14
	6	52	51.28	51.20	20.00	92	1,898	24.4	54.1	0	0.03	0.03	0.08	0.14
	5	52	51.28	51.20	24.00	97	1,880	26.6	54.0	0	0.04	0.03	0.09	0.16
	4	52	51.28	51.20	30.00	95	1,747	29.1	52.8	0	0.04	0.03	0.09	0.16
	3	52	51.28	51.20	40.00	69	1,067	26.1	37.4	0	0.03	0.01	0.06	0.10
u120.11	2	60	60.00	51.20	60.00	22	144	12.5	7.9	0	0.02	0.00	0.00	0.02
	7	49	48.39	48.31	17.14	91	1,683	23.6	49.5	0	0.04	0.02	0.07	0.13
	6	49	48.39	48.31	20.00	91	1,683	23.6	49.5	0	0.04	0.02	0.07	0.13
	5	49	48.39	48.31	24.00	89	1,628	24.3	48.7	0	0.04	0.02	0.07	0.13
	4	49	48.39	48.31	30.00	88	1,468	27.2	46.9	0	0.03	0.02	0.08	0.13
	3	49	48.39	48.31	40.00	67	934	26.0	35.4	0	0.03	0.01	0.04	0.08
u120.12	2	60	60.00	48.31	60.00	19	132	11.1	8.1	0	0.02	0.00	0.00	0.02
	7	48	47.87	47.87	17.14	100	2,175	26.2	56.3	0	0.04	0.04	0.14	0.22
	6	48	47.87	47.87	20.00	100	2,175	26.2	56.3	0	0.04	0.04	0.14	0.22
	5	48	47.87	47.87	24.00	99	2,107	27.0	55.2	0	0.04	0.04	0.15	0.23
	4	48	47.87	47.87	30.00	100	1,904	30.8	52.6	0	0.04	0.03	0.15	0.22
	3	48	47.87	47.87	40.00	70	1,061	26.8	34.2	0	0.04	0.02	0.12	0.18
u120.13	2	60	60.00	47.87	60.00	19	141	10.9	7.3	0	0.03	0.00	0.00	0.03
	7	49	48.01	48.01	17.14	100	1,939	24.9	53.2	0	0.04	0.03	0.13	0.20
	6	49	48.01	48.01	20.00	96	1,919	24.0	52.7	0	0.04	0.04	0.11	0.19
	5	49	48.01	48.01	24.00	97	1,894	25.8	52.9	0	0.04	0.03	0.12	0.19
	4	49	48.01	48.01	30.00	95	1,706	28.8	50.3	0	0.04	0.03	0.11	0.18
	3	49	48.01	48.01	40.00	65	988	24.8	34.1	0	0.03	0.01	0.06	0.10
u120.14	2	60	60.00	48.01	60.00	19	137	11.0	7.6	0	0.03	0.00	0.00	0.03
	7	50	49.17	49.15	17.14	93	1,797	24.9	52.3	0	0.04	0.03	0.08	0.15
	6	50	49.17	49.15	20.00	93	1,797	24.9	52.3	0	0.04	0.03	0.08	0.15
	5	50	49.17	49.15	24.00	90	1,720	25.1	50.7	0	0.03	0.03	0.09	0.15
	4	50	49.17	49.15	30.00	90	1,602	28.1	50.2	0	0.03	0.02	0.09	0.14
	3	50	49.17	49.15	40.00	67	1,005	26.1	36.9	0	0.03	0.01	0.06	0.10
u120.15	2	60	60.00	49.15	60.00	20	136	11.7	7.9	0	0.02	0.00	0.00	0.02
	7	48	47.38	47.35	17.14	96	1,944	23.8	50.0	0	0.04	0.03	0.13	0.20
	6	48	47.38	47.35	20.00	96	1,944	23.8	50.0	0	0.04	0.03	0.13	0.20
	5	48	47.38	47.35	24.00	99	1,883	26.1	49.6	0	0.04	0.03	0.11	0.18
	4	48	47.38	47.35	30.00	97	1,749	29.0	49.2	0	0.03	0.03	0.09	0.15
	3	48	47.38	47.35	40.00	66	980	24.8	32.6	0	0.03	0.01	0.06	0.10
u120.16	2	60	60.00	47.35	60.00	19	137	10.8	7.5	0	0.02	0.00	0.00	0.02
	7	52	51.33	51.25	17.14	90	1,815	24.6	54.4	0	0.03	0.03	0.09	0.15
	6	52	51.33	51.25	20.00	90	1,815	24.6	54.4	0	0.04	0.03	0.06	0.13
	5	52	51.33	51.25	24.00	87	1,767	24.4	53.3	0	0.03	0.03	0.07	0.13
	4	52	51.33	51.25	30.00	88	1,623	27.5	51.5	0	0.03	0.02	0.10	0.15
	3	52	51.33	51.25	40.00	67	1,094	25.9	40.1	0	0.03	0.01	0.05	0.09
	2	60	60.00	51.25	60.00	24	149	13.8	8.2	0	0.02	0.00	0.00	0.02

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u120_17	7	52	51.50	51.35	17.14	91	1,774	27.2	58.4	0	0.04	0.02	0.07	0.13
	6	52	51.50	51.35	20.00	91	1,774	27.2	58.4	0	0.03	0.02	0.07	0.12
	5	52	51.50	51.35	24.00	91	1,774	27.2	58.4	0	0.03	0.02	0.07	0.12
	4	52	51.50	51.35	30.00	84	1,630	27.5	55.2	0	0.03	0.02	0.07	0.12
	3	52	51.50	51.35	40.00	68	1,155	27.1	43.5	0	0.03	0.01	0.05	0.09
	2	60	60.00	51.35	60.00	21	142	12.4	7.8	0	0.02	0.00	0.01	0.03
u120_18	7	49	48.38	48.37	17.14	94	2,072	24.0	55.1	0	0.04	0.03	0.07	0.14
	6	49	48.38	48.37	20.00	94	2,072	24.0	55.1	0	0.04	0.03	0.07	0.14
	5	49	48.38	48.37	24.00	90	1,992	24.2	53.8	0	0.04	0.03	0.09	0.16
	4	49	48.38	48.37	30.00	96	1,893	29.2	53.7	0	0.04	0.03	0.10	0.17
	3	49	48.38	48.37	40.00	71	1,168	26.9	38.0	0	0.03	0.02	0.06	0.11
	2	60	60.00	48.37	60.00	22	151	12.4	7.5	0	0.03	0.00	0.00	0.03
u120_19	7	49	48.86	48.81	17.14	101	2,255	26.0	52.7	0	0.05	0.04	0.20	0.29
	6	49	48.86	48.81	20.00	101	2,255	26.0	52.7	0	0.05	0.04	0.19	0.28
	5	49	48.86	48.81	24.00	101	2,175	27.2	51.5	0	0.04	0.04	0.24	0.32
	4	49	48.86	48.81	30.00	97	1,936	29.4	48.4	0	0.04	0.03	0.13	0.20
	3	49	48.86	48.81	40.00	75	1,151	28.2	33.5	0	0.04	0.02	0.06	0.12
	2	60	60.00	48.81	60.00	20	149	11.2	7.0	0	0.03	0.00	0.00	0.03
u250_00	7	99	98.55	98.55	35.71	104	2,310	24.5	51.4	0	0.04	0.04	0.17	0.25
	6	99	98.55	98.55	41.67	100	2,282	23.9	50.9	0	0.04	0.05	0.15	0.24
	5	99	98.55	98.55	50.00	106	2,229	27.0	50.7	0	0.04	0.04	0.25	0.33
	4	99	98.55	98.55	62.50	101	2,039	29.4	49.4	0	0.04	0.04	0.14	0.22
	3	99	98.55	98.55	83.33	75	1,241	27.5	35.2	0	0.04	0.02	0.05	0.11
	2	125	125.00	98.55	125.00	23	159	12.6	7.1	0	0.02	0.00	0.01	0.03
u250_01	7	100	99.03	99.03	35.71	111	2,855	26.3	53.0	0	0.05	0.06	0.17	0.28
	6	100	99.03	99.03	41.67	108	2,824	25.8	52.5	0	0.05	0.06	0.23	0.34
	5	100	99.03	99.03	50.00	112	2,733	28.4	51.7	0	0.05	0.06	0.18	0.29
	4	100	99.03	99.03	62.50	109	2,488	31.4	50.0	0	0.05	0.05	0.20	0.30
	3	100	99.03	99.03	83.33	81	1,457	29.0	34.2	0	0.04	0.03	0.07	0.14
	2	125	125.00	99.03	125.00	26	178	13.7	6.4	0	0.02	0.00	0.01	0.03
u250_02	7	102	101.42	101.42	35.71	108	2,738	24.4	54.8	0	0.05	0.05	0.12	0.22
	6	102	101.42	101.42	41.67	104	2,708	24.0	54.5	0	0.05	0.05	0.11	0.21
	5	102	101.42	101.42	50.00	108	2,647	26.9	54.3	0	0.05	0.04	0.14	0.23
	4	102	101.42	101.42	62.50	107	2,459	30.5	53.3	0	0.05	0.04	0.17	0.26
	3	102	101.42	101.42	83.33	81	1,452	28.9	36.4	0	0.04	0.02	0.09	0.15
	2	125	125.00	101.42	125.00	26	175	13.8	6.7	0	0.02	0.00	0.01	0.03
u250_03	7	100	99.43	99.43	35.71	109	2,832	25.3	52.5	0	0.05	0.06	0.19	0.30
	6	100	99.43	99.43	41.67	106	2,803	24.8	52.1	0	0.05	0.05	0.23	0.33
	5	100	99.43	99.43	50.00	110	2,731	27.4	51.7	0	0.05	0.05	0.24	0.34
	4	100	99.43	99.43	62.50	109	2,484	30.9	49.9	0	0.05	0.04	0.18	0.27
	3	100	99.43	99.43	83.33	83	1,507	29.3	35.3	0	0.04	0.02	0.09	0.15
	2	125	125.00	99.43	125.00	28	183	14.6	6.5	0	0.02	0.00	0.01	0.03
u250_04	7	101	100.61	100.61	35.71	110	2,663	24.9	52.7	0	0.04	0.05	0.20	0.29
	6	101	100.61	100.61	41.67	105	2,624	24.3	52.1	0	0.05	0.05	0.19	0.29
	5	101	100.61	100.61	50.00	109	2,582	27.1	52.4	0	0.05	0.04	0.19	0.28
	4	101	100.61	100.61	62.50	107	2,363	30.5	50.8	0	0.04	0.04	0.17	0.25
	3	101	100.61	100.61	83.33	81	1,401	28.9	35.1	0	0.04	0.02	0.07	0.13
	2	125	125.00	100.61	125.00	26	174	13.8	6.7	0	0.02	0.00	0.01	0.03
u250_05	7	101	100.83	100.83	35.71	109	2,768	26.0	53.4	0	0.05	0.05	0.23	0.33
	6	101	100.83	100.83	41.67	107	2,738	25.7	52.9	0	0.05	0.05	0.21	0.31
	5	101	100.83	100.83	50.00	111	2,675	28.2	52.7	0	0.05	0.05	0.16	0.26
	4	101	100.83	100.83	62.50	109	2,469	31.4	51.5	0	0.04	0.05	0.15	0.24
	3	101	100.83	100.83	83.33	82	1,487	29.4	36.0	0	0.04	0.03	0.21	0.28
	2	125	125.00	100.83	125.00	27	179	14.2	6.6	0	0.02	0.00	0.01	0.03
u250_06	7	102	101.03	101.03	35.71	112	2,878	25.6	53.3	0	0.05	0.05	0.18	0.28
	6	102	101.03	101.03	41.67	108	2,845	25.1	52.8	0	0.05	0.05	0.21	0.31
	5	102	101.03	101.03	50.00	111	2,753	27.5	52.1	0	0.05	0.05	0.21	0.31
	4	102	101.03	101.03	62.50	109	2,510	30.9	50.4	0	0.05	0.04	0.18	0.27
	3	102	101.03	101.03	83.33	84	1,473	29.8	34.5	0	0.04	0.02	0.10	0.16
	2	125	125.00	101.03	125.00	26	178	13.6	6.4	0	0.02	0.00	0.01	0.03
u250_07	7	103	102.89	102.79	35.71	107	2,687	25.3	53.2	0	0.05	0.04	0.19	0.28

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u250_08	6	103	102.89	102.79	41.67	105	2,672	24.9	52.9	0	0.04	0.05	0.13	0.22
	5	103	102.89	102.79	50.00	112	2,612	28.3	52.8	0	0.04	0.04	0.14	0.22
	4	103	102.89	102.79	62.50	107	2,422	30.7	52.0	0	0.04	0.04	0.16	0.24
	3	103	102.89	102.79	83.33	82	1,473	29.3	36.6	0	0.04	0.02	0.15	0.21
	2	125	125.00	102.79	125.00	27	178	14.2	6.7	0	0.02	0.00	0.01	0.03
	7	105	104.92	104.91	35.71	105	2,733	26.2	55.8	0	0.05	0.04	0.13	0.22
	6	105	104.92	104.91	41.67	105	2,733	26.2	55.8	0	0.05	0.04	0.13	0.22
	5	105	104.92	104.91	50.00	109	2,676	28.7	55.3	0	0.05	0.04	0.17	0.26
	4	105	104.92	104.91	62.50	106	2,472	31.4	53.5	0	0.05	0.04	0.18	0.27
	3	105	104.92	104.91	83.33	83	1,548	30.3	38.2	0	0.04	0.02	0.28	0.34
u250_09	2	125	125.00	104.91	125.00	28	182	14.9	6.7	0	0.02	0.00	0.01	0.03
	7	101	100.20	100.20	35.71	109	2,805	25.9	53.4	0	0.05	0.05	0.12	0.22
	6	101	100.20	100.20	41.67	107	2,798	25.7	53.3	0	0.05	0.05	0.18	0.28
	5	101	100.20	100.20	50.00	107	2,709	27.3	52.4	0	0.05	0.05	0.11	0.21
	4	101	100.20	100.20	62.50	107	2,495	30.9	50.9	0	0.05	0.04	0.12	0.21
	3	101	100.20	100.20	83.33	85	1,576	30.5	37.0	0	0.04	0.02	0.09	0.15
	2	125	125.00	100.20	125.00	29	187	15.2	6.6	0	0.02	0.00	0.01	0.03
	7	105	104.39	104.37	35.71	108	2,705	25.5	53.2	0	0.05	0.04	0.12	0.21
	6	105	104.39	104.37	41.67	104	2,675	24.7	52.7	0	0.04	0.04	0.13	0.21
	5	105	104.39	104.37	50.00	108	2,614	27.3	52.5	0	0.05	0.04	0.19	0.28
u250_10	4	105	104.39	104.37	62.50	108	2,429	30.9	51.8	0	0.04	0.04	0.16	0.24
	3	105	104.39	104.37	83.33	81	1,467	28.9	36.3	0	0.04	0.02	0.10	0.16
	2	125	125.00	104.37	125.00	27	177	14.2	6.7	0	0.02	0.00	0.01	0.03
	7	101	100.71	100.71	35.71	109	2,879	24.7	53.7	0	0.05	0.06	0.24	0.35
	6	101	100.71	100.71	41.67	108	2,862	24.9	53.5	0	0.05	0.07	0.25	0.37
	5	101	100.71	100.71	50.00	110	2,771	27.2	52.7	0	0.05	0.06	0.23	0.34
	4	101	100.71	100.71	62.50	110	2,579	31.0	52.0	0	0.05	0.05	0.16	0.26
	3	101	100.71	100.71	83.33	84	1,572	29.6	36.7	0	0.04	0.02	0.10	0.16
	2	125	125.00	100.71	125.00	29	187	15.0	6.6	0	0.02	0.00	0.01	0.03
	7	105	104.98	104.93	35.71	102	2,591	24.8	55.4	0	0.05	0.05	0.45	0.55
u250_12	6	105	104.98	104.93	41.67	102	2,591	24.8	55.4	0	0.05	0.05	0.47	0.57
	5	105	104.98	104.93	50.00	106	2,557	27.2	55.4	0	0.05	0.04	0.35	0.44
	4	105	104.98	104.93	62.50	105	2,335	30.5	53.2	0	0.05	0.04	0.29	0.38
	3	105	104.98	104.93	83.33	81	1,476	29.2	38.7	0	0.04	0.02	0.22	0.28
	2	125	125.00	104.93	125.00	28	178	14.9	6.9	0	0.02	0.00	0.01	0.03
	7	103	102.04	101.96	35.71	109	2,663	25.1	52.3	0	0.05	0.04	0.14	0.23
	6	103	102.04	101.96	41.67	106	2,635	24.8	51.9	0	0.05	0.04	0.11	0.20
	5	103	102.04	101.96	50.00	107	2,528	26.8	50.7	0	0.05	0.04	0.09	0.18
	4	103	102.04	101.96	62.50	106	2,323	30.3	49.6	0	0.05	0.03	0.12	0.20
	3	103	102.04	101.96	83.33	79	1,353	28.3	33.8	0	0.04	0.02	0.06	0.12
u250_13	2	125	125.00	101.96	125.00	23	165	12.2	6.5	0	0.02	0.00	0.01	0.03
	7	100	99.17	99.17	35.71	112	2,845	25.6	52.2	0	0.05	0.06	0.17	0.28
	6	100	99.17	99.17	41.67	110	2,809	25.6	51.7	0	0.05	0.06	0.18	0.29
	5	100	99.17	99.17	50.00	110	2,712	27.4	51.0	0	0.05	0.05	0.19	0.29
	4	100	99.17	99.17	62.50	109	2,503	30.9	50.0	0	0.05	0.05	0.21	0.31
	3	100	99.17	99.17	83.33	81	1,480	28.6	34.5	0	0.04	0.02	0.08	0.14
	2	125	125.00	99.17	125.00	27	181	14.1	6.5	0	0.02	0.00	0.01	0.03
	7	105	104.86	104.81	35.71	105	2,751	25.1	54.3	0	0.05	0.05	0.17	0.27
	6	105	104.86	104.81	41.67	105	2,751	25.1	54.3	0	0.05	0.05	0.17	0.27
	5	105	104.86	104.81	50.00	108	2,679	27.3	53.7	0	0.05	0.05	0.16	0.26
u250_15	4	105	104.86	104.81	62.50	107	2,450	30.7	51.9	0	0.04	0.04	0.14	0.22
	3	105	104.86	104.81	83.33	84	1,507	29.9	36.9	0	0.04	0.02	0.23	0.29
	2	125	125.00	104.81	125.00	28	181	14.7	6.7	0	0.02	0.00	0.01	0.03
	7	97	96.51	96.51	35.71	112	2,779	25.8	52.3	0	0.05	0.06	0.19	0.30
	6	97	96.51	96.51	41.67	108	2,740	25.3	51.7	0	0.05	0.06	0.25	0.36
	5	97	96.51	96.51	50.00	111	2,637	27.8	50.8	0	0.05	0.05	0.25	0.35
	4	97	96.51	96.51	62.50	107	2,384	30.6	48.9	0	0.05	0.04	0.20	0.29
	3	97	96.51	96.51	83.33	78	1,376	27.9	33.1	0	0.04	0.02	0.08	0.14
	2	125	125.00	96.51	125.00	26	175	13.8	6.6	0	0.02	0.00	0.01	0.03
	7	100	99.17	99.17	35.71	109	2,740	25.8	53.4	0	0.05	0.06	0.17	0.28
u250_17	6	100	99.17	99.17	41.67	106	2,711	25.4	53.0	0	0.05	0.06	0.17	0.28

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u250.18	5	100	99.17	99.17	50.00	109	2,617	27.8	52.2	0	0.05	0.05	0.17	0.27
	4	100	99.17	99.17	62.50	107	2,405	31.0	50.9	0	0.04	0.04	0.19	0.27
	3	100	99.17	99.17	83.33	83	1,425	30.0	35.1	0	0.04	0.02	0.10	0.16
	2	125	125.00	99.17	125.00	25	173	13.3	6.6	0	0.02	0.00	0.01	0.03
	7	100	99.70	99.70	35.71	110	2,700	25.7	51.5	0	0.05	0.05	0.38	0.48
	6	100	99.70	99.70	41.67	107	2,659	25.3	50.8	0	0.05	0.06	0.15	0.26
	5	100	99.70	99.70	50.00	111	2,577	28.0	50.3	0	0.05	0.05	0.19	0.29
	4	100	99.70	99.70	62.50	106	2,319	30.5	48.2	0	0.04	0.05	0.23	0.32
	3	100	99.70	99.70	83.33	80	1,376	28.7	33.6	0	0.04	0.02	0.12	0.18
	2	125	125.00	99.70	125.00	25	173	13.2	6.6	0	0.02	0.00	0.01	0.03
u250.19	7	102	101.36	101.36	35.71	108	2,942	26.0	53.4	0	0.05	0.06	0.17	0.28
	6	102	101.36	101.36	41.67	108	2,942	26.0	53.4	0	0.05	0.06	0.16	0.27
	5	102	101.36	101.36	50.00	116	2,871	29.4	53.0	0	0.06	0.06	0.21	0.33
	4	102	101.36	101.36	62.50	111	2,640	31.8	51.6	0	0.05	0.05	0.17	0.27
	3	102	101.36	101.36	83.33	88	1,592	31.2	36.0	0	0.05	0.03	0.08	0.16
	2	125	125.00	101.36	125.00	29	189	15.0	6.4	0	0.02	0.00	0.01	0.03
u500.00	7	198	197.58	197.58	71.43	112	2,959	25.1	52.2	0	0.05	0.06	0.27	0.38
	6	198	197.58	197.58	83.33	110	2,925	25.2	51.7	0	0.05	0.06	0.20	0.31
	5	198	197.58	197.58	100.00	116	2,857	28.5	51.6	0	0.05	0.06	0.24	0.35
	4	198	197.58	197.58	125.00	111	2,621	31.2	50.4	0	0.05	0.05	0.16	0.26
	3	198	197.58	197.58	166.67	85	1,581	29.8	35.4	0	0.04	0.02	0.11	0.17
	2	250	250.00	197.58	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
u500.01	7	201	200.85	200.85	71.43	112	2,960	24.9	52.3	0	0.05	0.05	0.30	0.40
	6	201	200.85	200.85	83.33	108	2,922	24.7	51.8	0	0.05	0.05	0.23	0.33
	5	201	200.85	200.85	100.00	113	2,852	27.8	51.7	0	0.05	0.05	0.33	0.43
	4	201	200.85	200.85	125.00	111	2,623	31.2	50.6	0	0.05	0.04	0.16	0.25
	3	201	200.85	200.85	166.67	85	1,581	29.8	35.5	0	0.04	0.03	0.17	0.24
	2	250	250.00	200.85	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
u500.02	7	202	201.44	201.44	71.43	112	2,887	25.1	51.7	0	0.05	0.05	0.14	0.24
	6	202	201.44	201.44	83.33	108	2,845	24.7	51.2	0	0.05	0.05	0.20	0.30
	5	202	201.44	201.44	100.00	115	2,787	28.3	51.3	0	0.05	0.05	0.27	0.37
	4	202	201.44	201.44	125.00	110	2,555	31.0	50.0	0	0.05	0.04	0.20	0.29
	3	202	201.44	201.44	166.67	84	1,531	29.6	35.0	0	0.04	0.02	0.10	0.16
	2	250	250.00	201.44	250.00	28	185	14.5	6.5	0	0.02	0.00	0.01	0.03
u500.03	7	204	203.81	203.81	71.43	112	2,960	25.0	52.3	0	0.06	0.06	0.35	0.47
	6	204	203.81	203.81	83.33	108	2,921	24.7	51.8	0	0.05	0.06	0.25	0.36
	5	204	203.81	203.81	100.00	116	2,861	28.5	51.8	0	0.05	0.06	0.22	0.33
	4	204	203.81	203.81	125.00	111	2,621	31.2	50.5	0	0.05	0.04	0.22	0.31
	3	204	203.81	203.81	166.67	85	1,580	29.8	35.4	0	0.04	0.02	0.18	0.24
	2	250	250.00	203.81	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
u500.04	7	206	205.11	205.11	71.43	110	2,803	26.1	52.8	0	0.05	0.05	0.19	0.29
	6	206	205.11	205.11	83.33	107	2,791	25.7	52.7	0	0.05	0.05	0.17	0.27
	5	206	205.11	205.11	100.00	110	2,721	28.1	52.2	0	0.05	0.05	0.19	0.29
	4	206	205.11	205.11	125.00	107	2,500	30.9	50.6	0	0.05	0.04	0.17	0.26
	3	206	205.11	205.11	166.67	84	1,577	30.1	36.9	0	0.05	0.02	0.08	0.15
	2	250	250.00	205.11	250.00	29	187	15.2	6.5	0	0.02	0.00	0.01	0.03
u500.05	7	206	205.09	205.09	71.43	112	2,961	25.0	52.3	0	0.05	0.06	0.18	0.29
	6	206	205.09	205.09	83.33	110	2,927	25.1	51.9	0	0.05	0.05	0.23	0.33
	5	206	205.09	205.09	100.00	113	2,851	27.8	51.7	0	0.05	0.05	0.18	0.28
	4	206	205.09	205.09	125.00	111	2,621	31.2	50.5	0	0.05	0.05	0.21	0.31
	3	206	205.09	205.09	166.67	83	1,575	29.1	35.3	0	0.04	0.02	0.09	0.15
	2	250	250.00	205.09	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
u500.06	7	207	206.91	206.89	71.43	112	2,968	25.0	52.7	0	0.06	0.05	0.19	0.30
	6	207	206.91	206.89	83.33	108	2,936	24.7	52.3	0	0.05	0.05	0.20	0.30
	5	207	206.91	206.89	100.00	113	2,860	27.8	52.0	0	0.06	0.05	0.17	0.28
	4	207	206.91	206.89	125.00	111	2,623	31.2	50.6	0	0.05	0.04	0.18	0.27
	3	207	206.91	206.89	166.67	87	1,589	30.5	35.7	0	0.04	0.02	0.24	0.30
	2	250	250.00	206.89	250.00	29	189	14.9	6.4	0	0.03	0.00	0.01	0.04
u500.07	7	204	203.98	203.98	71.43	112	2,906	25.2	51.8	0	0.05	0.06	0.20	0.31
	6	204	203.98	203.98	83.33	110	2,871	25.2	51.4	0	0.06	0.06	0.24	0.36
	5	204	203.98	203.98	100.00	112	2,785	27.6	51.0	0	0.05	0.06	0.15	0.26

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u500_08	4	204	203.98	203.98	125.00	110	2,562	31.0	50.0	0	0.05	0.05	0.19	0.29
	3	204	203.98	203.98	166.67	84	1,536	29.6	35.0	0	0.05	0.02	0.06	0.13
	2	250	250.00	203.98	250.00	28	185	14.5	6.4	0	0.02	0.00	0.01	0.03
	7	196	195.68	195.68	71.43	112	2,962	25.3	52.3	0	0.06	0.06	0.22	0.34
	6	196	195.68	195.68	83.33	110	2,928	25.3	51.9	0	0.05	0.05	0.27	0.37
	5	196	195.68	195.68	100.00	116	2,857	28.6	51.7	0	0.06	0.06	0.18	0.30
	4	196	195.68	195.68	125.00	111	2,621	31.2	50.5	0	0.05	0.05	0.18	0.28
	3	196	195.68	195.68	166.67	85	1,581	29.8	35.5	0	0.04	0.02	0.09	0.15
u500_09	2	250	250.00	195.68	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	202	201.06	201.06	71.43	112	2,963	25.5	52.3	0	0.05	0.06	0.23	0.34
	6	202	201.06	201.06	83.33	110	2,927	25.5	51.9	0	0.23	0.06	0.23	0.52
	5	202	201.06	201.06	100.00	116	2,858	28.7	51.7	0	0.05	0.06	0.23	0.34
	4	202	201.06	201.06	125.00	111	2,620	31.3	50.4	0	0.05	0.05	0.16	0.26
	3	202	201.06	201.06	166.67	85	1,581	29.8	35.4	0	0.05	0.02	0.08	0.15
	2	250	250.00	201.06	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	200	199.07	199.07	71.43	112	2,959	24.9	52.2	0	0.06	0.05	0.20	0.31
u500_10	6	200	199.07	199.07	83.33	110	2,926	25.1	51.8	0	0.06	0.05	0.17	0.28
	5	200	199.07	199.07	100.00	112	2,845	27.5	51.5	0	0.05	0.05	0.16	0.26
	4	200	199.07	199.07	125.00	111	2,622	31.2	50.5	0	0.05	0.05	0.12	0.22
	3	200	199.07	199.07	166.67	85	1,580	29.8	35.4	0	0.04	0.02	0.09	0.15
	2	250	250.00	199.07	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	200	199.43	199.43	71.43	112	2,963	25.3	52.3	0	0.05	0.06	0.15	0.26
	6	200	199.43	199.43	83.33	110	2,924	25.3	51.8	0	0.05	0.05	0.20	0.30
	5	200	199.43	199.43	100.00	116	2,857	28.6	51.7	0	0.06	0.05	0.18	0.29
u500_11	4	200	199.43	199.43	125.00	111	2,618	31.2	50.4	0	0.05	0.05	0.15	0.25
	3	200	199.43	199.43	166.67	85	1,582	29.8	35.5	0	0.04	0.02	0.10	0.16
	2	250	250.00	199.43	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	199	198.62	198.62	71.43	112	2,959	24.9	52.1	0	0.06	0.05	0.19	0.30
	6	199	198.62	198.62	83.33	110	2,926	25.1	51.8	0	0.05	0.06	0.23	0.34
	5	199	198.62	198.62	100.00	113	2,849	27.8	51.5	0	0.05	0.05	0.23	0.33
	4	199	198.62	198.62	125.00	111	2,621	31.2	50.4	0	0.05	0.05	0.19	0.29
	3	199	198.62	198.62	166.67	85	1,581	29.8	35.4	0	0.04	0.02	0.18	0.24
u500_12	2	250	250.00	198.62	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	196	195.59	195.59	71.43	112	2,957	24.9	52.2	0	0.05	0.05	0.18	0.28
	6	196	195.59	195.59	83.33	110	2,927	25.1	51.9	0	0.05	0.05	0.17	0.27
	5	196	195.59	195.59	100.00	116	2,855	28.5	51.8	0	0.05	0.05	0.25	0.35
	4	196	195.59	195.59	125.00	111	2,621	31.2	50.6	0	0.05	0.05	0.23	0.33
	3	196	195.59	195.59	166.67	86	1,585	30.2	35.6	0	0.05	0.02	0.12	0.19
	2	250	250.00	195.59	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	204	203.03	203.03	71.43	112	2,962	24.9	52.2	0	0.05	0.06	0.21	0.32
u500_13	6	204	203.03	203.03	83.33	110	2,927	25.1	51.8	0	0.05	0.06	0.17	0.28
	5	204	203.03	203.03	100.00	116	2,859	28.5	51.7	0	0.05	0.06	0.16	0.27
	4	204	203.03	203.03	125.00	111	2,621	31.2	50.5	0	0.05	0.05	0.15	0.25
	3	204	203.03	203.03	166.67	85	1,581	29.8	35.4	0	0.05	0.03	0.17	0.25
	2	250	250.00	203.03	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	201	200.13	200.13	71.43	112	2,952	25.2	52.5	0	0.06	0.06	0.21	0.33
	6	201	200.13	200.13	83.33	108	2,912	24.8	52.0	0	0.05	0.06	0.22	0.33
	5	201	200.13	200.13	100.00	116	2,850	28.6	52.0	0	0.05	0.06	0.21	0.32
u500_14	4	201	200.13	200.13	125.00	111	2,609	31.3	50.6	0	0.05	0.05	0.17	0.27
	3	201	200.13	200.13	166.67	85	1,554	29.9	35.2	0	0.04	0.02	0.10	0.16
	2	250	250.00	200.13	250.00	28	186	14.5	6.4	0	0.03	0.00	0.01	0.04
	7	202	201.01	201.01	71.43	112	2,954	24.9	52.0	0	0.05	0.05	0.15	0.25
	6	202	201.01	201.01	83.33	110	2,921	25.1	51.7	0	0.05	0.05	0.23	0.33
	5	202	201.01	201.01	100.00	116	2,855	28.5	51.7	0	0.05	0.05	0.23	0.33
	4	202	201.01	201.01	125.00	111	2,619	31.2	50.5	0	0.05	0.05	0.19	0.29
	3	202	201.01	201.01	166.67	85	1,581	29.8	35.5	0	0.04	0.02	0.08	0.14
u500_15	2	250	250.00	201.01	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	198	197.43	197.43	71.43	112	2,957	25.0	52.1	0	0.05	0.06	0.29	0.40
	6	198	197.43	197.43	83.33	110	2,924	25.1	51.7	0	0.05	0.06	0.27	0.38
	5	198	197.43	197.43	100.00	116	2,858	28.5	51.7	0	0.05	0.05	0.26	0.36
	4	198	197.43	197.43	125.00	111	2,621	31.2	50.4	0	0.05	0.04	0.18	0.27
	7	202	201.01	201.01	71.43	112	2,954	24.9	52.0	0	0.05	0.05	0.15	0.25
	6	202	201.01	201.01	83.33	110	2,921	25.1	51.7	0	0.05	0.05	0.23	0.33
	5	202	201.01	201.01	100.00	116	2,855	28.5	51.7	0	0.05	0.05	0.23	0.33

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instance	C	z^*	lb^{LP}	lb^{SP}	lb^{CRD}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{PP}	t^{LP}	t^{IP}	t^{TOT}
u500_18	3	198	197.43	197.43	166.67	83	1,575	29.1	35.3	0	0.04	0.02	0.09	0.15
	2	250	250.00	197.43	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	202	201.29	201.29	71.43	112	2,962	25.3	52.3	0	0.05	0.06	0.15	0.26
	6	202	201.29	201.29	83.33	108	2,919	24.8	51.7	0	0.06	0.05	0.21	0.32
	5	202	201.29	201.29	100.00	116	2,858	28.6	51.7	0	0.05	0.05	0.18	0.28
	4	202	201.29	201.29	125.00	111	2,621	31.2	50.5	0	0.06	0.05	0.20	0.31
u500_19	3	202	201.29	201.29	166.67	85	1,581	29.8	35.4	0	0.04	0.02	0.09	0.15
	2	250	250.00	201.29	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	196	195.63	195.63	71.43	112	2,960	25.3	52.2	0	0.05	0.06	0.24	0.35
	6	196	195.63	195.63	83.33	110	2,925	25.3	51.8	0	0.05	0.06	0.29	0.40
	5	196	195.63	195.63	100.00	113	2,847	27.8	51.5	0	0.05	0.06	0.25	0.36
	4	196	195.63	195.63	125.00	111	2,620	31.2	50.4	0	0.06	0.05	0.23	0.34
u1000_00	3	196	195.63	195.63	166.67	85	1,582	29.8	35.5	0	0.04	0.02	0.47	0.53
	2	250	250.00	195.63	250.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	399	398.43	398.43	142.86	112	2,956	24.9	52.0	0	0.05	0.05	0.26	0.36
	6	399	398.43	398.43	166.67	110	2,923	25.1	51.6	0	0.05	0.05	0.23	0.33
	5	399	398.43	398.43	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.26	0.36
	4	399	398.43	398.43	250.00	111	2,621	31.2	50.4	0	0.05	0.05	0.17	0.27
u1000_01	3	399	398.43	398.43	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.19	0.25
	2	500	500.00	398.43	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	406	405.25	405.25	142.86	112	2,957	24.9	52.0	0	0.05	0.06	0.18	0.29
	6	406	405.25	405.25	166.67	108	2,917	24.7	51.5	0	0.05	0.05	0.16	0.26
	5	406	405.25	405.25	200.00	116	2,857	28.5	51.6	0	0.05	0.06	0.22	0.33
	4	406	405.25	405.25	250.00	111	2,621	31.2	50.4	0	0.05	0.04	0.19	0.28
u1000_02	3	406	405.25	405.25	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.10	0.16
	2	500	500.00	405.25	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	411	410.20	410.20	142.86	112	2,956	25.0	52.0	0	0.05	0.05	0.16	0.26
	6	411	410.20	410.20	166.67	110	2,923	25.1	51.6	0	0.06	0.05	0.18	0.29
	5	411	410.20	410.20	200.00	116	2,857	28.5	51.6	0	0.06	0.05	0.19	0.30
	4	411	410.20	410.20	250.00	111	2,621	31.2	50.4	0	0.05	0.05	0.17	0.27
u1000_03	3	411	410.20	410.20	333.33	85	1,581	29.8	35.4	0	0.05	0.02	0.10	0.17
	2	500	500.00	410.20	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	411	410.87	410.87	142.86	112	2,956	24.9	52.0	0	0.06	0.06	0.29	0.41
	6	411	410.87	410.87	166.67	110	2,923	25.1	51.6	0	0.05	0.06	0.21	0.32
	5	411	410.87	410.87	200.00	116	2,857	28.5	51.6	0	0.05	0.06	0.26	0.37
	4	411	410.87	410.87	250.00	111	2,621	31.2	50.4	0	0.05	0.05	0.17	0.27
u1000_04	3	411	410.87	410.87	333.33	85	1,581	29.8	35.4	0	0.05	0.03	0.16	0.24
	2	500	500.00	410.87	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	397	396.74	396.74	142.86	112	2,956	24.9	52.0	0	0.05	0.06	0.27	0.38
	6	397	396.74	396.74	166.67	110	2,923	25.1	51.6	0	0.07	0.05	0.28	0.40
	5	397	396.74	396.74	200.00	116	2,857	28.5	51.6	0	0.06	0.05	0.16	0.27
	4	397	396.74	396.74	250.00	111	2,621	31.2	50.4	0	0.05	0.05	0.14	0.24
u1000_05	3	397	396.74	396.74	333.33	85	1,581	29.8	35.4	0	0.05	0.02	0.17	0.24
	2	500	500.00	396.74	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	399	398.49	398.49	142.86	112	2,956	24.9	52.0	0	0.06	0.05	0.28	0.39
	6	399	398.49	398.49	166.67	110	2,923	25.1	51.6	0	0.05	0.05	0.28	0.38
	5	399	398.49	398.49	200.00	116	2,857	28.5	51.6	0	0.06	0.05	0.17	0.28
	4	399	398.49	398.49	250.00	111	2,621	31.2	50.4	0	0.05	0.05	0.18	0.28
u1000_06	3	399	398.49	398.49	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.18	0.24
	2	500	500.00	398.49	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	395	394.21	394.21	142.86	112	2,956	24.9	52.0	0	0.05	0.05	0.23	0.33
	6	395	394.21	394.21	166.67	110	2,923	25.1	51.6	0	0.06	0.05	0.23	0.34
	5	395	394.21	394.21	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.14	0.24
	4	395	394.21	394.21	250.00	111	2,621	31.2	50.4	0	0.05	0.04	0.18	0.27
u1000_07	3	395	394.21	394.21	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.18	0.24
	2	500	500.00	394.21	500.00	29	189	14.9	6.4	0	0.03	0.00	0.01	0.04
	7	404	403.16	403.16	142.86	112	2,956	24.9	52.0	0	0.05	0.06	0.28	0.39
	6	404	403.16	403.16	166.67	110	2,923	25.1	51.6	0	0.05	0.06	0.23	0.34
	5	404	403.16	403.16	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.15	0.25
	4	404	403.16	403.16	250.00	111	2,621	31.2	50.4	0	0.05	0.05	0.15	0.25
	3	404	403.16	403.16	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.09	0.15

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u1000_08	2	500	500.00	403.16	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	399	398.43	398.43	142.86	112	2,956	24.9	52.0	0	0.05	0.05	0.17	0.27
	6	399	398.43	398.43	166.67	110	2,923	25.1	51.6	0	0.06	0.05	0.21	0.32
	5	399	398.43	398.43	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.16	0.26
	4	399	398.43	398.43	250.00	111	2,621	31.2	50.4	0	0.06	0.05	0.22	0.33
	3	399	398.43	398.43	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.09	0.15
u1000_09	2	500	500.00	398.43	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	397	396.93	396.93	142.86	112	2,956	24.9	52.0	0	0.05	0.06	0.26	0.37
	6	397	396.93	396.93	166.67	110	2,923	25.1	51.6	0	0.05	0.05	0.23	0.33
	5	397	396.93	396.93	200.00	116	2,857	28.5	51.6	0	0.06	0.05	0.18	0.29
	4	397	396.93	396.93	250.00	111	2,621	31.2	50.4	0	0.05	0.04	0.20	0.29
	3	397	396.93	396.93	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.26	0.32
u1000_10	2	500	500.00	396.93	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	400	399.34	399.34	142.86	112	2,956	24.9	52.0	0	0.05	0.05	0.28	0.38
	6	400	399.34	399.34	166.67	110	2,923	25.1	51.6	0	0.05	0.05	0.27	0.37
	5	400	399.34	399.34	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.18	0.28
	4	400	399.34	399.34	250.00	111	2,621	31.2	50.4	0	0.05	0.04	0.13	0.22
	3	400	399.34	399.34	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.10	0.16
u1000_11	2	500	500.00	399.34	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	401	400.52	400.52	142.86	112	2,957	24.9	52.0	0	0.05	0.05	0.23	0.33
	6	401	400.52	400.52	166.67	108	2,917	24.7	51.5	0	0.05	0.06	0.26	0.37
	5	401	400.52	400.52	200.00	116	2,857	28.5	51.6	0	0.06	0.05	0.21	0.32
	4	401	400.52	400.52	250.00	111	2,621	31.2	50.4	0	0.05	0.05	0.19	0.29
	3	401	400.52	400.52	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.19	0.25
u1000_12	2	500	500.00	400.52	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	393	392.24	392.24	142.86	112	2,956	24.9	52.0	0	0.05	0.05	0.20	0.30
	6	393	392.24	392.24	166.67	110	2,923	25.1	51.6	0	0.05	0.05	0.22	0.32
	5	393	392.24	392.24	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.20	0.30
	4	393	392.24	392.24	250.00	111	2,621	31.2	50.4	0	0.05	0.04	0.14	0.23
	3	393	392.24	392.24	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.11	0.17
u1000_13	2	500	500.00	392.24	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	396	395.27	395.27	142.86	112	2,956	24.9	52.0	0	0.05	0.05	0.25	0.35
	6	396	395.27	395.27	166.67	110	2,923	25.1	51.6	0	0.06	0.06	0.22	0.34
	5	396	395.27	395.27	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.24	0.34
	4	396	395.27	395.27	250.00	111	2,621	31.2	50.4	0	0.05	0.04	0.21	0.30
	3	396	395.27	395.27	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.09	0.15
u1000_14	2	500	500.00	395.27	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	394	393.89	393.89	142.86	112	2,956	24.9	52.0	0	0.05	0.05	0.21	0.31
	6	394	393.89	393.89	166.67	110	2,923	25.1	51.6	0	0.05	0.05	0.26	0.36
	5	394	393.89	393.89	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.19	0.29
	4	394	393.89	393.89	250.00	111	2,621	31.2	50.4	0	0.05	0.05	0.20	0.30
	3	394	393.89	393.89	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.10	0.16
u1000_15	2	500	500.00	393.89	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	402	401.81	401.81	142.86	112	2,956	24.9	52.0	0	0.05	0.05	0.18	0.28
	6	402	401.81	401.81	166.67	110	2,923	25.1	51.6	0	0.06	0.05	0.29	0.40
	5	402	401.81	401.81	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.25	0.35
	4	402	401.81	401.81	250.00	111	2,621	31.2	50.4	0	0.05	0.04	0.14	0.23
	3	402	401.81	401.81	333.33	85	1,581	29.8	35.4	0	0.05	0.02	0.17	0.24
u1000_16	2	500	500.00	401.81	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	404	403.03	403.03	142.86	112	2,956	24.9	52.0	0	0.05	0.06	0.15	0.26
	6	404	403.03	403.03	166.67	110	2,923	25.1	51.6	0	0.05	0.06	0.17	0.28
	5	404	403.03	403.03	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.15	0.25
	4	404	403.03	403.03	250.00	111	2,621	31.2	50.4	0	0.05	0.04	0.14	0.23
	3	404	403.03	403.03	333.33	85	1,581	29.8	35.4	0	0.05	0.02	0.09	0.16
u1000_17	2	500	500.00	403.03	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
	7	404	403.80	403.80	142.86	112	2,956	24.9	52.0	0	0.05	0.05	0.24	0.34
	6	404	403.80	403.80	166.67	110	2,923	25.1	51.6	0	0.05	0.05	0.29	0.39
	5	404	403.80	403.80	200.00	116	2,857	28.5	51.6	0	0.06	0.05	0.26	0.37
	4	404	403.80	403.80	250.00	111	2,621	31.2	50.4	0	0.05	0.04	0.13	0.22
	3	404	403.80	403.80	333.33	85	1,581	29.8	35.4	0	0.05	0.02	0.08	0.15
	2	500	500.00	403.80	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
u1000-18	7	399	398.19	398.19	142.86	112	2,957	24.9	52.0	0	0.05	0.05	0.20	0.30
	6	399	398.19	398.19	166.67	110	2,924	25.1	51.7	0	0.05	0.05	0.18	0.28
	5	399	398.19	398.19	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.17	0.27
	4	399	398.19	398.19	250.00	111	2,621	31.2	50.4	0	0.05	0.04	0.15	0.24
	3	399	398.19	398.19	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.08	0.14
	2	500	500.00	398.19	500.00	29	189	14.9	6.4	0	0.02	0.00	0.01	0.03
u1000-19	7	400	399.33	399.33	142.86	112	2,956	24.9	52.0	0	0.05	0.05	0.23	0.33
	6	400	399.33	399.33	166.67	110	2,923	25.1	51.6	0	0.05	0.05	0.30	0.40
	5	400	399.33	399.33	200.00	116	2,857	28.5	51.6	0	0.05	0.05	0.17	0.27
	4	400	399.33	399.33	250.00	111	2,621	31.2	50.4	0	0.05	0.04	0.17	0.26
	3	400	399.33	399.33	333.33	85	1,581	29.8	35.4	0	0.04	0.02	0.11	0.17
	2	500	500.00	399.33	500.00	29	189	14.9	6.4	0	0.03	0.00	0.01	0.04

Table A.7: Cardinality constrained bin packing results in triplets classes.

instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t60-00	4	20	20.00	20.00	15.00	55	670	7.9	12.2	0	0.04	0.01	0.06	0.11
	3	20	20.00	20.00	20.00	55	670	7.9	12.2	0	0.04	0.01	0.06	0.11
	2	30	30.00	20.00	30.00	4	101	0.9	5.8	0	0.02	0.00	0.00	0.02
t60-01	4	20	20.00	20.00	15.00	57	772	8.2	11.9	0	0.05	0.01	0.06	0.12
	3	20	20.00	20.00	20.00	57	772	8.2	11.9	0	0.05	0.01	0.06	0.12
	2	30	30.00	20.00	30.00	4	113	0.9	5.5	0	0.02	0.00	0.00	0.02
t60-02	4	20	20.00	20.00	15.00	52	619	7.4	11.5	0	0.04	0.01	0.02	0.07
	3	20	20.00	20.00	20.00	52	619	7.4	11.5	0	0.04	0.01	0.02	0.07
	2	30	30.00	20.00	30.00	4	98	0.9	6.0	0	0.02	0.00	0.00	0.02
t60-03	4	20	20.00	20.00	15.00	56	698	7.9	13.6	0	0.04	0.01	0.03	0.08
	3	20	20.00	20.00	20.00	56	698	7.9	13.6	0	0.04	0.01	0.03	0.08
	2	30	30.00	20.00	30.00	4	99	0.9	5.8	0	0.02	0.00	0.00	0.02
t60-04	4	20	20.00	20.00	15.00	50	681	7.0	13.9	0	0.04	0.01	0.02	0.07
	3	20	20.00	20.00	20.00	50	681	7.0	13.9	0	0.04	0.01	0.02	0.07
	2	30	30.00	20.00	30.00	4	97	0.9	5.9	0	0.02	0.00	0.00	0.02
t60-05	4	20	20.00	20.00	15.00	53	692	7.7	12.9	0	0.04	0.01	0.03	0.08
	3	20	20.00	20.00	20.00	53	692	7.7	12.9	0	0.04	0.01	0.03	0.08
	2	30	30.00	20.00	30.00	4	101	0.9	5.9	0	0.02	0.00	0.00	0.02
t60-06	4	20	20.00	20.00	15.00	53	726	7.7	13.7	0	0.05	0.01	0.06	0.12
	3	20	20.00	20.00	20.00	53	726	7.7	13.7	0	0.05	0.01	0.06	0.12
	2	30	30.00	20.00	30.00	4	103	0.9	5.8	0	0.02	0.00	0.00	0.02
t60-07	4	20	20.00	20.00	15.00	55	710	8.0	13.2	0	0.04	0.01	0.03	0.08
	3	20	20.00	20.00	20.00	55	710	8.0	13.2	0	0.04	0.01	0.03	0.08
	2	30	30.00	20.00	30.00	4	101	0.9	5.9	0	0.02	0.00	0.00	0.02
t60-08	4	20	20.00	20.00	15.00	52	657	7.5	12.9	0	0.04	0.01	0.02	0.07
	3	20	20.00	20.00	20.00	52	657	7.5	12.9	0	0.04	0.01	0.02	0.07
	2	30	30.00	20.00	30.00	4	99	0.9	5.9	0	0.02	0.00	0.00	0.02
t60-09	4	20	20.00	20.00	15.00	55	675	8.1	13.5	0	0.04	0.01	0.02	0.07
	3	20	20.00	20.00	20.00	55	675	8.1	13.5	0	0.04	0.01	0.02	0.07
	2	30	30.00	20.00	30.00	4	99	0.9	5.9	0	0.02	0.00	0.00	0.02
t60-10	4	20	20.00	20.00	15.00	57	775	8.0	13.7	0	0.04	0.01	0.08	0.13
	3	20	20.00	20.00	20.00	57	775	8.0	13.7	0	0.04	0.01	0.08	0.13
	2	30	30.00	20.00	30.00	4	107	0.9	5.6	0	0.02	0.00	0.00	0.02
t60-11	4	20	20.00	20.00	15.00	52	631	7.6	13.9	0	0.04	0.01	0.02	0.07
	3	20	20.00	20.00	20.00	52	631	7.6	13.9	0	0.04	0.01	0.02	0.07
	2	30	30.00	20.00	30.00	4	93	0.9	6.1	0	0.02	0.00	0.00	0.02
t60-12	4	20	20.00	20.00	15.00	57	747	8.0	12.7	0	0.05	0.01	0.03	0.09
	3	20	20.00	20.00	20.00	57	747	8.0	12.7	0	0.05	0.01	0.03	0.09
	2	30	30.00	20.00	30.00	4	107	0.9	5.6	0	0.02	0.00	0.00	0.02
t60-13	4	20	20.00	20.00	15.00	54	730	7.5	12.5	0	0.04	0.01	0.03	0.08
	3	20	20.00	20.00	20.00	54	730	7.5	12.5	0	0.04	0.01	0.03	0.08

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t60_14	2	30	30.00	20.00	30.00	4	103	0.8	5.7	0	0.02	0.00	0.00	0.02
	4	20	20.00	20.00	15.00	56	671	7.8	11.7	0	0.04	0.01	0.03	0.08
	3	20	20.00	20.00	20.00	56	671	7.8	11.7	0	0.04	0.01	0.03	0.08
	2	30	30.00	20.00	30.00	4	101	0.9	5.8	0	0.02	0.00	0.00	0.02
t60_15	4	20	20.00	20.00	15.00	56	685	7.8	11.1	0	0.05	0.01	0.03	0.09
	3	20	20.00	20.00	20.00	56	685	7.8	11.1	0	0.05	0.01	0.03	0.09
	2	30	30.00	20.00	30.00	4	103	0.9	5.7	0	0.02	0.00	0.00	0.02
	2	30	30.00	20.00	30.00	4	103	0.9	5.7	0	0.02	0.00	0.00	0.02
t60_16	4	20	20.00	20.00	15.00	47	615	6.7	12.5	0	0.04	0.01	0.02	0.07
	3	20	20.00	20.00	20.00	47	615	6.7	12.5	0	0.04	0.01	0.02	0.07
	2	30	30.00	20.00	30.00	4	93	0.9	6.0	0	0.02	0.00	0.00	0.02
	2	30	30.00	20.00	30.00	4	93	0.9	6.0	0	0.02	0.00	0.00	0.02
t60_17	4	20	20.00	20.00	15.00	54	657	7.6	12.5	0	0.04	0.01	0.02	0.07
	3	20	20.00	20.00	20.00	54	657	7.6	12.5	0	0.04	0.01	0.02	0.07
	2	30	30.00	20.00	30.00	4	97	0.9	5.9	0	0.02	0.00	0.00	0.02
	2	30	30.00	20.00	30.00	4	97	0.9	5.9	0	0.02	0.00	0.00	0.02
t60_18	4	20	20.00	20.00	15.00	56	729	7.8	13.8	0	0.04	0.01	0.03	0.08
	3	20	20.00	20.00	20.00	56	729	7.8	13.8	0	0.04	0.01	0.03	0.08
	2	30	30.00	20.00	30.00	4	101	0.9	5.8	0	0.02	0.00	0.00	0.02
	2	30	30.00	20.00	30.00	4	101	0.9	5.8	0	0.02	0.00	0.00	0.02
t60_19	4	20	20.00	20.00	15.00	56	666	8.0	11.4	0	0.04	0.01	0.03	0.08
	3	20	20.00	20.00	20.00	56	666	8.0	11.4	0	0.04	0.01	0.03	0.08
	2	30	30.00	20.00	30.00	4	103	0.9	5.8	0	0.02	0.00	0.00	0.02
	2	30	30.00	20.00	30.00	4	103	0.9	5.8	0	0.02	0.00	0.00	0.02
t120_00	4	40	40.00	40.00	30.00	96	1,917	11.9	17.5	0	0.08	0.03	0.16	0.27
	3	40	40.00	40.00	40.00	96	1,917	11.9	17.5	0	0.08	0.03	0.16	0.27
	2	60	60.00	40.00	60.00	4	174	0.7	4.0	0	0.04	0.00	0.00	0.04
	2	60	60.00	40.00	60.00	4	174	0.7	4.0	0	0.04	0.00	0.00	0.04
t120_01	4	40	40.00	40.00	30.00	93	1,978	11.7	19.9	0	0.08	0.03	0.16	0.27
	3	40	40.00	40.00	40.00	91	1,974	11.5	19.8	0	0.07	0.03	0.18	0.28
	2	60	60.00	40.00	60.00	4	171	0.7	4.1	0	0.03	0.00	0.00	0.03
	2	60	60.00	40.00	60.00	4	171	0.7	4.1	0	0.03	0.00	0.00	0.03
t120_02	4	40	40.00	40.00	30.00	101	2,117	12.3	17.8	0	0.09	0.03	0.24	0.36
	3	40	40.00	40.00	40.00	99	2,113	12.1	17.7	0	0.09	0.03	0.11	0.23
	2	60	60.00	40.00	60.00	4	183	0.7	3.8	0	0.04	0.00	0.00	0.04
	2	60	60.00	40.00	60.00	4	183	0.7	3.8	0	0.04	0.00	0.00	0.04
t120_03	4	40	40.00	40.00	30.00	95	1,894	12.0	17.2	0	0.09	0.03	0.22	0.34
	3	40	40.00	40.00	40.00	95	1,894	12.0	17.2	0	0.09	0.03	0.22	0.34
	2	60	60.00	40.00	60.00	4	174	0.7	4.0	0	0.04	0.00	0.00	0.04
	2	60	60.00	40.00	60.00	4	174	0.7	4.0	0	0.04	0.00	0.00	0.04
t120_04	4	40	40.00	40.00	30.00	99	2,214	11.9	18.1	0	0.09	0.03	0.09	0.21
	3	40	40.00	40.00	40.00	99	2,214	11.9	18.1	0	0.09	0.03	0.09	0.21
	2	60	60.00	40.00	60.00	4	185	0.7	3.8	0	0.04	0.00	0.00	0.04
	2	60	60.00	40.00	60.00	4	185	0.7	3.8	0	0.04	0.00	0.00	0.04
t120_05	4	40	40.00	40.00	30.00	96	2,005	11.7	17.6	0	0.09	0.03	0.22	0.34
	3	40	40.00	40.00	40.00	96	2,005	11.7	17.6	0	0.09	0.03	0.22	0.34
	2	60	60.00	40.00	60.00	4	177	0.7	3.9	0	0.04	0.00	0.00	0.04
	2	60	60.00	40.00	60.00	4	177	0.7	3.9	0	0.04	0.00	0.00	0.04
t120_06	4	40	40.00	40.00	30.00	93	1,955	11.8	18.6	0	0.08	0.03	0.16	0.27
	3	40	40.00	40.00	40.00	93	1,955	11.8	18.6	0	0.07	0.03	0.17	0.27
	2	60	60.00	40.00	60.00	4	173	0.7	4.0	0	0.04	0.00	0.00	0.04
	2	60	60.00	40.00	60.00	4	173	0.7	4.0	0	0.04	0.00	0.00	0.04
t120_07	4	40	40.00	40.00	30.00	95	2,061	11.7	19.4	0	0.08	0.03	0.26	0.37
	3	40	40.00	40.00	40.00	93	2,056	11.5	19.3	0	0.08	0.03	0.26	0.37
	2	60	60.00	40.00	60.00	4	175	0.7	4.0	0	0.04	0.00	0.00	0.04
	2	60	60.00	40.00	60.00	4	175	0.7	4.0	0	0.04	0.00	0.00	0.04
t120_08	4	40	40.00	40.00	30.00	97	2,003	12.1	19.0	0	0.08	0.03	0.43	0.54
	3	40	40.00	40.00	40.00	97	2,003	12.1	19.0	0	0.08	0.03	0.43	0.54
	2	60	60.00	40.00	60.00	4	173	0.7	4.0	0	0.04	0.00	0.00	0.04
	2	60	60.00	40.00	60.00	4	173	0.7	4.0	0	0.04	0.00	0.00	0.04
t120_09	4	40	40.00	40.00	30.00	93	1,902	11.6	18.7	0	0.08	0.03	0.08	0.19
	3	40	40.00	40.00	40.00	91	1,898	11.4	18.6	0	0.07	0.03	0.16	0.26
	2	60	60.00	40.00	60.00	4	171	0.7	4.0	0	0.04	0.00	0.00	0.04
	2	60	60.00	40.00	60.00	4	171	0.7	4.0	0	0.04	0.00	0.00	0.04
t120_10	4	40	40.00	40.00	30.00	92	1,809	11.5	17.4	0	0.07	0.02	0.08	0.17
	3	40	40.00	40.00	40.00	92	1,809	11.5	17.4	0	0.07	0.02	0.08	0.17
	2	60	60.00	40.00	60.00	4	168	0.7	4.1	0	0.03	0.00	0.00	0.03
	2	60	60.00	40.00	60.00	4	168	0.7	4.1	0	0.03	0.00	0.00	0.03
t120_11	4	40	40.00	40.00	30.00	93	1,890	11.6	17.3	0	0.08	0.03	0.23	0.34
	3	40	40.00	40.00	40.00	93	1,890	11.6	17.3	0	0.09	0.03	0.23	0.35
	2	60	60.00	40.00	60.00	4	173	0.7	4.0	0	0.04	0.00	0.00	0.04
	2	60	60.00	40.00	60.00	4	173	0.7	4.0	0	0.04	0.00	0.00	0.04
t120_12	4	40	40.00	40.00	30.00	92	1,894	11.2	19.1	0	0.08	0.03	0.14	0.25
	3	40	40.00	40.00	40.00	90	1,890	11.0	19.0	0	0.08	0.03	0.16	0.27
	2	60	60.00	40.00	60.00	4	165	0.7	4.1	0	0.03	0.00	0.00	0.03
	2	60	60.00	40.00	60.00	4	165	0.7	4.1	0	0.03	0.00	0.00	0.03

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t120_13	4	40	40.00	40.00	30.00	94	1,985	12.0	18.6	0	0.08	0.03	0.10	0.21
	3	40	40.00	40.00	40.00	94	1,985	12.0	18.6	0	0.08	0.03	0.10	0.21
	2	60	60.00	40.00	60.00	4	175	0.7	4.0	0	0.04	0.00	0.00	0.04
t120_14	4	40	40.00	40.00	30.00	94	1,754	11.9	16.6	0	0.08	0.03	0.07	0.18
	3	40	40.00	40.00	40.00	94	1,754	11.9	16.6	0	0.07	0.03	0.08	0.18
	2	60	60.00	40.00	60.00	4	168	0.7	4.1	0	0.03	0.00	0.00	0.03
t120_15	4	40	40.00	40.00	30.00	92	1,694	11.5	16.2	0	0.08	0.03	0.07	0.18
	3	40	40.00	40.00	40.00	92	1,694	11.5	16.2	0	0.08	0.03	0.07	0.18
	2	60	60.00	40.00	60.00	4	163	0.7	4.2	0	0.03	0.00	0.00	0.03
t120_16	4	40	40.00	40.00	30.00	100	2,049	12.5	19.1	0	0.08	0.03	0.20	0.31
	3	40	40.00	40.00	40.00	100	2,049	12.5	19.1	0	0.08	0.03	0.19	0.30
	2	60	60.00	40.00	60.00	4	175	0.7	4.0	0	0.04	0.00	0.00	0.04
t120_17	4	40	40.00	40.00	30.00	98	2,180	12.0	18.6	16	0.09	0.03	0.77	0.89
	3	40	40.00	40.00	40.00	98	2,180	12.0	18.6	16	0.09	0.04	0.78	0.91
	2	60	60.00	40.00	60.00	4	181	0.7	3.9	0	0.04	0.00	0.00	0.04
t120_18	4	40	40.00	40.00	30.00	91	1,983	11.2	18.8	0	0.08	0.03	0.09	0.20
	3	40	40.00	40.00	40.00	89	1,979	10.9	18.7	0	0.08	0.03	0.19	0.30
	2	60	60.00	40.00	60.00	4	171	0.7	4.0	0	0.04	0.00	0.00	0.04
t120_19	4	40	40.00	40.00	30.00	98	2,133	11.8	19.7	0	0.08	0.03	0.19	0.30
	3	40	40.00	40.00	40.00	98	2,133	11.8	19.7	0	0.08	0.03	0.19	0.30
	2	60	60.00	40.00	60.00	4	175	0.7	4.0	0	0.04	0.00	0.00	0.04
t249_00	4	83	83.00	83.00	62.25	140	5,143	15.8	27.7	0	0.14	0.10	0.45	0.69
	3	83	83.00	83.00	83.00	138	5,139	15.6	27.6	0	0.15	0.10	0.61	0.86
	2	125	124.50	83.00	124.50	4	269	0.6	2.8	0	0.07	0.00	0.00	0.07
t249_01	4	83	83.00	83.00	62.25	153	5,609	17.3	29.1	0	0.16	0.10	0.70	0.96
	3	83	83.00	83.00	83.00	151	5,605	17.1	29.0	0	0.15	0.11	0.52	0.78
	2	125	124.50	83.00	124.50	4	281	0.6	2.7	0	0.08	0.02	0.00	0.10
t249_02	4	83	83.00	83.00	62.25	147	5,370	16.7	27.8	0	0.16	0.11	0.55	0.82
	3	83	83.00	83.00	83.00	145	5,366	16.5	27.7	0	0.15	0.11	0.29	0.55
	2	125	124.50	83.00	124.50	4	279	0.6	2.7	0	0.08	0.00	0.00	0.08
t249_03	4	83	83.00	83.00	62.25	152	5,945	17.1	30.4	0	0.16	0.12	1.09	1.37
	3	83	83.00	83.00	83.00	150	5,941	16.9	30.4	0	0.16	0.11	0.67	0.94
	2	125	124.50	83.00	124.50	4	285	0.6	2.6	0	0.08	0.00	0.00	0.08
t249_04	4	83	83.00	83.00	62.25	146	5,136	16.6	27.6	0	0.15	0.10	0.21	0.46
	3	83	83.00	83.00	83.00	144	5,132	16.4	27.5	0	0.15	0.10	0.50	0.75
	2	125	124.50	83.00	124.50	4	269	0.6	2.8	0	0.07	0.00	0.00	0.07
t249_05	4	83	83.00	83.00	62.25	154	5,926	17.3	29.3	0	0.16	0.12	0.61	0.89
	3	83	83.00	83.00	83.00	152	5,922	17.1	29.3	0	0.17	0.12	0.70	0.99
	2	125	124.50	83.00	124.50	4	291	0.6	2.6	0	0.09	0.00	0.00	0.09
t249_06	4	83	83.00	83.00	62.25	149	5,671	16.8	30.4	0	0.16	0.10	0.48	0.74
	3	83	83.00	83.00	83.00	147	5,667	16.6	30.3	0	0.15	0.10	0.28	0.53
	2	125	124.50	83.00	124.50	4	277	0.6	2.7	0	0.08	0.00	0.00	0.08
t249_07	4	83	83.00	83.00	62.25	141	5,458	15.9	29.3	0	0.15	0.11	0.81	1.07
	3	83	83.00	83.00	83.00	139	5,454	15.7	29.3	0	0.15	0.10	0.69	0.94
	2	125	124.50	83.00	124.50	4	275	0.6	2.7	0	0.08	0.00	0.00	0.08
t249_08	4	83	83.00	83.00	62.25	145	5,459	16.5	28.5	0	0.16	0.11	0.32	0.59
	3	83	83.00	83.00	83.00	143	5,455	16.2	28.5	0	0.16	0.11	0.76	1.03
	2	125	124.50	83.00	124.50	4	280	0.6	2.7	0	0.08	0.00	0.00	0.08
t249_09	4	83	83.00	83.00	62.25	148	5,641	16.9	29.2	0	0.16	0.10	0.32	0.58
	3	83	83.00	83.00	83.00	146	5,637	16.7	29.2	0	0.16	0.11	0.62	0.89
	2	125	124.50	83.00	124.50	4	283	0.6	2.6	0	0.08	0.00	0.00	0.08
t249_10	4	83	83.00	83.00	62.25	153	5,642	17.3	28.5	0	0.16	0.11	0.52	0.79
	3	83	83.00	83.00	83.00	151	5,638	17.1	28.5	0	0.16	0.11	0.80	1.07
	2	125	124.50	83.00	124.50	4	281	0.6	2.7	0	0.08	0.00	0.00	0.08
t249_11	4	83	83.00	83.00	62.25	154	5,728	17.5	29.2	0	0.16	0.14	0.89	1.19
	3	83	83.00	83.00	83.00	154	5,728	17.5	29.2	0	0.16	0.14	0.86	1.16
	2	125	124.50	83.00	124.50	4	283	0.6	2.6	0	0.08	0.00	0.00	0.08

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t249_12	4	83	83.00	83.00	62.25	156	5,592	17.7	28.8	0	0.16	0.11	0.77	1.04
	3	83	83.00	83.00	83.00	154	5,588	17.5	28.8	0	0.16	0.11	0.51	0.78
	2	125	124.50	83.00	124.50	4	283	0.6	2.6	0	0.08	0.00	0.00	0.08
t249_13	4	83	83.00	83.00	62.25	150	5,785	17.0	29.9	0	0.16	0.12	1.43	1.71
	3	83	83.00	83.00	83.00	148	5,781	16.8	29.8	0	0.15	0.11	0.73	0.99
	2	125	124.50	83.00	124.50	4	283	0.6	2.6	0	0.08	0.00	0.00	0.08
t249_14	4	83	83.00	83.00	62.25	159	5,707	18.0	27.7	0	0.17	0.12	2.05	2.34
	3	83	83.00	83.00	83.00	159	5,707	18.0	27.7	0	0.17	0.12	2.04	2.33
	2	125	124.50	83.00	124.50	4	291	0.6	2.6	0	0.09	0.00	0.00	0.09
t249_15	4	83	83.00	83.00	62.25	154	5,576	17.3	27.3	0	0.17	0.11	1.14	1.42
	3	83	83.00	83.00	83.00	152	5,572	17.1	27.3	0	0.17	0.10	1.63	1.90
	2	125	124.50	83.00	124.50	4	285	0.6	2.6	0	0.08	0.00	0.00	0.08
t249_16	4	83	83.00	83.00	62.25	151	5,908	17.0	28.7	0	0.17	0.12	0.59	0.88
	3	83	83.00	83.00	83.00	149	5,904	16.8	28.7	0	0.17	0.12	0.54	0.83
	2	125	124.50	83.00	124.50	4	289	0.6	2.6	0	0.09	0.00	0.00	0.09
t249_17	4	83	83.00	83.00	62.25	159	5,874	17.8	29.1	0	0.17	0.12	1.18	1.47
	3	83	83.00	83.00	83.00	157	5,870	17.6	29.1	0	0.16	0.12	0.58	0.86
	2	125	124.50	83.00	124.50	4	291	0.6	2.6	0	0.09	0.00	0.00	0.09
t249_18	4	83	83.00	83.00	62.25	149	5,360	16.8	28.0	0	0.16	0.11	0.96	1.23
	3	83	83.00	83.00	83.00	147	5,356	16.6	28.0	0	0.16	0.11	0.64	0.91
	2	125	124.50	83.00	124.50	4	277	0.6	2.7	0	0.08	0.00	0.00	0.08
t249_19	4	83	83.00	83.00	62.25	147	5,480	16.6	28.5	0	0.15	0.10	0.51	0.76
	3	83	83.00	83.00	83.00	145	5,476	16.4	28.5	0	0.16	0.09	0.88	1.13
	2	125	124.50	83.00	124.50	4	273	0.6	2.7	0	0.08	0.00	0.00	0.08
t501_00	4	167	167.00	167.00	125.25	199	11,550	21.1	39.9	0	0.27	0.27	1.06	1.60
	3	167	167.00	167.00	167.00	197	11,546	21.0	39.9	0	0.26	0.27	2.54	3.07
	2	251	250.50	167.00	250.50	4	382	0.6	2.0	0	0.15	0.00	0.00	0.15
t501_01	4	167	167.00	167.00	125.25	201	11,479	21.4	38.5	0	0.26	0.28	1.53	2.07
	3	167	167.00	167.00	167.00	199	11,475	21.2	38.5	0	0.26	0.26	2.84	3.36
	2	251	250.50	167.00	250.50	4	385	0.6	2.0	0	0.15	0.00	0.00	0.15
t501_02	4	167	167.00	167.00	125.25	197	10,886	21.0	36.9	0	0.27	0.25	1.50	2.02
	3	167	167.00	167.00	167.00	195	10,882	20.8	36.9	0	0.26	0.26	1.72	2.24
	2	251	250.50	167.00	250.50	4	381	0.6	2.0	0	0.15	0.00	0.00	0.15
t501_03	4	167	167.00	167.00	125.25	207	12,155	21.8	38.7	0	0.29	0.30	2.67	3.26
	3	167	167.00	167.00	167.00	205	12,151	21.6	38.7	0	0.29	0.29	4.46	5.04
	2	251	250.50	167.00	250.50	4	399	0.6	1.9	0	0.16	0.00	0.00	0.16
t501_04	4	167	167.00	167.00	125.25	202	11,846	21.3	39.1	0	0.28	0.26	1.86	2.40
	3	167	167.00	167.00	167.00	200	11,842	21.1	39.1	0	0.28	0.27	2.62	3.17
	2	251	250.50	167.00	250.50	4	392	0.6	2.0	0	0.16	0.00	0.00	0.16
t501_05	4	167	167.00	167.00	125.25	200	11,909	21.2	39.3	0	0.27	0.28	1.34	1.89
	3	167	167.00	167.00	167.00	198	11,905	21.0	39.3	0	0.28	0.27	1.29	1.84
	2	251	250.50	167.00	250.50	4	392	0.6	2.0	0	0.15	0.00	0.00	0.15
t501_06	4	167	167.00	167.00	125.25	206	12,059	21.7	39.8	0	0.28	0.27	1.84	2.39
	3	167	167.00	167.00	167.00	204	12,055	21.5	39.7	0	0.29	0.28	1.83	2.40
	2	251	250.50	167.00	250.50	4	393	0.6	2.0	0	0.15	0.00	0.00	0.15
t501_07	4	167	167.00	167.00	125.25	198	11,521	21.0	38.9	0	0.26	0.27	1.51	2.04
	3	167	167.00	167.00	167.00	196	11,517	20.8	38.9	0	0.26	0.26	1.78	2.30
	2	251	250.50	167.00	250.50	4	386	0.6	2.0	0	0.15	0.00	0.00	0.15
t501_08	4	167	167.00	167.00	125.25	208	12,152	21.9	39.7	0	0.28	0.26	1.97	2.51
	3	167	167.00	167.00	167.00	206	12,148	21.8	39.7	0	0.27	0.26	1.91	2.44
	2	251	250.50	167.00	250.50	4	393	0.6	2.0	0	0.15	0.00	0.00	0.15
t501_09	4	167	167.00	167.00	125.25	196	11,062	20.9	38.1	0	0.26	0.25	0.63	1.14
	3	167	167.00	167.00	167.00	194	11,058	20.7	38.0	0	0.27	0.26	2.25	2.78
	2	251	250.50	167.00	250.50	4	379	0.6	2.0	0	0.15	0.00	0.00	0.15
t501_10	4	167	167.00	167.00	125.25	196	11,288	20.9	38.7	0	0.26	0.23	2.92	3.41
	3	167	167.00	167.00	167.00	194	11,284	20.7	38.7	0	0.26	0.23	1.47	1.96
	2	251	250.50	167.00	250.50	4	382	0.6	2.0	0	0.15	0.00	0.00	0.15

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instance	C	z^*	lb^{lp}	lb^{sp}	lb^{crd}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
t501_11	4	167	167.00	167.00	125.25	205	12,023	21.7	40.0	0	0.27	0.28	1.34	1.89
	3	167	167.00	167.00	167.00	203	12,019	21.5	40.0	1	0.27	0.26	4.79	5.32
	2	251	250.50	167.00	250.50	4	391	0.6	2.0	0	0.16	0.00	0.00	0.16
t501_12	4	167	167.00	167.00	125.25	193	11,238	20.6	38.8	0	0.26	0.25	2.89	3.40
	3	167	167.00	167.00	167.00	191	11,234	20.4	38.8	0	0.26	0.26	1.16	1.68
	2	251	250.50	167.00	250.50	4	379	0.6	2.0	0	0.15	0.00	0.00	0.15
t501_13	4	167	167.00	167.00	125.25	205	12,384	21.6	40.1	0	0.28	0.29	1.81	2.38
	3	167	167.00	167.00	167.00	203	12,380	21.4	40.1	0	0.28	0.27	1.51	2.06
	2	251	250.50	167.00	250.50	4	397	0.6	1.9	0	0.16	0.00	0.00	0.16
t501_14	4	167	167.00	167.00	125.25	212	12,890	22.2	40.1	0	0.29	0.26	1.82	2.37
	3	167	167.00	167.00	167.00	210	12,886	22.0	40.0	0	0.29	0.28	1.45	2.02
	2	251	250.50	167.00	250.50	4	407	0.6	1.9	0	0.17	0.00	0.00	0.17
t501_15	4	167	167.00	167.00	125.25	207	12,043	21.8	39.1	0	0.28	0.29	1.87	2.44
	3	167	167.00	167.00	167.00	205	12,039	21.6	39.1	0	0.28	0.29	1.88	2.45
	2	251	250.50	167.00	250.50	4	396	0.6	1.9	0	0.16	0.00	0.00	0.16
t501_16	4	167	167.00	167.00	125.25	204	12,338	21.5	39.9	0	0.28	0.29	1.28	1.85
	3	167	167.00	167.00	167.00	202	12,334	21.3	39.9	0	0.29	0.30	1.82	2.41
	2	251	250.50	167.00	250.50	4	397	0.6	1.9	0	0.16	0.00	0.00	0.16
t501_17	4	167	167.00	167.00	125.25	206	12,194	21.8	39.9	0	0.29	0.28	1.35	1.92
	3	167	167.00	167.00	167.00	204	12,190	21.6	39.9	0	0.29	0.26	1.73	2.28
	2	251	250.50	167.00	250.50	4	394	0.6	2.0	0	0.15	0.00	0.00	0.15
t501_18	4	167	167.00	167.00	125.25	202	11,745	21.4	39.3	0	0.27	0.26	0.76	1.29
	3	167	167.00	167.00	167.00	200	11,741	21.2	39.3	0	0.27	0.24	0.91	1.42
	2	251	250.50	167.00	250.50	4	388	0.6	2.0	0	0.15	0.00	0.00	0.15
t501_19	4	167	167.00	167.00	125.25	200	11,784	21.2	40.3	0	0.26	0.25	1.49	2.00
	3	167	167.00	167.00	167.00	198	11,780	21.0	40.3	0	0.27	0.26	2.28	2.81
	2	251	250.50	167.00	250.50	4	386	0.6	2.0	0	0.15	0.00	0.00	0.15

Table A.8: Results for two-constraint bin packing results with $n = 25$.

class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	1	6	5.69	5.64	5.65	5.64	5.65	389	2,315	2.7	7.8	0	0.13	0.04	0.07	0.24
	2	7	6.23	6.23	5.30	6.23	5.30	266	1,734	2.6	8.5	0	0.10	0.03	0.05	0.18
	3	7	6.16	5.30	6.16	5.30	6.16	266	1,791	2.8	9.0	0	0.10	0.03	0.16	0.29
	4	7	6.16	6.16	5.45	6.16	5.45	279	1,788	2.8	8.7	0	0.10	0.03	0.06	0.19
	5	7	6.02	5.45	6.02	5.45	6.02	288	1,832	2.9	8.9	0	0.10	0.03	0.07	0.20
	6	7	6.02	6.02	5.31	6.02	5.31	298	1,822	2.4	7.3	0	0.12	0.03	0.07	0.22
	7	7	6.08	5.31	6.08	5.31	6.08	253	1,667	2.5	8.0	0	0.10	0.02	0.06	0.18
	8	7	6.08	6.08	5.35	6.08	5.35	291	1,839	2.6	8.1	0	0.11	0.03	0.07	0.21
	9	7	6.10	5.35	6.10	5.35	6.10	251	1,683	2.5	8.2	0	0.10	0.03	0.04	0.17
	10	7	6.10	6.10	5.25	6.10	5.25	294	1,851	2.6	8.1	0	0.11	0.03	0.05	0.19
2	1★	13	12.50	12.00	11.00	10.48	10.53	70	281	7.2	14.4	0	0.02	0.00	0.01	0.03
	2★	14	14.00	11.00	13.00	10.53	11.04	43	166	9.1	17.7	0	0.02	0.00	0.00	0.02
	3★	14	13.50	13.00	12.00	11.04	11.43	56	219	8.2	16.0	0	0.01	0.00	0.01	0.02
	4★	14	14.00	12.00	13.00	11.43	10.93	40	156	9.2	18.0	0	0.02	0.00	0.00	0.02
	5★	13	13.00	13.00	11.00	10.93	10.61	64	248	8.3	16.1	0	0.01	0.00	0.01	0.02
	6★	14	14.00	11.00	13.50	10.61	11.37	43	174	8.3	16.8	0	0.02	0.00	0.00	0.02
	7★	14	13.50	13.50	10.25	11.37	9.95	68	264	7.9	15.4	0	0.01	0.00	0.01	0.02
	8★	15	15.00	10.25	14.50	9.95	11.92	45	175	8.6	16.7	0	0.02	0.00	0.00	0.02
	9★	15	14.50	14.50	10.00	11.92	9.68	53	210	8.0	15.9	0	0.02	0.00	0.00	0.02
	10★	16	16.00	10.00	15.50	9.68	12.46	39	148	9.3	17.8	0	0.02	0.00	0.00	0.02
3	1★	13	12.50	12.25	11.62	11.29	11.31	29	124	11.9	25.6	0	0.02	0.00	0.00	0.02
	2★	14	14.00	11.62	13.00	11.31	11.62	24	99	13.2	27.5	0	0.01	0.00	0.00	0.01
	3★	14	13.50	13.00	12.38	11.62	11.85	26	110	12.9	27.6	0	0.02	0.00	0.00	0.02
	4★	14	14.00	12.38	13.00	11.85	11.55	22	92	12.7	26.9	0	0.02	0.00	0.00	0.02
	5★	13	13.00	13.00	11.88	11.55	11.36	26	112	12.4	27.1	0	0.02	0.00	0.00	0.02
	6★	14	14.00	11.88	13.50	11.36	11.82	25	105	13.7	29.0	0	0.01	0.00	0.00	0.01

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class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
4	7★	14	13.50	13.50	11.38	11.82	10.96	27	121	12.6	28.4	0	0.02	0.00	0.00	0.02
	8★	15	15.00	11.38	14.50	10.96	12.15	23	99	12.6	27.5	0	0.01	0.00	0.00	0.01
	9★	15	14.50	14.50	11.25	12.15	10.80	22	104	11.4	27.2	0	0.02	0.00	0.00	0.02
	10★	16	16.00	11.25	15.50	10.80	12.47	21	89	13.0	27.8	0	0.01	0.00	0.00	0.01
	1★	3	2.83	2.82	2.83	2.82	2.83	22,206	143,516	10.3	15.5	0	7.07	239.61	914.95	1,161.63
	2★	3	2.90	2.83	2.90	2.83	2.90	16,158	104,078	6.9	11.1	0	6.90	71.12	50.03	128.05
	3★	3	2.96	2.90	2.96	2.90	2.96	18,737	120,104	9.2	14.2	0	6.25	108.81	885.01	1,000.07
	4★	3	2.96	2.96	2.89	2.96	2.89	14,121	92,642	6.4	10.7	0	6.31	53.07	428.81	488.19
	5★	3	2.89	2.89	2.84	2.89	2.84	20,400	129,771	9.3	14.0	0	6.87	160.30	106.63	273.80
	6★	3	2.96	2.84	2.96	2.84	2.96	15,675	98,079	7.1	11.0	0	6.46	68.78	524.87	600.11
6	7★	3	2.96	2.96	2.74	2.96	2.74	18,655	122,069	8.9	13.7	0	6.55	141.91	94.21	242.67
	8★	4	3.04	2.74	3.04	2.74	3.04	14,028	90,358	6.5	10.5	0	6.15	57.65	52.00	115.80
	9★	4	3.04	3.04	2.70	3.04	2.70	15,178	104,816	7.2	12.2	0	6.17	86.83	83.66	176.66
	10★	4	3.12	2.70	3.12	2.70	3.12	11,208	76,519	5.2	9.3	0	5.68	30.36	32.56	68.60
	1	10	9.33	8.94	8.95	8.90	8.93	65	340	7.3	18.3	0	0.02	0.00	0.01	0.03
	2	10	9.60	8.95	9.26	8.93	9.20	42	269	6.5	20.5	0	0.02	0.00	0.01	0.03
	3	10	9.88	9.26	9.50	9.20	9.41	48	267	6.5	17.9	0	0.02	0.00	0.01	0.03
	4	10	9.80	9.50	9.18	9.41	9.14	44	268	7.1	21.5	0	0.01	0.00	0.01	0.02
	5	10	9.47	9.18	9.02	9.14	8.97	54	298	6.8	18.5	0	0.02	0.00	0.01	0.03
	6	10	9.60	9.02	9.43	8.97	9.38	50	289	7.5	21.2	0	0.02	0.00	0.01	0.03
7	7	10	9.50	9.43	8.63	9.38	8.61	54	306	6.5	18.0	0	0.02	0.00	0.01	0.03
	8	10	9.80	8.63	9.72	8.61	9.67	49	284	7.4	21.0	0	0.02	0.00	0.01	0.03
	9	10	9.88	9.72	8.47	9.67	8.47	49	281	6.8	19.3	0	0.02	0.00	0.01	0.03
	10	11	10.10	8.47	10.03	8.47	9.96	43	260	7.5	22.4	0	0.01	0.00	0.01	0.02
	1	9	8.94	8.94	8.64	8.90	8.63	67	430	5.1	13.4	0	0.02	0.01	0.02	0.05
	2	9	8.98	8.95	8.74	8.93	8.73	95	582	7.1	17.7	0	0.02	0.01	0.02	0.05
	3	10	9.33	9.26	9.07	9.20	9.05	60	376	5.2	13.7	0	0.02	0.00	0.02	0.04
	4	10	9.54	9.50	9.23	9.41	9.19	75	446	6.8	17.1	0	0.02	0.01	0.01	0.04
	5	10	9.33	9.18	8.91	9.14	8.88	60	392	5.1	14.2	0	0.02	0.00	0.02	0.04
	6	10	9.07	9.02	8.84	8.97	8.81	97	621	7.3	18.6	0	0.02	0.01	0.03	0.06
8	7	10	9.50	9.43	9.07	9.38	9.03	47	335	4.8	14.9	0	0.02	0.00	0.02	0.04
	8	9	8.67	8.63	8.55	8.61	8.53	123	775	8.0	19.4	0	0.03	0.01	0.03	0.07
	9	10	9.83	9.72	9.35	9.67	9.29	44	301	5.4	16.3	0	0.02	0.00	0.01	0.03
	10	9	8.57	8.47	8.48	8.47	8.46	156	940	9.3	20.8	0	0.04	0.01	0.03	0.08
	1★	13	12.50	8.94	10.50	8.90	10.06	19	135	8.6	29.7	0	0.02	0.00	0.00	0.02
	2★	13	12.50	8.95	10.16	8.93	10.07	17	138	7.8	31.1	0	0.02	0.00	0.00	0.02
	3★	13	12.50	9.26	10.25	9.20	9.82	19	134	8.5	29.4	0	0.02	0.00	0.00	0.02
	4★	13	12.50	9.50	9.63	9.41	9.58	17	135	7.8	30.3	0	0.02	0.00	0.00	0.02
	5★	13	12.50	9.18	10.25	9.14	9.83	18	135	8.0	29.1	0	0.02	0.00	0.00	0.02
	6★	13	12.50	9.02	10.13	8.97	10.05	18	132	8.5	30.3	0	0.02	0.00	0.00	0.02
9	7★	13	12.50	9.43	9.92	9.38	9.54	15	124	6.3	25.6	0	0.02	0.00	0.00	0.02
	8★	13	12.50	8.63	10.58	8.61	10.43	19	133	9.4	31.9	0	0.01	0.00	0.01	0.02
	9★	13	12.50	9.72	9.43	9.67	9.23	16	129	6.7	26.3	0	0.02	0.00	0.00	0.02
	10★	13	12.50	8.47	11.00	8.47	10.61	19	131	9.8	32.8	0	0.01	0.00	0.01	0.02
	1	7	6.06	6.00	6.00	6.00	6.00	303	1,786	3.1	8.9	0	0.10	0.02	0.06	0.18
	2	7	6.07	6.00	6.00	6.00	6.00	236	1,512	3.0	9.5	0	0.07	0.02	0.06	0.15
	3	7	6.05	6.00	6.00	6.00	6.00	294	1,738	3.0	8.6	0	0.10	0.02	0.06	0.18
	4	7	6.06	6.00	5.99	6.00	5.99	258	1,642	3.1	9.8	0	0.08	0.02	0.05	0.15
	5	7	6.06	5.99	6.00	5.99	6.00	290	1,716	3.0	8.9	0	0.10	0.02	0.26	0.38
	6	7	6.05	6.00	5.99	6.00	5.99	269	1,725	3.2	9.9	0	0.08	0.02	0.07	0.17
10	7	7	6.04	5.99	6.00	5.99	6.00	302	1,825	3.1	9.1	0	0.09	0.03	0.05	0.17
	8	8	7.13	7.00	7.00	6.99	6.99	141	812	4.5	12.7	0	0.04	0.01	0.02	0.07
	9	8	7.12	7.00	7.00	6.99	6.99	148	868	4.2	12.2	0	0.04	0.01	0.03	0.08
	10	8	7.14	7.00	7.00	6.99	6.99	133	800	4.4	13.1	0	0.04	0.01	0.02	0.07

Table A.9: Results for two-constraint bin packing with $n = 50$.

class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	1	13	12.60	12.60	11.21	12.60	11.21	2,010	19,217	2.1	7.2	0	1.74	1.03	1.68	4.45

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class	inst.	z^*	lb^p	lb^{p1}	lb^{p2}	lb^{sp1}	lb^{sp2}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	2	13	12.80	11.21	12.80	11.21	12.80	1,560	16,383	2.0	8.5	0	1.18	0.61	1.16	2.95
	3	13	12.80	12.80	11.20	12.80	11.20	1,786	17,388	2.0	7.4	0	1.52	0.77	1.49	3.78
	4	13	12.76	11.20	12.76	11.20	12.76	1,590	16,678	2.0	8.5	0	1.22	0.65	1.26	3.13
	5	13	12.76	12.76	11.13	12.76	11.13	1,797	17,655	2.0	7.4	0	1.56	0.82	1.61	3.99
	6	14	13.03	11.13	13.03	11.13	13.03	1,521	15,903	2.0	8.6	0	1.16	0.57	0.94	2.67
	7	14	13.03	13.03	11.01	13.03	11.01	1,599	15,932	1.9	7.1	0	1.42	0.62	1.29	3.33
	8	14	13.12	11.01	13.12	11.01	13.12	1,496	15,835	2.0	8.7	0	1.11	0.60	1.13	2.84
	9	14	13.12	13.12	11.10	13.12	11.10	1,469	15,328	1.8	7.5	0	1.25	0.52	1.14	2.91
	10	14	13.36	13.36	11.27	13.36	11.27	1,302	13,807	1.9	8.4	0	1.00	0.44	0.98	2.42
2	1★	30	30.00	29.50	21.00	25.35	20.74	208	997	4.3	10.3	0	0.05	0.01	0.02	0.08
	2★	31	30.50	21.00	30.50	20.74	26.02	103	527	5.6	14.2	0	0.03	0.01	0.01	0.05
	3★	31	31.00	30.50	21.00	26.02	20.71	171	810	4.6	10.9	0	0.04	0.01	0.02	0.07
	4★	31	30.50	21.00	30.50	20.71	25.91	105	537	5.5	14.1	0	0.03	0.01	0.01	0.05
	5★	31	31.00	30.50	20.67	25.91	20.45	172	819	4.6	10.9	0	0.04	0.01	0.02	0.07
	6★	32	31.50	20.67	31.50	20.45	26.78	102	517	5.5	13.9	0	0.03	0.00	0.02	0.05
	7★	32	32.00	31.50	20.22	26.78	20.07	166	788	4.6	10.8	0	0.04	0.01	0.01	0.06
	8★	32	32.00	20.22	32.00	20.07	27.09	105	519	5.7	13.9	0	0.02	0.00	0.02	0.04
	9★	33	32.50	32.00	20.50	27.09	20.35	135	648	4.7	11.2	0	0.04	0.01	0.01	0.06
	10★	32	32.00	20.50	32.00	20.35	26.95	106	522	6.0	14.8	0	0.02	0.00	0.02	0.04
3	1★	30	30.00	29.50	23.25	25.20	22.42	44	300	6.5	22.1	0	0.02	0.00	0.01	0.03
	2★	31	30.50	23.25	30.50	22.42	25.60	47	263	9.8	27.5	0	0.01	0.00	0.01	0.02
	3★	31	31.00	30.50	23.25	25.60	22.41	40	277	6.7	23.2	0	0.01	0.00	0.01	0.02
	4★	31	30.50	23.25	30.50	22.41	25.53	46	266	9.5	27.7	0	0.01	0.00	0.01	0.02
	5★	31	31.00	30.50	23.00	25.53	22.25	41	279	6.8	23.2	0	0.01	0.00	0.01	0.02
	6★	32	31.50	23.00	31.50	22.25	26.06	46	266	9.7	28.3	0	0.01	0.00	0.01	0.02
	7★	32	32.00	31.50	22.75	26.06	22.02	38	262	6.5	22.4	0	0.01	0.00	0.01	0.02
	8★	32	32.00	22.75	32.00	22.02	26.24	44	268	9.2	28.2	0	0.01	0.00	0.01	0.02
	9★	33	32.50	32.00	23.00	26.24	22.19	37	251	6.8	23.0	0	0.01	0.00	0.01	0.02
	10★	32	32.00	23.00	32.00	22.19	26.16	44	264	9.6	28.8	0	0.01	0.00	0.01	0.02
6	1	21	20.30	20.23	17.70	20.16	17.70	140	1,197	5.2	18.1	0	0.04	0.01	0.05	0.10
	2	21	20.69	17.70	20.60	17.70	20.52	108	1,006	5.3	21.8	0	0.04	0.01	0.03	0.08
	3	21	20.68	20.60	17.68	20.52	17.68	120	1,059	4.9	18.1	0	0.03	0.01	0.05	0.09
	4	21	20.61	17.68	20.53	17.68	20.46	108	1,022	5.2	21.9	0	0.03	0.01	0.09	0.13
	5	21	20.61	20.53	17.54	20.46	17.54	119	1,075	4.8	18.2	0	0.04	0.01	0.04	0.09
	6	22	21.08	17.54	21.02	17.54	20.93	105	989	5.2	21.7	0	0.03	0.01	0.04	0.08
	7★	22	21.07	21.02	17.33	20.93	17.33	112	1,016	4.7	18.0	0	0.03	0.01	0.04	0.08
	8	22	21.21	17.33	21.18	17.33	21.09	108	996	5.3	21.7	0	0.03	0.01	0.03	0.07
	9	22	21.24	21.18	17.49	21.09	17.49	107	984	4.9	19.1	0	0.04	0.01	0.03	0.08
	10	22	21.13	17.49	21.10	17.49	21.01	106	1,001	5.3	22.3	0	0.03	0.01	0.03	0.07
7	1	21	20.27	20.23	19.60	20.16	19.58	139	1,290	6.1	15.5	0	0.05	0.02	0.04	0.11
	2	18	17.89	17.70	17.82	17.70	17.82	441	3,908	13.8	23.9	0	0.11	0.08	0.18	0.37
	3	21	20.67	20.60	19.96	20.52	19.93	126	1,191	5.8	15.4	0	0.06	0.02	0.05	0.13
	4	18	17.87	17.68	17.79	17.68	17.79	447	4,002	13.7	24.1	0	0.12	0.08	0.16	0.36
	5	21	20.67	20.53	19.86	20.46	19.84	125	1,198	5.7	15.4	0	0.06	0.02	0.05	0.13
	6	18	17.82	17.54	17.77	17.54	17.77	447	4,028	13.7	24.1	0	0.11	0.07	0.15	0.33
	7★	22	21.17	21.02	20.28	20.93	20.25	95	952	4.8	14.5	0	0.05	0.01	0.04	0.10
	8	18	17.65	17.33	17.61	17.33	17.61	457	4,190	14.0	24.6	0	0.12	0.08	0.23	0.43
	9	22	21.33	21.18	20.48	21.09	20.45	95	946	5.0	15.8	0	0.04	0.01	0.04	0.09
	10	18	17.80	17.49	17.74	17.49	17.74	437	3,944	13.6	23.9	0	0.12	0.08	0.15	0.35
8	1★	25	25.00	20.23	17.74	20.16	17.73	31	402	4.6	25.2	0	0.02	0.00	0.01	0.03
	2★	25	25.00	17.70	20.67	17.70	20.53	27	406	4.4	27.8	0	0.02	0.00	0.01	0.03
	3★	25	25.00	20.60	17.37	20.52	17.37	30	400	4.6	25.6	0	0.02	0.00	0.01	0.03
	4★	25	25.00	17.68	20.67	17.68	20.54	27	408	4.4	28.0	0	0.02	0.00	0.01	0.03
	5★	25	25.00	20.53	17.41	20.46	17.41	31	392	4.7	24.9	0	0.02	0.00	0.01	0.03
	6★	25	25.00	17.54	20.92	17.54	20.74	28	406	4.6	27.8	0	0.02	0.00	0.01	0.03
	7★	25	25.00	21.02	16.91	20.93	16.91	30	374	4.7	24.2	0	0.02	0.00	0.01	0.03
	8★	25	25.00	17.33	21.17	17.33	20.97	28	413	4.6	28.4	0	0.02	0.00	0.01	0.03
	9★	25	25.00	21.18	16.77	21.09	16.77	31	388	4.8	25.0	0	0.02	0.00	0.01	0.03
	10★	25	25.00	17.49	20.92	17.49	20.81	28	417	4.5	28.3	0	0.02	0.00	0.01	0.03
9	1	14	13.04	12.99	12.99	12.99	12.99	1,279	12,195	2.1	8.3	0	0.89	0.40	0.79	2.08

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class	inst.	z^*	lb^l	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	2	14	13.05	12.99	12.99	12.99	12.99	1,094	11,102	2.3	10.1	0	0.63	0.29	0.49	1.41
	3	14	13.04	12.99	13.00	12.99	13.00	1,209	11,594	2.1	8.4	0	0.83	0.33	0.69	1.85
	4	14	13.05	13.00	13.00	13.00	13.00	1,092	11,176	2.3	10.2	0	0.63	0.25	0.54	1.42
	5	14	13.06	13.00	13.00	13.00	13.00	1,220	11,828	2.2	8.7	0	0.81	0.33	0.68	1.82
	6★	15	14.07	13.99	13.99	13.99	13.99	738	6,769	2.6	10.8	0	0.34	0.16	0.20	0.70
	7	15	14.07	13.99	13.99	13.99	13.99	808	7,139	2.4	9.3	0	0.41	0.13	0.23	0.77
	8	15	14.07	13.99	13.99	13.99	13.99	777	7,213	2.7	11.3	0	0.35	0.14	0.32	0.81
	9	15	14.07	13.99	13.99	13.99	13.99	792	7,251	2.4	9.7	0	0.40	0.13	0.34	0.87
	10	15	14.07	13.99	13.99	13.99	13.99	735	6,983	2.6	11.2	0	0.34	0.13	0.30	0.77

Table A.10: Results for two-constraint bin packing with $n = 100$.

class	inst.	z^*	lb^l	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	1★	25	24.96	24.96	24.26	24.96	24.26	10,090	162,623	4.6	11.0	0	15.00	43.50	50.23	108.73
	2	26	25.09	24.26	25.09	24.26	25.09	10,724	172,688	4.9	12.2	0	14.36	44.79	49.69	108.84
	3	26	25.09	25.09	24.31	25.09	24.31	9,393	155,608	4.3	11.1	0	14.43	40.55	55.89	110.87
	4★	25	24.91	24.31	24.91	24.31	24.91	10,910	173,748	4.9	11.7	0	14.95	53.10	50.35	118.40
	5★	25	24.91	24.91	24.28	24.91	24.28	9,548	157,997	4.4	11.0	0	14.46	43.11	50.23	107.80
	6★	25	25.00	24.28	25.00	24.28	25.00	10,854	173,252	5.0	11.7	0	15.04	47.59	193.05	255.68
	7★	25	25.00	25.00	24.34	25.00	24.34	9,360	155,642	4.3	11.1	11	14.22	42.31	143.10	199.63
	8	26	25.11	24.34	25.11	24.34	25.11	10,653	170,230	4.9	11.8	0	14.51	45.66	53.94	114.11
	9	26	25.11	25.11	24.47	25.11	24.47	8,816	150,548	4.2	11.3	0	13.34	36.35	35.10	84.79
	10	26	25.12	24.47	25.12	24.47	25.12	10,094	164,536	4.7	11.8	0	14.08	45.52	50.09	109.69
2	1★	62	62.00	53.50	60.00	49.62	55.38	320	1,958	4.8	14.6	0	0.07	0.02	0.05	0.14
	2★	57	57.00	50.00	54.00	47.60	50.34	759	4,529	3.7	10.6	0	0.22	0.07	0.11	0.40
	3★	56	56.00	54.00	50.00	50.34	47.75	562	3,501	3.4	10.1	0	0.17	0.04	0.08	0.29
	4★	57	57.00	50.00	53.00	47.75	49.76	850	5,202	3.5	10.2	0	0.26	0.08	0.18	0.52
	5★	56	56.00	53.00	50.00	49.76	47.66	568	3,537	3.4	10.2	0	0.17	0.05	0.09	0.31
	6★	57	57.00	50.00	53.00	47.66	50.05	846	5,189	3.4	10.2	0	0.25	0.08	0.14	0.47
	7★	56	56.00	53.00	50.00	50.05	47.88	566	3,505	3.4	10.2	0	0.17	0.05	0.07	0.29
	8★	58	58.00	50.00	53.50	47.88	50.42	836	5,120	3.4	10.1	0	0.26	0.08	0.17	0.51
	9★	57	57.00	53.50	50.75	50.42	48.30	497	3,076	3.5	10.5	0	0.15	0.04	0.08	0.27
	10★	58	58.00	50.75	53.50	48.30	50.45	753	4,565	3.5	10.3	0	0.22	0.07	0.13	0.42
3	1★	56	56.00	53.00	50.50	49.91	48.53	102	882	5.0	21.5	0	0.03	0.01	0.02	0.06
	2★	57	57.00	50.50	54.00	48.53	50.17	113	975	5.6	24.2	0	0.04	0.01	0.01	0.06
	3★	57	56.50	54.00	50.50	50.17	48.62	97	850	5.1	22.3	0	0.04	0.01	0.01	0.06
	4★	57	57.00	50.50	53.00	48.62	49.82	115	1,000	5.3	23.2	0	0.04	0.01	0.01	0.06
	5★	56	56.00	53.00	50.50	49.82	48.56	98	859	5.1	22.3	0	0.03	0.01	0.02	0.06
	6★	57	57.00	50.50	53.00	48.56	49.99	115	992	5.3	22.9	0	0.04	0.01	0.01	0.06
	7★	56	56.00	53.00	50.50	49.99	48.70	99	862	5.2	22.7	0	0.03	0.01	0.02	0.06
	8★	58	58.00	50.50	53.50	48.70	50.22	114	985	5.3	23.0	0	0.04	0.01	0.01	0.06
	9★	57	57.00	53.50	51.00	50.22	48.95	98	825	5.4	22.8	0	0.03	0.01	0.01	0.05
	10★	58	58.00	51.00	53.50	48.95	50.24	112	952	5.5	23.5	0	0.04	0.01	0.01	0.06
6	1★	41	40.26	39.92	38.69	39.90	38.68	321	4,317	6.2	21.1	0	0.15	0.06	0.12	0.33
	2	41	40.52	38.69	40.15	38.68	40.13	382	4,978	7.3	24.8	0	0.15	0.08	0.11	0.34
	3	41	40.50	40.15	38.76	40.13	38.75	294	4,071	5.8	21.2	0	0.14	0.06	0.14	0.34
	4★	41	40.28	38.76	39.84	38.75	39.83	397	5,145	7.4	24.1	0	0.16	0.08	0.14	0.38
	5★	41	40.25	39.84	38.72	39.83	38.71	296	4,102	5.8	21.1	0	0.14	0.07	0.12	0.33
	6★	41	40.42	38.72	39.99	38.71	39.98	398	5,155	7.5	24.2	0	0.16	0.08	0.13	0.37
	7★	41	40.41	39.99	38.84	39.98	38.83	295	4,074	5.8	21.2	0	0.13	0.06	0.13	0.32
	8	41	40.62	38.84	40.20	38.83	40.18	398	5,130	7.5	24.4	0	0.16	0.07	1.26	1.49
	9	41	40.69	40.20	39.07	40.18	39.05	274	3,924	5.6	21.6	0	0.13	0.06	0.11	0.30
	10	41	40.75	39.07	40.22	39.05	40.19	372	4,886	7.2	24.4	0	0.15	0.07	0.53	0.75
7	1★	41	40.08	39.92	39.55	39.90	39.55	678	9,713	17.9	29.8	0	0.27	0.22	0.44	0.93
	2★	39	38.93	38.69	38.73	38.68	38.71	1,001	13,841	23.7	32.4	0	0.35	0.45	5.01	5.81
	3	41	40.35	40.15	39.81	40.13	39.81	666	9,363	17.8	30.0	0	0.24	0.21	0.36	0.81
	4★	39	38.97	38.76	38.73	38.75	38.71	967	13,420	23.0	31.9	0	0.34	0.40	2.68	3.42
	5★	41	40.03	39.84	39.49	39.83	39.49	725	9,975	19.0	30.5	0	0.25	0.24	0.41	0.90
	6★	39	38.94	38.72	38.73	38.71	38.70	967	13,443	23.0	31.9	0	0.34	0.39	2.06	2.79

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class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
8	7★	41	40.20	39.99	39.67	39.98	39.67	639	9,058	17.1	29.0	0	0.25	0.19	0.36	0.80
	8★	40	39.10	38.84	38.91	38.83	38.87	967	13,411	23.0	31.9	0	0.33	0.44	0.73	1.50
	9	41	40.43	40.20	39.92	40.18	39.92	631	8,790	17.0	29.4	0	0.23	0.18	0.32	0.73
	10	40	39.35	39.07	39.14	39.05	39.10	948	12,942	22.6	31.5	0	0.33	0.43	0.88	1.64
	1★	50	50.00	39.92	36.29	39.90	36.29	58	1,512	4.6	31.7	0	0.04	0.01	0.03	0.08
	2★	50	50.00	38.69	37.71	38.68	37.71	55	1,350	4.8	31.6	0	0.04	0.01	0.03	0.08
	3★	50	50.00	40.15	36.07	40.13	36.07	56	1,482	4.5	31.1	0	0.04	0.01	0.03	0.08
	4★	50	50.00	38.76	37.59	38.75	37.59	54	1,350	4.6	31.3	0	0.04	0.01	0.03	0.08
	5★	50	50.00	39.84	36.37	39.83	36.37	56	1,452	4.5	30.5	0	0.04	0.01	0.04	0.09
	6★	50	50.00	38.72	37.66	38.71	37.66	55	1,333	4.8	30.9	0	0.04	0.01	0.03	0.08
9	7★	50	50.00	39.99	36.23	39.98	36.23	56	1,434	4.4	29.9	0	0.04	0.01	0.04	0.09
	8★	50	50.00	38.84	37.57	38.83	37.57	55	1,331	4.8	31.1	0	0.04	0.01	0.03	0.08
	9★	50	50.00	40.20	36.06	40.18	36.06	57	1,473	4.5	30.1	0	0.04	0.01	0.04	0.09
	10★	50	50.00	39.07	37.34	39.05	37.34	56	1,346	4.9	31.6	0	0.04	0.01	0.03	0.08
	1★	26	25.00	24.98	24.99	24.98	24.99	9,057	146,717	4.4	11.1	0	13.25	35.18	20.83	69.26
	2★	27	26.02	25.98	26.00	25.98	26.00	7,158	117,794	4.1	12.3	0	9.18	20.94	10.74	40.86
	3★	27	26.03	26.00	26.00	26.00	26.00	6,522	107,874	3.7	11.5	0	9.05	16.84	15.73	41.62
	4★	26	25.00	24.98	24.99	24.98	24.99	9,689	156,518	4.7	11.8	0	13.29	41.93	19.44	74.66
	5★	26	25.00	24.99	24.98	24.99	24.98	8,521	141,101	4.2	11.1	0	12.88	36.95	24.27	74.10
	6★	27	26.02	25.99	25.99	25.99	25.99	7,244	117,249	4.1	11.9	0	9.54	18.91	10.68	39.13
10	7★	27	26.02	25.99	25.98	25.99	25.98	6,383	107,206	3.7	11.5	0	9.07	17.14	7.95	34.16
	8★	27	26.02	25.98	26.00	25.98	26.00	7,291	118,975	4.1	12.0	0	9.73	19.90	11.08	40.71
	9★	27	26.02	26.00	25.98	26.00	25.98	6,329	108,190	3.7	11.7	0	8.75	15.84	7.16	31.75
	10★	27	26.01	25.98	25.98	25.98	25.98	7,142	117,923	4.0	12.0	0	9.51	19.44	10.28	39.23

Table A.11: Results for two-constraint bin packing with $n = 200$.

class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
1	1★	50	49.65	49.65	49.54	49.65	49.54	52,685	1,496,846	15.5	22.1	0	92.89	3,742.35	5,985.97	9,821.21
	2★	50	49.66	49.54	49.66	49.54	49.66	55,504	1,539,030	16.3	23.5	0	88.64	3,843.68	5,280.94	9,213.26
	3★	50	49.66	49.66	49.61	49.66	49.61	52,350	1,491,635	15.4	22.2	0	92.05	3,971.50	4,891.44	8,954.99
	4★	50	49.62	49.61	49.62	49.61	49.62	55,491	1,540,284	16.3	23.5	0	88.88	5,614.72	574.71	6,278.31
	5★	50	49.62	49.62	49.40	49.62	49.40	52,735	1,512,466	15.5	22.2	0	93.06	3,326.05	5,909.37	9,328.48
	6★	50	49.86	49.40	49.86	49.40	49.86	55,236	1,534,424	16.3	23.5	0	88.77	2,943.66	5,090.21	8,122.64
	7★	50	49.86	49.86	49.34	49.86	49.34	52,207	1,496,385	15.4	22.2	0	92.47	3,169.20	4,675.45	7,937.12
	8	51	50.04	49.34	50.04	49.34	50.04	54,890	1,524,070	16.3	23.5	0	87.41	2,697.41	4,803.42	7,588.24
	9	51	50.04	50.04	49.48	50.04	49.48	50,781	1,460,781	15.1	22.2	0	90.95	2,642.43	5,444.30	8,177.68
	10	51	50.26	49.48	50.26	49.48	50.26	53,207	1,479,991	15.9	23.4	0	85.20	2,396.35	4,854.64	7,336.19
2	1★	111	111.00	100.50	102.50	98.93	98.60	7,544	65,542	4.1	11.0	0	4.50	10.19	9.04	23.73
	2★	114	114.00	102.50	100.50	98.60	98.94	12,040	98,904	6.3	16.8	0	4.46	22.90	16.01	43.37
	3★	111	111.00	100.50	102.50	98.94	98.83	7,250	61,943	4.1	11.0	0	4.21	10.94	10.52	25.67
	4★	115	114.50	102.50	100.33	98.83	98.81	12,052	99,023	6.3	16.8	0	4.43	24.45	26.50	55.38
	5★	110	110.00	100.33	101.50	98.81	98.12	7,266	62,206	4.1	11.0	0	4.23	9.59	5.38	19.20
	6★	116	115.50	101.50	101.33	98.12	99.64	12,047	98,971	6.3	16.8	0	4.48	22.09	33.02	59.59
	7★	111	111.00	101.33	101.00	99.64	97.93	7,238	61,924	4.1	11.0	0	4.25	9.36	13.07	26.68
	8★	117	116.50	101.00	102.00	97.93	100.22	12,045	98,757	6.3	16.8	0	4.42	22.57	17.24	44.23
	9★	112	112.00	102.00	102.00	100.22	98.38	6,721	56,885	4.0	10.8	0	3.96	9.43	7.06	20.45
	10★	118	117.50	102.00	103.00	98.38	100.97	11,345	91,633	6.1	16.4	0	4.20	19.47	20.91	44.58
3	1★	111	111.00	102.25	102.50	99.29	99.10	267	3,406	2.8	17.1	0	0.13	0.04	0.07	0.24
	2★	114	114.00	102.50	102.25	99.10	99.30	254	3,503	2.7	18.4	0	0.12	0.03	0.08	0.23
	3★	111	111.00	102.25	102.50	99.30	99.24	265	3,401	2.8	17.6	0	0.12	0.03	0.07	0.22
	4★	115	114.50	102.50	102.25	99.24	99.22	252	3,491	2.7	18.3	0	0.12	0.03	0.11	0.26
	5★	110	110.00	102.25	101.75	99.22	98.81	264	3,428	2.8	17.6	0	0.12	0.04	0.08	0.24
	6★	116	115.50	101.75	103.25	98.81	99.72	251	3,472	2.7	18.2	0	0.12	0.03	0.10	0.25
	7★	111	111.00	103.25	101.50	99.72	98.70	264	3,403	2.8	17.6	0	0.12	0.04	0.07	0.23
	8★	117	116.50	101.50	103.50	98.70	100.07	253	3,471	2.7	18.3	0	0.13	0.03	0.10	0.26
	9★	112	112.00	103.50	102.00	100.07	98.97	259	3,318	2.8	17.8	0	0.12	0.03	0.08	0.23
	10★	118	117.50	102.00	104.25	98.97	100.52	249	3,370	2.8	18.5	0	0.12	0.03	0.09	0.24
6	1★	81	80.35	79.35	79.19	79.35	79.19	1,149	21,602	14.1	26.8	0	0.75	0.65	1.04	2.44

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class	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	2★	81	80.50	79.19	79.36	79.19	79.36	1,221	23,938	14.9	30.9	0	0.71	0.61	1.42	2.74
	3★	81	80.41	79.36	79.30	79.36	79.30	1,137	21,507	14.1	27.2	0	0.72	0.59	1.09	2.40
	4★	81	80.52	79.30	79.29	79.30	79.29	1,221	23,963	15.0	30.9	0	0.74	0.61	9.07	10.42
	5★	81	80.21	79.29	78.93	79.29	78.93	1,142	21,620	14.1	27.1	0	0.74	0.65	1.11	2.50
	6★	81	80.67	78.93	79.74	78.93	79.74	1,222	23,933	15.0	31.0	0	0.72	0.57	4.86	6.15
	7★	81	80.49	79.74	78.83	79.74	78.83	1,135	21,511	14.1	27.2	0	0.72	0.60	2.53	3.85
	8	81	80.84	78.83	80.05	78.83	80.05	1,221	23,858	15.0	31.1	0	0.70	0.63	7.98	9.31
	9	81	80.84	80.05	79.07	80.05	79.07	1,082	20,726	13.5	27.0	0	0.72	0.55	7.38	8.65
	10★	82	81.25	79.07	80.45	79.07	80.45	1,181	23,009	14.6	30.8	0	0.69	0.53	1.42	2.64
	1	80	79.55	79.35	79.15	79.35	79.15	1,738	50,967	33.4	49.7	0	1.02	2.55	2.98	6.55
7	2	80	79.37	79.19	79.03	79.19	79.03	1,885	53,165	34.9	47.3	0	1.12	4.52	3.28	8.92
	3	80	79.57	79.36	79.19	79.36	79.19	1,731	50,743	33.3	49.8	0	1.10	2.47	2.90	6.47
	4	80	79.48	79.30	79.13	79.30	79.13	1,860	52,128	34.5	47.2	0	1.12	3.61	16.03	20.76
	5	80	79.49	79.29	79.03	79.29	79.03	1,731	50,786	33.3	49.8	0	1.02	2.85	2.99	6.86
	6★	80	79.14	78.93	78.87	78.93	78.87	1,861	52,137	34.5	47.2	0	1.12	4.45	4.37	9.94
	7★	80	79.95	79.74	79.45	79.74	79.45	1,667	48,327	32.4	48.8	0	1.00	2.55	31.17	34.72
	8★	80	79.07	78.83	78.85	78.83	78.85	1,861	52,174	34.5	47.2	0	1.10	4.00	3.29	8.39
	9	81	80.29	80.05	79.81	80.05	79.81	1,658	47,404	32.3	48.9	0	1.00	2.10	3.08	6.18
	10	80	79.38	79.07	79.19	79.07	79.19	1,847	51,192	34.3	46.8	0	1.09	4.26	3.17	8.52
	1	100	100.00	79.35	73.23	79.35	73.23	101	4,960	6.5	34.7	0	0.14	0.04	0.09	0.27
8	2★	100	100.00	79.19	73.42	79.19	73.42	92	4,799	6.2	39.4	0	0.12	0.05	0.09	0.26
	3★	100	100.00	79.36	73.23	79.36	73.23	101	4,909	6.5	34.4	0	0.13	0.04	0.12	0.29
	4★	100	100.00	79.30	73.29	79.30	73.29	92	4,823	6.2	39.5	0	0.12	0.04	0.09	0.25
	5★	100	100.00	79.29	73.25	79.29	73.25	101	4,847	6.5	33.9	0	0.14	0.04	0.09	0.27
	6★	100	100.00	78.93	73.72	78.93	73.72	90	4,752	6.1	38.9	0	0.11	0.04	0.09	0.24
	7★	100	100.00	79.74	72.79	79.74	72.79	101	4,848	6.5	33.9	0	0.14	0.04	0.12	0.30
	8★	100	100.00	78.83	73.85	78.83	73.85	91	4,775	6.2	39.0	0	0.11	0.04	0.09	0.24
	9★	100	100.00	80.05	72.52	80.05	72.52	101	4,848	6.5	33.9	0	0.13	0.04	0.09	0.26
	10★	100	100.00	79.07	73.67	79.07	73.67	91	4,757	6.2	39.0	0	0.12	0.04	0.10	0.26

Table A.12: Results for two-constraint bin packing in the class 10.

n	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
24	1	8	8.00	8.00	8.00	8.00	8.00	52	354	6.5	20.5	0	0.02	0.00	0.01	0.03
	2	8	8.00	8.00	8.00	8.00	8.00	54	376	6.6	20.5	0	0.02	0.00	0.01	0.03
	3	8	8.00	8.00	8.00	8.00	8.00	59	381	7.6	22.7	0	0.02	0.00	0.01	0.03
	4	8	8.00	8.00	8.00	8.00	8.00	58	393	7.7	23.2	0	0.02	0.00	0.01	0.03
	5	8	8.00	8.00	8.00	8.00	8.00	58	380	7.1	21.1	0	0.02	0.00	0.01	0.03
	6	8	8.00	8.00	8.00	8.00	8.00	59	402	7.5	22.5	0	0.02	0.00	0.01	0.03
	7	8	8.00	8.00	8.00	8.00	8.00	53	359	7.1	22.3	0	0.02	0.00	0.01	0.03
	8	8	8.00	8.00	8.00	8.00	8.00	51	347	6.7	21.4	0	0.02	0.00	0.01	0.03
	9	8	8.00	8.00	8.00	8.00	8.00	54	362	6.5	20.1	0	0.02	0.00	0.01	0.03
	10	8	8.00	8.00	8.00	8.00	8.00	59	411	7.3	22.7	0	0.02	0.00	0.01	0.03
51	1	17	17.00	17.00	17.00	17.00	17.00	205	1,994	9.1	25.5	0	0.05	0.02	0.06	0.13
	2	17	17.00	17.00	17.00	17.00	17.00	187	2,000	9.2	25.2	0	0.06	0.03	0.04	0.13
	3	17	17.00	17.00	17.00	17.00	17.00	189	1,822	8.7	24.3	0	0.05	0.02	0.06	0.13
	4	17	17.00	17.00	17.00	17.00	17.00	154	1,687	7.5	23.1	0	0.06	0.02	0.02	0.10
	5	17	17.00	17.00	17.00	17.00	17.00	199	1,984	9.0	25.2	0	0.06	0.02	0.03	0.11
	6	17	17.00	17.00	17.00	17.00	17.00	204	2,218	9.8	25.5	0	0.07	0.03	0.05	0.15
	7	17	17.00	17.00	17.00	17.00	17.00	193	1,883	8.8	24.7	0	0.05	0.02	0.06	0.13
	8	17	17.00	17.00	17.00	17.00	17.00	167	1,776	8.0	23.6	0	0.05	0.02	0.05	0.12
	9	17	17.00	17.00	17.00	17.00	17.00	225	2,101	9.7	25.0	0	0.06	0.03	0.03	0.12
	10	17	17.00	17.00	17.00	17.00	17.00	211	2,256	10.1	25.7	0	0.06	0.03	0.06	0.15
99	1	33	33.00	33.00	33.00	33.00	33.00	915	11,322	24.1	40.8	0	0.24	0.26	0.37	0.87
	2	33	33.00	33.00	33.00	33.00	33.00	884	13,478	25.3	46.2	0	0.25	0.39	0.42	1.06
	3	33	33.00	33.00	33.00	33.00	33.00	1,020	12,330	27.3	45.8	0	0.24	0.29	0.38	0.91
	4	33	33.00	33.00	33.00	33.00	33.00	820	10,905	22.8	38.5	0	0.26	0.23	0.34	0.83
	5	33	33.00	33.00	33.00	33.00	33.00	874	11,019	23.4	40.7	0	0.24	0.26	0.36	0.86
	6	33	33.00	33.00	33.00	33.00	33.00	892	13,782	25.3	46.2	0	0.28	0.40	0.53	1.21
	7	33	33.00	33.00	33.00	33.00	33.00	965	11,731	26.3	45.1	0	0.24	0.29	0.36	0.89

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★ - previously open instance

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n	inst.	z^*	lb^{lp}	lb^{lp1}	lb^{lp2}	lb^{sp1}	lb^{sp2}	$\#v$	$\#a$	$\%v$	$\%a$	n^{bb}	t^{pp}	t^{lp}	t^{ip}	t^{tot}
	8	33	33.00	33.00	33.00	33.00	33.00	804	10,416	22.4	37.6	0	0.24	0.24	0.35	0.83
	9	33	33.00	33.00	33.00	33.00	33.00	878	10,720	23.6	40.4	0	0.23	0.26	0.36	0.85
	10	33	33.00	33.00	33.00	33.00	33.00	906	13,859	25.2	45.7	0	0.27	0.38	0.55	1.20
201	1	67	67.00	66.81	67.00	66.81	67.00	2,743	66,382	50.3	73.7	0	1.05	9.85	23.24	34.14
	2	67	67.00	66.96	67.00	66.96	67.00	2,067	56,787	41.7	64.3	115	1.00	5.36	73.74	80.10
	3	67	67.00	66.86	67.00	66.86	67.00	2,588	61,825	49.0	72.7	0	1.00	7.34	38.70	47.04
	4	67	67.00	66.68	67.00	66.68	67.00	2,097	53,435	41.8	58.9	0	1.00	4.34	30.86	36.20
	5	67	67.00	66.95	67.00	66.95	67.00	2,684	65,817	49.8	74.1	8	1.08	10.57	116.78	128.43
	6	67	67.00	66.81	67.00	66.81	67.00	2,132	59,772	42.7	65.2	8	1.09	5.44	133.40	139.93
	7	67	67.00	66.80	67.00	66.80	67.00	2,740	65,532	50.4	73.3	0	1.04	9.02	27.27	37.33
	8	67	67.00	66.84	67.00	66.84	67.00	2,138	54,249	42.5	59.1	0	1.01	5.33	27.15	33.49
	9	67	67.00	66.93	67.00	66.93	67.00	2,779	67,027	50.9	73.9	94	1.07	10.39	83.96	95.42
	10	67	67.00	66.76	67.00	66.76	67.00	2,208	61,306	43.2	64.2	8	1.07	5.26	68.92	75.25